Ibn Sīnā on patterns of proofs

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This section 9.3 of Ibn Sīnā's *Qiyās* is a commentary on sections 25 and 26 of Aristotle's *Prior Analytics i*, pages 41b36–43a19. NB the parallel passage in Ibn Sīnā's *Burhān* iii.2, p. 136ff (Badawi).

The paper below is what Wikipedia would call a stub. I will add and correct as time allows. I make the paper available now because it contains a complete translation of *Qiyās* section 9.3, which is a prerequisite for Ibn Sīnā's section 9.6 on proof search, whose translation is already on this website. The relevant material is about the relationships between two kinds of compound syllogism, which Ibn Sīnā refers to as 'connected' and 'separated' compound syllogisms.

The section contains historically interesting material on inductively defined classes and methods for proving their properties. The paper will discuss this material when I can get a better hold on what ideas in this area were already available in Ibn Sīnā's time. I would be grateful for any leads on this. (An obvious place to look is the linguistic tradition starting with the *Kitāb al-ᶜayn*.)

The section also contains an unusually large amount of low-grade material; some of it looks like preliminary notes by students. Sifting out this material will be a major editorial chore. For this reason among others, the translation below is highly provisional. But I am hugely grateful to Amirouche Moktefi who went through the translation with me and made many improvements; we did this sitting in a cafe some forty miles from Aristotle's birthplace.

1 The number of ways of proving a proposition

Conclusion	First figure	Second figure	Third figure
Univ. affirmative	1	0	0
Univ. negative	1	2	0
Exist. affirmative	1	0	3
Exist. negative	1	2	3

Conclusion	First figure	Second figure	Third figure
A(A, C)	1. A(A, B), A(B, C)		
E(A, C)	2. A(A, B), E(B, C)	3. A(A, B), E(C, B) (2cp) 4. E(A, B), A(C, B)	
I(A, C)	5. I(A, B), A(B, C)		6. A(B, A), A(B, C) (8w) 7. A(B, A), I(B, C) 8. I(B, A), A(B, C) (5cp)
O(A, C)	9. I(A, B), E(B, C)	10. I(A, B), E(C, B) (9cp) 12. O(A, B), A(C, B)	11. A(B, A), E(B, C) (14w) 13. A(B, A), O(B, C) 14. I(B, A), E(B, C) (9cp)

2 Translation of *Qiyās 9.3*

IX.3 On syllogisms composed of more than two premises, and a proof that there are many such syllogisms and they are compound

[9.3.1] It has been made clear to you that there is no recombinant syllo- 433.5
gism with a single premise, nor is there one with more than two premises. It remains for you to raise a doubt and say: We have sometimes seen syllogistic discourse in which a proof is devised which has a single goal but more than two premises in it. There are demonstrations of this kind in the Book of Elements in geometry, and elsewhere.

[9.3.2] So we say: Syllogisms have many — i.e. more than two — premises 433.9
in any one of three cases. (1) Either these premises are not premises of the 433.10
proximate syllogism, but rather they are premises from which the premises of a more proximate syllogism follow. Or (2) they are introduced by way of induction or illustration. Thus they are not premises of the syllogism itself, but premises of an induction to explain the legitimacy of a premise. Or (3) they are not strictly necessary, though their usefulness is not far from necessity. This can take several forms. One is that [the premise] is intro-
duced as a stratagem; another is that it is introduced for decoration, and 433.15

another is that it is introduced to clarify the proof. It is introduced as a
stratagem when the intention is to draw a veil over the entailment, in a
case where if the necessary premises were introduced neat, then one would 434.1
guess what conclusion [the argument] was headed for, and one would see
how it was going to get there. Something makes the conclusion difficult
to accept; so one hides the drift so that it seems that [the argument] is go-
ing nowhere, particularly when it does contain a useless element — and
[thus] you bypass what made [the conclusion] difficult to accept. This [is
useful] in debate and in examination, and something like this can occur in
feigned ignorance and in dressing-up and in using details to distract atten-
tion. When the purpose is decoration, premises are devised which improve 434.5
the discourse, to make it more attractive, or to extricate oneself — these are
premises whose presence or absence one desires for reasons of social stand-
ing. When the purpose is explanation, there are for example similes that are
not part of the argument but are introduced just to fix ideas. There are also
quotations that are not part of the argument, and division of the expression,
and translation of one expression into another, and other things discussed
in the book *Jadal*.

[9.3.3] The proximate syllogism can't have more than two premises. 434.9
[[But rather its minor term must be, either potentially or actually, included 434.10
entirely in the content of the major.]] So if there are more premises, and 434.11
not because of induction or any other of the cases above, it is because the
syllogism is compound. The meaning of 'the syllogism is compound' is
that the [proximate] syllogism is composed of two premises, one or both
of which needs a syllogism to prove it. So two syllogisms are packed to-
gether, one of them yielding the [proximate] premise and the other the goal.
The goal necessarily has an even number of premises [to prove it], and the 435.1
premises entailing one of these two premises are an even number. There are
an even number of premises to entail the two conclusions, since it is twice
the number that entailed a single [conclusion], and even plus even is even.
Therefore both simple and compound syllogisms have an even number of
premises. So if the number of them is odd then either there is a shortfall or
there is an excess. Or else the syllogism is invalid — if it can't be completed
by adding a premise, and an equivalent syllogism can't be made by leaving
out [a premise].

[9.3.4] There are two kinds of syllogism with a shortfall of premises. 435.4
In one kind the major premise has dropped out because its general accep- 435.5
tance made it unnecessary to state it explicitly; or else [the missing premise]

gives an impression that it didn't need to be stated explicitly, though if it
had been made explicit its falsehood would have been clear — as happens
in sophistry and rhetoric. Or the minor premise has dropped out for one of
these same reasons. In the other kind the premise drops out for the reason
that it is not needed — not because it is or appears to be obvious in itself
or [was introduced] for a strategem, but because it is entailed by an array
consisting of two premises that make it so clear that there is no need to
state it after them as a premise. So the conclusion drawn from those two 435.10
premises drops out, and those two premises together with the other [prox-
imate] premise form three premises from which the goal follows. When
both of the [proximate] premises are the conclusions of syllogisms, then
one wouldn't expect to find both these premises dropping out as conclu-
sions that don't need to be stated explicitly. If one of them drops out, then
[it would be] the one whose syllogistic proof comes later. It's as if the [prox-
imate premise] whose syllogistic proof comes earlier is finished when work
begins on the one whose syllogistic proof comes later. So the [proximate
premise] that is more appropriate not to be mentioned is the one which is
the conclusion of the [preliminary] syllogism that is closer in time [to the
conclusion].

[9.3.5] When there is an extra premise, this will be one of the cases which 435.14
were described to you earlier. If it was because the argument is not valid, 435.15
then the odd number of premises can't be restored to an even number in
any way, either by taking away or by adding.

[9.3.6] Every compound syllogism is either connected or separated. A 436.1
connected [compound syllogism] is one in which the conclusions that come
before the goal and are premises for the goal are explicitly mentioned. It
[can be] a compound [syllogism] either because one of the two premises
[for the goal] needs a syllogism [to prove it], and [the two syllogisms]
make a single compound [syllogism]; or equally well because each of the
premises [for the goal] needs [a syllogism to prove it], so that a compound 436.5
is formed by adding something.

[9.3.7] I have already talked about conclusions as conclusions, and then 436.5
I have talked about [conclusions] as premises. The way it goes is that one
begins from the premises that are furthest from the goal. [The premises] are
associated in pairs so that they entail a conclusion which is also a premise.
Thus if another premise needed to be proved, then [we would attach two
premises to prove it]. If no [other premise] needed [to be proved], then

we would take the premise [which was proved] and the other [proximate] premise and deduce [the goal] from them; so there would be four premises and two conclusions.

[9.3.8] In the case where the other [proximate premise] has to be derived 436.9
[as well], a syllogism with two premises is introduced in order to derive it. 436.10
Then at one level there are four premises and two conclusions, and at the second level there are two premises and a single conclusion. So the compound [syllogism] contains six premises altogether and three conclusions altogether. The number of conclusions is half the number of premises. Each of the [simple] syllogisms contains three terms and a conclusion. Suppose in fact that each [proximate] premise [is proved by] a syllogism, and the
two [proximate] premises share a term. Then there are six terms, except 436.15
that the one of them is shared in the middle, so there are five terms. The shared term and the term at one end of the five give rise to one proximate premise, and the shared term and the other end term give rise to the other [proximate] premise. The two end terms of the five give rise to the goal which is the target of the compound syllogism.

[9.3.9] If just one premise [of the proximate syllogism] is deduced from 437.1
a syllogism, then in that case [the compound syllogism] consist of just two [simple] syllogisms. Thus there are four premises: two premise for the premise [of the proximate syllogism] and two for [its] conclusion. One of the two [premises of the proximate syllogism] is the conclusion of the first syllogism; the other is not its conclusion. The goal is entailed by these two [premises]. So given that more than one [of the propositions] count as conclusions, the number of premises is four and the number of conclusions is two, since the number of premises is twice the number of conclusions.
Turning to the number of terms, in this case it is the same as the number of 437.5
premises. An example:

Every C is a B. Every B is a D.

Every C is a D. Every D is an H.

(1) Every C is an H.

Thus the terms are C, B, D and H.

[9.3.10] The starting point for this is that when the syllogism is a sin- 437.8

gle [simple syllogism], the premises are formed from three terms. Next,
if the syllogism is two [simple syllogisms] and the second is at the same
level as the first — i.e. no part of the second syllogism is a conclusion from
the first syllogism, but rather [the two simple syllogisms] entail two com- 437.10
pletely different conclusions — [then] there are four premises, and there
are not four but six terms. But if the two syllogisms entail the two premises
of another syllogism, and thus share a term, then [the number of terms will
be] five. Next, if there are three [simple] syllogisms on a single level and
their conclusions are completely different, then there are six premises and
nine terms. But if each pair of adjacent conclusions has a term in common,
then there are seven terms. Thus in each case the number of terms in ad-
jacent simple syllogisms is the number of premises plus one; there are an 437.15
even number of premises and an odd number of terms. Twice the number
of conclusions is the number of premises. [This number of conclusions]
can be either even or odd, because half an even number may be even and
it may be odd.

[9.3.11] Next we consider the case where two syllogisms are connected 438.1
in a different way, namely where one of the two syllogisms is at an earlier
level than the second syllogism, so that the first yields one of the premises
of the second. Then the first syllogism as a whole has three terms. The
second syllogism introduces another premise and another term. When the
two syllogisms are set out [separately] they have six terms. But two of these
six, which are terms of the first syllogism, [should be subtracted] leaving 438.5
four terms for the two syllogisms [together]. Thus the number of terms is
equal to the number of premises, and the [number of] conclusions is half
as many. Then if a third syllogism is introduced, which yields a premise
associated with the conclusion of the second [syllogism], this adds a term.
So the premises, including the conclusions at the first level, make six; there
are three conclusions and five terms. So when there were four premises
there were four terms; but now when another term is added, there are six
premises, and one conclusion [and one premise] in addition to what was 438.10
there before. Then if we add a term, this adds a syllogism, so that there
are eight premises, four conclusions and six terms. So the first [compound]
syllogism has one more term than premise. The second syllogism has equal
numbers of premises and terms, as if the premises catch up with the terms.
In all the subsequent [compound syllogisms] there are more premises than
terms, since with every [added] term two premises are added. In fact there
are three terms at the outset. Then one term is added making four terms, 438.15
and two premises are added to the two premises, making four. Then when

[another] term is added, the result is that there are six premises and five
terms. And so on and so on. There are always an even number of premises.
At the outset there are an odd number of terms, viz. three, in the second 439.1
compound [syllogism] the number of terms is even, and in the third it is
odd. This is how it goes.

[9.3.12] And if the compound is mixed, it doesn't preserve the first or- 439.2
dering or the second ordering. As for the first ordering, [[because]] even
if there continue to be an even number of premises, the terms won't stay
an odd number and they won't be in good order. As for the second order-
ing, there are always an even number of premises but the increase in the 439.5
number of terms doesn't stay in line with the increase in the number [of
premises] as more and more are added.

[9.3.13] The only case where all the compound syllogisms of this type 439.6
consist of syllogisms from just one figure — and we include the further construction that we will mention below — is where the goal is universally quantified and affirmative. In fact the [proximate] syllogisms to [a goal of this form], and the syllogisms to the [proximate] premises, will be in the first mood of the first figure. I am referring here to predicative [syllogisms].

[9.3.14] If the goal is negative and universally quantified, one of its two 439.9
[proximate] premises is universally quantified affirmative, and a syllogism 439.10
proving this will be in the first figure. A syllogism proving its second
premise can be in the first figure or the second, [[with exactly the same
terms]]. Suppose for example that the goal is 'No C is an A', proved by
the simplest compound syllogism, namely where [each of the proximate]
premises is derived by a syllogism. There are several cases.

[9.3.15] (1) The first case is that the minor premise is affirmative and 439.13
the major premise is negative. I am referring to the proximate syllogism,
which is in the first figure. You will find that the minor premise can be 439.15
proved only in the first figure. But the major can be proved in either of two
figures — in fact it can be proved in two ways in the second figure. [In the
first of these three ways] the major premise is proved in the first figure: 440.1

Every C is a B. Every B is a D. Every D is an H. No H is an A.

Every C is a D. No D is an A.

(2) No C is an A.

In the second way the major [premise is proved with] a second figure [syllogism] whose minor premise is affirmative:

Every C is a B. Every B is a D. Every D is an H. No A is an H.

Every C is a D. No D is an A.

(3) No C is an A.

In the third way the major [premise is proved with] a second figure [syllogism] whose minor premise is negative: 440.5

Every C is a B. Every B is a D. No D is an H. Every A is an H.

Every C is a D. No D is an A.

(4) No C is an A.

[9.3.16] (2) The next case is that the minor premise of the proximate 440.7 syllogism is affirmative and its major premise is negative, where the [proximate] syllogism is in the second figure. Then the minor can be proved only in the first figure also, while the major can be proved in either [of the first two] figures. In the first way the syllogism [proving] the major [premise] is in first figure: 440.10

Every C is a B. Every B is a D. Every A is an H. No H is a D.

Every C is a D. No A is a D.

(5) No C is an A.

In the second way the syllogism [proving] the major [premise] is in the second figure and has an affirmative minor premise:

Every C is a B. Every B is a D. Every A is an H. No D is an H.

Every C is a D. No A is a D.

(6) No C is an A.

In the third way the syllogism [proving] the major [premise is in] second 441.1
figure and it has a negative minor premise:

Every C is a B. Every B is a D. No A is an H. Every D is an H.

Every C is a D. No A is a D.

(7) No C is an A.

[9.3.17] (3) The last case is where the minor premise is negative. [[In this 441.4
case [the syllogism] can be completed only in the second figure.]] There are
three ways to do this case, and they are the converses of the aforementioned 441.5
three ways. You can inform yourself of this.

[9.3.18] If the goal is existentially quantified affirmative, then its prox- 441.6
imate syllogism has two affirmative premises [[and just one of the two is universally quantified]]. If its form was in the first figure, then the syllogism proving the universally quantified affirmative major premise [can] only be in the first figure, and the syllogism proving the minor premise is either in the first figure [[and the minor premise has to be existentially

quantified affirmative]] or else it is in the third (figure). in which case it is 441.10
proved either from two universally quantified [premises] [[and the two re-
mote syllogisms can be in the first figure but not any other]], or else from an
existentially quantified [premise] and a universally quantified [premise],
where the existentially quantified [premise] can be either the minor premise
or the major. If the proximate syllogism is in the third figure and the mi-
nor premise is affirmative existentially quantified, then the syllogism prov-
ing the major premise is in the first figure, and the one proving the minor
premise is either in the first [figure] (as you know), or else in the third [[in
one of two ways]]. And if the minor [proximate] premise is universally
quantified, the syllogism proving its minor premise is in the first figure,
and the one proving its major premise can be either in the first figure or in 441.15
one of at least three moods of the third.

[9.3.19] If the goal is existentially quantified negative [as in "Some C 441.16
is not an A"], then the proximate syllogism proving it can be in either the
first figure or the second or the third. If the proximate syllogism proving
it is in the first figure, then the syllogism proving the major premise of
this syllogism can only be in the first figure; and the one proving its minor
premise can be in one mood of the first figure or in one of three moods of 442.1
the third figure. If the syllogism proving it is in second figure and its minor
premise is affirmative and its major premise is universally quantified, then
the [[proximate]] syllogism proving its major premise can be in the first
figure or one of two moods of the second figure, and the one proving its
minor premise can be in the first [figure] or in one of three moods of the
third figure. [[And the pairings in it are compounded, so it is eight.]] And 442.5
if [the syllogism proving it has] negative minor premise, its major [premise]
can be proved in one mood of the first figure, and its minor can be proved
either in one mood of the first figure or in either of two moods of the second
figure or in any of three moods of the third. [[So there are twenty-four
constructions]].

[9.3.20] Next we consider the compound [syllogism] which separates 442.8
the consequences from the premises, in the sense that the premises are ex-
plicit and the conclusions except for the final conclusion are completely
omitted. An example is

(8) Every C is a D. Every D is an H. Every H is a Z. Every Z is an I. Therefore every C is an I.

[There is a] first [separated] syllogism, and its [conclusions] have to be ex- 442.10

plicit; this syllogism has two premises. [[The second [such] syllogism — in the example we gave, the major premise [for the final inference] is explicit.]] Then whenever we add a term, it adds a premise. So when we add a fourth term, it adds a third premise, and when we add a fifth term, we get a fourth premise. So the number of premises is one less than the number of terms. Thus if there was an even number of premises, there was an odd number of terms; and if there was an odd number of premises, there was 442.15
an even number of terms. And so on [as more terms are added].

[9.3.21] The addition of a term adds a possible conclusion potentially, I 442.16
mean a conclusion that is helpful for the goal. So whenever a term is added, this adds a conclusion, so the number of additional conclusions which are helpful for the goal is the same as the number of [added] terms. In some 443.1
instances this number is even, [in some it is] odd. When we say 'consequence that is helpful for the goal', this means potentially. For example the compound (8) entails conclusions which are not helpful for the goal. The conclusions which are helpful for the goal in this example of ours are for example 'Every C is an H', and 'Every C is a D'. A [conclusion] that is not helpful for the goal is for example when we say (drawing syllogistic 443.5
conclusions from these premises): Every D is an H and every H is a Z, so it follows that every D is a Z. This conclusion is not helpful for the goal in our chosen arrangement [of the connected syllogism]. If we had the option of choosing a different ordering and a different arrangement, we would make it that the premise 'C is H' is clear and the premise '[Every] C is a Z' is not clear, so then we prove it. Then we add to it the premise '[Every] Z [is an] I' on the basis that [this premise] is clear. But then we would have altered the arrangement which we chose in this example. But [separated] syllogisms don't yield a [new conclusion] whenever a term is added. 443.10

[9.3.22] [Converting this example] to the other kind [of compound syl- 443.10
logism], the first [added] conclusion is 'Every C is an H'. Then we add 'Every H is a Z', and this entails 'Every C is a Z'. Then we add 'Every Z is an I', and this entails 'Every C is an I'. As for 'Every D is Z' and similar sentences, these play no role at all in this ordering of the syllogism.

[9.3.23] Know that the new term can be added just before the lesser 443.14
term, or just after the greater term, or between the two. 443.15

[9.3.24] For a universally quantified affirmative [proposition], the only 443.15
compound syllogisms that prove it are [entirely] in the first figure. [[The construction which occurs in this case is of the kind which you already

know from the example which we gave.]] In the case of the universally quantified negative, we described what kind of connected compound syllogism proves it. [A separated syllogism] equivalent to the first kind is: 444.1

(9) 'Every C is a B', 'Every B is a D', 'Every D is an H', 'No H is an A'. So 'No C is an A'.

One equivalent to the second kind is:

(10) 'Every C is a B', 'Every B is a D', 'Every D is an H', 'No A is an H'. So 'No C is an A'.

And one equivalent to the third kind is:

(11) 'Every C is a B', 'Every B is a D', 'No D is an H', 'Every A is an H'. So 'No C is an A'.

These are typical of the kinds [of syllogism] in which the [intermediate] conclusions are not expressed at all; this is satisfactory, and [these conclusions] are merely potential, so that we mention explicitly only the final [conclusion]. 444.6

[9.3.25] These things will make it clear to you that it is very difficult to find a syllogism whose conclusion is a [given] universally quantified affirmative proposition, regardless of whether the syllogism is atomic or compound, since [such a syllogism] can exist only in a single mood of a single figure. It is very easy to find a syllogism proving the opposite [kind of proposition], because it can be proved in any of six different moods. By 'opposite' here I mean the existentially quantified negative proposition, which can be proved through very many moods of compound syllogism; we counted them for you. In terms of difficulty the universally quantified negative proposition is like the universally quantified affirmative proposition. This can be confirmed along the lines of the discussion above. In terms of difficulty the universally quantified negative proposition comes close to comparison with the existentially quantified affirmative proposition. This also can be confirmed along the same lines. 444.7 444.10

[9.3.26] Know that in the separated compound [syllogism], when it concludes with negative premises after the affirmative ones, then the ordering splits here unless there is a sound link. When [the compound syllogism] begins with negative premises and then a number — any number — of affirmatives come into play, then the syllogism conforms to the separated construction throughout. [A compound syllogism] can be constructed out 444.15 445.1

of both duplicative and recombinant syllogisms. The entailment can include either recombinant syllogisms (both meet-like and difference-like) or duplicative syllogisms.

3 Notes

[9.3]

Title 'many': This is understated. In fact Ibn Sīnā presents in a crude form a process that generates an infinite class of syllogisms (unless we run out of terms).

[9.3.1]

433.8 'Book of Elements': Euclid's Elements. Note the clear statement that Euclid's book is 'syllogistic discourse'. Today it's a commonplace that syllogistic logic is inadequate for formalising any significant amount of mathematics. In *J. Philosophical Logic* (forthcoming) I argue that Ibn Sīnā has in mind a different notion of formalising, which was universal in logic before the mid 19th century; I call it 'local formalising'. I also argue there that it is not absurd to believe that Euclid's Elements can be locally formalised in syllogisms.

[9.3.2]

433.10 'proximate syllogism': This is the bottom simple syllogism, which proves the main conclusion (the 'goal'). The premises of the proximate syllogism are the 'proximate premises' which Ibn Sīnā often refers to.

434.3 'dressing-up' (*talbīs*): al-Jurjānī *Ta^{c}rīfāt* defines this as 'veiling of the explicit truth of the matter by means of the opposite of what the argument is aiming at'. Probably 'feigned ignorance' and 'using details to distract attention' are names of standard moves in debate and rhetoric.

[9.3.3]

434.9f 'But rather ... both': Aristotle sometimes seems to assume that every proposition is of the form 'Every A is a B', for example at ***. This clause seems to repeat that assumption. It has nothing to do with the present topic, and it interrupts the argument. Delete.

435.3 'both simple and compound': This needs a proof by induction. The proof offered in the preceding few lines applies only to a compound of two simple syllogisms. But the same argument applies generally. In fact Ibn Sīnā makes sure that it does, by speaking of 'even' rather than using the exact numbers 2 and 4 that apply with the compound of two simple syllogisms.

[9.3.4]

435.11 'from which the goal follows': It seems Ibn Sīnā assumed that the missing premise was one of the proximate premises.

[9.3.5]

435.15 'If it was ...': This seems a remarkably stupid sentence. I can't suggest a repair.

[9.3.6]

Ibn Sīnā defines the inductive class by describing the base case and the operations for generating new elements. He does so by describing just two cases, namely the base case and the result of applying the operations at most once each. This gives the impression that the inductive class consists just of these cases; later examples show that Ibn Sīnā is well aware that this is not so. Strictly his account is faulty, and the same fault is endemic in descriptions of inductive classes before modern times. From the parallel case of recursion we should be cautious about assuming the faulty exposition implies a faulty understanding.

[9.3.7]

436.8 'we would attach ... prove it': This is a guess. There are two consecutive sentences of the form 'If X then Y'. In the first the Y has gone missing, and 'then' (*fa-*) has been completed to a word which makes little sense here. The overall sense is that the simplest non-simple compound syllogism is got from a simple syllogism (the proximate syllogism) by adding two premises to prove one of the premises of the proximate syllogism, and all others are got by adding pairs of premises to prove unproved premises.

[9.3.8]

436.10 'level': For Ibn Sīnā the levels are inhabited by simple syllogisms. Today most logicians would automatically count the levels of sentences, not the levels of syllogisms. I think a real difference of perception lies behind this. For Ibn Sīnā a compound syllogism is not an array of sentences that are related by rules of derivation. Instead it is a collection of atomic inferences, some of which feed their conclusions into the premises of others.

[9.3.9]

437.2 Read ^{c}*alā l-muqaddamati* for ^{c}*alā l-muqaddami*, as required by the sense.

[9.3.11]

The paragraph claims to introduce a new kind of compound syllogism, where the simple syllogisms are not all at the same level. But this was already the situation in paragraph [9.3.9].

[9.3.12]

The paragraph seems to rest on the distinction made in paragraphs [9.3.8]–[9.3.11] between syllogisms which are all on one level and syllogisms with two or more levels. It makes some remarks about numbers of premises and terms in the two cases. But the point of the paragraph is obscure to me.

439.3 I read *fa-'anna* in place of *fa-li-'anna* ('because') which seems ungrammatical.

[9.3.13]

The lumpy syntax in this paragraph suggests that several marginal comments have been incorporated into the text. There are two obvious candidates

439.6 'this type': Connected compound syllogisms. The 'further construction' is separated compound syllogisms.

[9.3.14]

439.10 'second premise': This is careless exposition. Ibn Sīnā means the other premise. But readers may take him to mean the major premise, which he normally puts second. As it happens, the premise which is not universally quantified affirmative is major premise in two of the syllogisms proving a conclusion of this form, but not in the third. In the first and second case it has the same terms, but not in the third. Maybe the false information in the next line (which should be deleted) was a marginal note by a reader who was misled by Ibn Sīnā's choice of words.

439.13 Read *ᶜalā muqaddamatayhi* with one manuscript.

[9.3.15]

439.13 For *'ammā 'in* read *'immā 'an*.

439.16 *aḥaduhumā* is certainly wrong, because this is the first of three cases. The critical apparatus reports that four manuscripts have a different text, but I can't make out what it is. Could it be *al-wajh al-awwal yujarr* 'the first way proceeds'? This would work.

440.4 The conclusion appears in some manuscripts as 'No A is a C', while in the remainder it is missing altogether. It certainly has to be 'No C is an A' from Ibn Sīnā's description of the case under consideration. I would amend to *fa-lā šay'a min j a*, assuming that one scribe transposed the letters, and then another scribe saw this was wrong and left the clause out.

[9.3.17]

441.4 For *'ammā 'in* read *'immā 'an*. Also the second sentence is a gratuitous falsehood; delete.

441.5 'the aforementioned three ways': I.e. those in paragraph [9.3.16]. In that paragraph the proximate premises were 'Every C is a D' and 'No A is a D'. Here they are 'No C is a D' and 'Every A is a D'; so there are three possible proofs for the first of these premises, and they correspond to the three possible proofs of the second proximate premise in [9.3.16].

[9.3.18]

441.7 'and ... quantified': This is false, as Ibn Sīnā certainly knew well, cf. 441.10. Delete.

441.9 'and the minor ... affirmative': It's correct that if the proximate syllogism is in first figure then its minor premise is existentially quantified affirmative. But placed here, the comment implies that this holds only when the minor proximate premise is proved in first figure, which is absurd. Delete.

441.10 'and the two ... figure': The comment is correct but a pointless distraction here. Delete.

441.14 'in one of two ways': Again Ibn Sīnā knew well that it is three, not two. Delete.

441.15 'at least three': When referring to a known number, classical Arabic uses the plural only when the number is at least three. (For two it uses the dual.) But the 'at least' expressed by the plural doesn't carry any implication that the number could be more than three.

[9.3.19]

This whole section on counting numbers of proofs could be due to Ibn Sīnā without saddling him with this particular paragraph. Perhaps he wrote 'Do this case yourself', and a student's rough notes on a first attempt to do this somehow found their way into the text. They are clearly very rough notes, and the student gave up before reaching the case where the proximate syllogism is in third figure.

441.18 'can only be in the first figure': this is nonsense. The major premise in this case will be universally quantified negative, and such a proposition can be the conclusion of three forms of simple syllogism, as Ibn Sīnā certainly knew.

442.4 'the pairings in it are compounded': the meaning is unclear, but he may be reminding us that the total number of derivations is the number of ordered pairs consisting of a proof of the first proximate premise and a proof of the second proximate premise. However, this calculation gives $3 \times 4 = 12$ derivations, not the 8 stated.

442.7 I can't get 24 out of the data given. For an existentially quantified negative conclusion, the author has considered those proofs where the proximate syllogism is in first or second figure. The number in the first figure case is 12 (though the text asserts 3), and the number in the second figure case is 12 + 6 = 18. This gives a total of 30 (or 21 with the wrong figure for first figure). If one includes the proofs with proximate syllogism in third figure, this adds 21, giving a total of 51 proofs.

[9.3.20]

This paragraph comments on Aristotle *Prior Analytics* i25, 42b1.

442.9 For 'is an H' read 'is a D'. The H could be a faulty inference from line 443.4 below, where Ibn Sīnā is saying not that 'Every C is a H' is the first premise, but that it is the first *conclusion* as one fills in the connected syllogism starting from the left. The phrase *awwal al-qiyās* seems odd here; if the translation is correct, one expects *awwal qiyāsin*.

442.10f 'the example we gave': The comment fits the example at 437.6f.

442.11 This clause seems to be a reference to the example of a *connected* compound syllogism at 437.6f. That syllogism has three topmost premises and four terms (so it is next in line after the three-term syllogisms that Ibn Sīnā has just but first). There is one intermediate conclusion, which contains the first and third terms. This conclusion combines with the third premise to yield the main conclusion. So the major premise for the proximate syllogism is the third of the topmost premises, which is explicit in the corresponding separated syllogism, as the note says. If Ibn Sīnā wanted to make this point at all, the appropriate place would be in line 442.13 below where he turns to the four-term syllogism. Probably the note is a reader's marginal jotting.

442.14f The verb is in the perfect tense. This is normal for timeless statements; but if it's a timeless statement about separated syllogisms of all lengths, why does Ibn Sīnā add 'and so on' after it? Assuming Ibn Sīnā is maintaining his normal standards of precision, the perfect tense should probably be read as a statement about the cases already considered; and then 'and so on' means that the pattern continues as we add more terms.

This arrangement is interesting because the past-tense statement is a formulation of an induction hypothesis. Since odd cases alternate with even, the induction hypothesis can't be made plausible in the usual way, by taking a single typical case. Ibn Sīnā's arrangement is closer to the general pattern of a proof by induction on the natural numbers than any other example I've seen in any author before the 19th century. But of course Ibn Sīnā has not stated the general principle of induction here. That came with De Morgan.

[9.3.21]

Given a separated compound syllogism, there are in general many ways of completing it to a connected syllogism. In this paragraph Ibn Sīnā presents a default choice: namely we first draw a conclusion from the first two premises, then we draw a conclusion from this conclusion and the third premise, and so on from left to right through the premises. In his proof search procedure of section 9.3.6 he seems to assume that the student has learned this default choice.

There are some puzzles about his presentation. First, why does he introduce the default choice indirectly, as an answer to the question 'which potential conclusions are helpful for reaching the goal'? (In other words, why didn't he put paragraph [93.23] before [93.22]?) Second, there are some separated syllogisms where the default choice won't work, because it would involve fourth figure syllogisms, which Ibn Sīnā rejects. Why doesn't he mention this? His one example for this section, the separated syllogism (8), doesn't illustrate this possibility.

442.16 'The addition of a term': In (8) for example, if a new term J was added between D and H, the result would be to replace 'Every D is an H' by two premises 'Every D is a J' and 'Every J is an H' which entail it. In the default connected syllogism the effect would be that the conclusion 'Every C is an H' would be derived by first proving the new conclusion 'Every C is a J' and then combining this with 'Every J is an H'; so one more term gives one more conclusion.

443.7 '[Every] C is a Z': Ibn Sīnā simply writes the letters 'C Z'. He often uses this abbreviation for syllogistic sentences, leaving it to the reader to supply a quantifier and possibly a negation.

443.8 I read 'C is a Z' where the Cairo edition has 'H is Z'. Ibn Sīnā says that it is proved after 'Every C is an H' has been proved, and the obvious candidate for this position is 'Every C is a Z'

, which is proved from 'Every C is an H' and the (supposedly clear) premise 'Every H is a Z'. One can easily see how a scribe could have altered this to the premise found in (8).

[9.3.22]

443.10 'the other kind': In other words, connected syllogisms. The example that he chooses for showing how to convert a separated syllogism to a connected one is (8), as in the previous paragraph.

[9.3.23]

This paragraph has the air of an interpolated marginal comment, but it's harmless so it may as well stay.

443.14 'the lesser term': In a simple syllogism, this means the term that occurs as subject of the conclusion. Presumably for a connected compound syllogism it means the term that occurs as subject of the goal. The writer seems to think it will be the leftmost term in the corresponding separated syllogism; but this is false when the leftmost simple syllogism is in third figure. If the leftmost term is intended, maybe a better translation is 'least'; Arabic makes no distinction between 'the lesser' and 'the least'. These comments apply also to 'the greater term', but with 'predicate' for 'subject' , 'right' for 'left' and 'second figure' for 'third figure'.

[9.3.24]

The paragraph comments on Aristotle *Prior Analytics* i26, 42b27. The three separated syllogisms given correspond respectively to the connected syllogisms described earlier at 440.1f, 440.3f and 440.5f.

443.16f 'The construction . . . we gave': No such example was given, and there seems little point in giving an example anyway. Delete the sentence.

443.17 Grammatically, shouldn't the *yakūnu* be *takūnu*?

444.1 'what kind': Since Ibn Sīnā is about to mention three kinds, a plural would be better here, *al-wujūhi al-mad̠kūrati*.

444.4 'No D is an H': Corrected from the parallel passage at 440.5f. The Cairo text has nonsense at this point, though this may be a printing error since the critical apparatus seems to show a text closer to the correct version. Also the parallel passage confirms that we need 'Every A is an H', where the Cairo text and the critical apparatus have 'Every A is a D'. Finally the Cairo text has an unwanted 'a' before the last word of this line, but again this seems to be a printing error since it is missing in the critical apparatus.

444.6 'this is satisfactory': The reason for this comment is unclear.

[9.3.25]

444.8 Here and two lines lower, *wujūd* clearly means 'finding', not 'existence'.

444.11 'we counted them': This must be a reference to paragraph [9.3.19] above. But that paragraph doesn't remotely do the job; among other things it ignores all the cases where the proximate syllogism is in third figure.

[9.3.26]

This paragraph contains some closing observations on connected and separated compound syllogisms.

The first observation seems to have some interest. The author has noticed a discrepancy in passing between separated and connected syllogisms. The discrepancy takes the form of a break in the order of the separated syllogism, so presumably the problem lies in the passage from a connected syllogism to its associated separated syllogism. Though this passage is never described explicitly, the natural operation is to read the tips of the branches of the connected syllogism, passing from left to right (or in Arabic, from right to left). Presumably the break in the ordering is a place where two adjacent premises in the separated syllogism have no terms in common.

The author says that this can occur when the negative premises come after the affirmative ones. This is already puzzling; in a valid separated syllogism there can never be more than one negative premise. But in a moment we will see a possible explanation of this.

In fact this kind of disruption in the order of the separated syllogism occurs in exactly two cases: either the connected syllogism contains a second figure syllogism whose major premise is a conclusion; or the connected syllogism contains a third figure syllogism whose minor premise is a conclusion. (The two cases overlap.) Now second and third figure syllogisms appear only when the goal is either negative or existentially quantified (or both). The author of the note has evidently missed the existentially quantified case. I suspect the author has noticed the case where a negative goal is proved by a proximate syllogism in second figure with negative major premise. Then there is a discontinuity immediately to the left of all the premises that lie above the proximate major premise. Some of these premises will be affirmative; but the author may well think of them as being part of the 'negative area' of the compound syllogism, so he lumps them together as 'the negatives'.

The corresponding phenomenon at the lefthand side of the compound syllogism would arise when the proximate syllogism is in second figure with a negative minor premise. But in this case there is no reversal of terms (though one might occur higher up in the connected syllogism).

The author of the note was evidently a serious logician, though (if the reconstruction above is correct) he seems to have proceeded by staring at examples without any idea of the general theory behind them. But I doubt that the author was Ibn Sīnā himself. Ibn Sīnā was well aware that the phenomenon of reversal of the order of terms can occur with existentially quantified affirmative goals and at the lefthand side of the syllogism; he gives an example at 465.13 (Problem 33) in section 9.6 of *Al-Qiyās*.