

# Historical remarks on dependence and scope

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‘But what is the logic of dependence?  
... we set out to make a systematic logical study  
of this important concept.’

‘Formulas of dependence logic declare dependencies  
while formulas of first order logic state relations.  
These two roles of formulas are incompatible ...’

Jouko Väänänen, *Dependence Logic*

Creating a logic of dependence is as much a conceptual problem as a technical one.

How did the issues seem to earlier logicians before they had our technical equipment?

We go back to Ibn Sīnā, who in the early 11th century was the first logician to study two-quantifier sentences with universal and existential quantifiers, in the volume *Qiyās* of his major work *Shifā'*.

Ibn Sīnā (Avicenna), Uzbekistan and Persia 980–1037



Typical examples of sentences that he studied:

- ▶ Every horse is awake some of the time.
- ▶ Everything that breathes in breathes out.
- ▶ Everybody who travels from Ray to Baghdad passes through Kermanshah.

First problem: What are the logical forms of these sentences?

(But what did 'logical form' mean in the 1020s?)

Ibn Sīnā's view (stated with remarkable clarity in his *Shifā'*):

Logic studies the meanings of compound phrases. These compound meanings are built up from the meanings of single words, by repeated attachments.

'The form of the compound can vary from one language to another.

For example in the possessive construction the possessor comes second in some languages and first in others.

Likewise there is nothing in nature to make the subject and the predicate appear in one particular order.'

Ibn Sīnā, *ʿIbāra* 31.3–5

So the attachments form a tree, not a string.

'In first order logic the order in which quantifiers are written determines the mutual dependence relations between the variables.'

Jouko Väänänen, *Dependence Logic*

Ibn Sīnā couldn't say this, because the meanings of the quantifiers don't come in any order.

In short he *can't use syntactic scope to determine semantic scope*.



Remark 1: Ibn Sīnā's inability to use the order of quantifiers is the result of linguistic knowledge, not of ignorance.

Remark 2: In spite of what they tell you about De Morgan and Frege inventing quantifiers, Ibn Sīnā had no problem whatever in talking about quantifiers.

'In "Every horse watches", "every horse" quantifies over the class of horses, not the class of horses and the class of times together.'

Ibn Sīnā, *Qiyās* 44.11.

(In fact Ibn Sīnā knows how to pack two universal quantifiers into a single quantifier over ordered pairs; but here the quantifiers are of different types.)

Instead Ibn Sīnā uses *functions* to determine the quantifier dependencies.

For example with ‘Every human breathes’, there is an implicit existential quantifier over times, and it is represented by a function taking humans to (sets of) times.

In most cases the function is indeterminate (in effect, quantified over). But in important cases it is definable within the theory. More to follow on this.

Note that if the time quantifier is a function quantifier, the order of attachment of the quantifier meanings is irrelevant, as with Henkin branching quantifiers.

Second problem: How do we negate the sentence?

‘Suppose we say “Every  $B$  is an  $A$ ” and each individual  $B$  has a different time [for being  $A$ ] which we can’t specify. Then if we say “Not every  $B$  is an  $A$ ”, it’s not possible for us to make this sentence point to the time which is specific to each individual.

How could the sentence signify times that have not been specified?’

Ibn Sīnā, *Qiyās* 39.16ff

His problem is:

1. Negating a sentence attaches negation without altering anything else. So the semantics of not- $\phi$  should be the same as those of  $\phi$  except for the added 'not'.
2. Given 'Not every  $B$  is an  $A$ ' it makes no sense to ask 'At what time is each  $B$  an  $A$ ?', although this question makes perfect sense with 'Every  $B$  is an  $A$ '.

These two observations seem incompatible.

In modern terms, adding 'not' at the beginning swaps positive and negative occurrences of quantifiers.

But 'positive occurrence' is defined in terms of the scope of negations,  
and Ibn Sīnā has no notion of scope.

Nor can the scope of negation be translated into some kind of functional dependence.  
(Or can it??)

Ibn Sīnā separates off the case where the function quantifier is definable in the theory (i.e. in our background knowledge).

For example if it's known that the moon is eclipsed at times  $T$ , then for 'The moon gets eclipsed' we can write

For every time  $t$ , if  $T(t)$  then the moon is eclipsed at  $t$ .

Here there is no existential quantifier, so the problem above disappears.

Remark: For Ibn Sīnā, progress in science largely consists of replacing existential quantifiers by universal ones.

## Conclusion One:

Ideas can be introduced in many different orders,  
but only some orders make sense in practice.

For example you can't handle Skolem functions before  
you understand the scopes of negations.

## Conclusion Two:

Ibn Sīnā's speculations certainly helped to prevent any of his logic from being translated into Latin.  
So his questions in logic never reached the Scholastics.

Lagerlund 2009: 'The Western thirteenth-century interpreters of Aristotle's modal logic ... have the same kind of problems as Avicenna and they try to solve them in much the same way.'

This shows the huge gap between Ibn Sīnā's conservative interpretation of Aristotle's modal logic and his own radical advances in the *Shifā'*.



### Conclusion Three:

The sense we can make of earlier logicians very much depends on what we understand now.

If it hadn't been for logicians like Henkin and Hintikka on anomalies of scope,  
or linguists like Reinhart on breakdown of anaphoric islands,  
we wouldn't have had a clue what Ibn Sīnā was talking about.

A translation of Ibn Sīnā's *Qiyās* 1.5  
(as far as I know, the first into any western language)  
is on my website at

<http://wilfridhodges.co.uk>

together with related material like *Qiyās* 1.3  
and my lecture on it in Hamburg.

Western logic is totally independent of Ibn Sīnā's.  
But Ibn Sīnā's work still influences modern Qur'anic interpretation. For example *Qur'an* 49.6:

If a person of bad character brings you a report,  
you should scrutinize it carefully.

Modern Islamic jurists (e.g. Ayatollah Khomeini) debate what is implied if a report comes from someone not of bad character.

The debate is conducted in terms familiar from Ibn Sīnā's logic: 'identifying the subject', 'necessitating condition', 'the meaning in context'. Note the scope anomaly.

Jouko:

Thanks for your good character  
and the many good reports you have brought us.

We wish you many more of the same.

