

Ibn Sīnā's view of the practice of logic

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In the last half century Ibrahim Madkour revolutionised the study of Arabic logic by making available a modern edition of the text of the Logic section of Ibn Sīnā's *Šifā'*. Ibn Sīnā's account of logic in the *Šifā'* is much fuller than any of his other surviving accounts; it runs to some two thousand pages. It is also — in my view — more radical and more independent of Aristotle than his other accounts of logic, though the Logic section of his *Mašriqiyyūn* comes close. In this talk I want to take up some important aspects of logic that are discussed in several places in the *Šifā'* but hardly at all in Ibn Sīnā's other known logical works.

The basic question is this. Logic is a skill, and Ibn Sīnā means to teach this skill. But how does Ibn Sīnā expect his students to apply this skill in practice? What exactly are they supposed to be able to do after this teaching that they couldn't do before? What is the aim of the exercise? A fundamental principle in history of science is that you can't properly assess the work of a past scientist if you don't know what questions that scientist was aiming to answer. So I claim that the topic of this paper is a prerequisite for any sound assessment of Ibn Sīnā's logic.

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1 Ibn Sīnā's account of analysis

Ibn Sīnā mentions analysis (*taḥlīl*) in many places. Near the beginning of *Qiyās* ([11] 8.9) he says that the science of syllogisms is commonly called the 'science of analysis', and a page later ([11] 9.2) he connects this with the name *Prior Analytics* of Aristotle's work on syllogisms [20]. Near the end of *Jadal* he lists the skills that a debater should take from logic, and high in his list is

analysis of definitions and syllogisms down to the principles and bases through practical application of the book 'Analytics'. ([13] 335.7f) (1)

There are also a dozen or so references to analysis in *Burhan* [12], where he contrasts it with synthesis (*tarkīb*). When analysis and synthesis are taken as a pair, analysis means tracing back to first principles and synthesis means reconstructing from first principles.

But Ibn Sīnā's fullest treatment of analysis consists of four sections of *Qiyās*, [11] sections 9.6 to 9.9. He introduces these sections as follows:

Sometimes a person is addressed with a well-crafted and definitive syllogism, or he finds such a syllogism written in a book. But then [sometimes] the syllogism is not simple but compound; or it appears not as a connected whole but as scattered pieces. And sometimes moreover the pieces are jumbled out of their natural order, or a part of the syllogism is hidden, or something superfluous is added. [Even] when it is simple, sometimes it is jumbled out of its natural order, or missing a piece, or with a piece added. You already know how this happens. If we don't have rules to guide us, on how to seek with due deliberation the syllogism that proves a given goal, ... then the new information that the syllogism provides will escape us. ([11] 460.4–12) (2)

What Ibn Sīnā is referring to here is the use of logic to *validate arguments*. The logician is given — or perhaps produces himself — a *raw argument* in Arabic, and the task is to turn this raw argument into a form where it can be shown to be a valid syllogism or group of syllogisms. Ibn Sīnā describes this process of turning raw arguments into satisfactory logical form as *taḥlīl*. It forms one arm of what Ibn Sīnā frequently refers to as *taḥṣīl*, i.e. bringing concepts, definitions, axioms, syllogisms etc. into a form where they are fit for use in rational argument. (For example at *Qiyās* [11] 463.9–11 he describes a piece of analysis as 'making your syllogism *ḥāṣil*'.

See [6] Section 3 for a more detailed account of the role of *taḥṣīl* within Ibn Sīnā's logic, and Gutas [5] 188–193 on *taḥṣīl* as a component of Ibn Sīnā's philosophy more generally.)

Analysis rests directly on Aristotle's discoveries in logic. Aristotle noticed that many valid arguments conform to certain patterns, which I will refer to as the *sylogistic forms*. These patterns can be used to classify valid arguments. But they also have a more practical application: we can use them to set objective standards for reasoning. A raw argument can be tested against the patterns: does it conform to a sylogistic form? If not, can it be repaired so that it does? To pose and answer these questions is what Ibn Sīnā means by 'analysis'.

Analysis in this sense is not the same thing as analysis in the sense of tracing back to first principles, i.e. the converse of synthesis. One could reasonably include tracing back as a part of analysis in the sense of validating. You give me an argument: I check that your conclusions follow from your premises by some sylogistic form, and I also check that you had good arguments for your premises, and for the premises used to get those premises and so on back. But in *Qiyās*, analysis is mainly about validating rather than tracing back to first principles.

So the essence of analysis is a procedure for taking raw natural language arguments and relating them to valid argument forms. The collection of valid argument forms differs from one logic to another; I will refer to it as the *formal core* of the logic. For Aristotle the formal core consists of the valid predicative and modal sylogistic forms. For the kind of logic now commonly taught to undergraduates in the western tradition, the formal core consists of the valid sequents of first-order logic. We will ask below what Ibn Sīnā's formal core consists of; it certainly contains the predicative sylogistic forms. (Predicative sylogisms are often called 'categorical'; I thank Stephen Read for convincing me that 'predicative' is a better name.)

In the second quarter of the twentieth century, when logicians were becoming aware of the importance of formal systems, Jan Łukasiewicz [16] proposed a methodology for studying the logic of Aristotle. His approach was to study Aristotle's formal core as a formal system, i.e. as a set of expressions generated by some mechanisable inductive process. Aristotle himself gives arguments that reduce his sylogistic forms to first-figure predicative forms, and Łukasiewicz interpreted these arguments as the inductive process for generating the formal core. Later historians accepted the general methodology that Łukasiewicz had proposed, but gave a different account of the inductive generation of the formal core. The account

given by Timothy Smiley [21] is widely accepted as very close to Aristotle’s own formulations. In Arabic logic the chief exponent of Łukasiewicz’s methodology has been Nicholas Rescher.

In my view the successes of Łukasiewicz’s methodology have obscured the narrowness of its aims. Aristotle’s logic was always more than the formal core; in fact his account of the formal core takes up only a third of the *Prior Analytics*. The chief thing that Łukasiewicz left out was the process of analysis.

2 From informal to formal

Suppose then that we have a raw natural language argument N , and we want to analyse it. How do we proceed?

In modern logic we do the following. First we introduce symbols for the notions that appear in N , and we write a chart to show the correlation:

$$\begin{array}{ll}
 a & : \text{Zayd} \\
 f(x) & : \text{the father of } x \\
 Hxy & : x \text{ hit } y \\
 \vdots & \quad \quad \quad \vdots
 \end{array} \tag{3}$$

Then using the chart, we translate the sentences of N , one by one, into sentences of a formal language of logic — usually first-order logic. There will be a set T of formal sentences expressing the premises of N , and a formal sentence ψ expressing its conclusion. Our final step is to generate the sequent

$$T \vdash \psi \tag{4}$$

by giving a formal derivation of ψ from T in some standard logical calculus, for example a natural deduction calculus. This procedure is the main practical skill taught in undergraduate introductions to logic. The broad lines of it go back to George Boole [2], who used an early version of boolean algebra for his formal core.

Clearly this is not what Ibn Sīnā did, since he had no formal language. But before we discuss his procedure, it will be helpful to say a word about Aristotle’s own approach. Aristotle never sets it out in detail, but enough fragments of it survive in his writings for us to make a good guess at it.

First, Aristotle analysed one syllogism at a time. So perhaps his first step was to take N and break it down into syllogisms; or perhaps he never considered the analysis of complex arguments as a practical question. Then,

given a single syllogism, Aristotle would write a chart similar to (3) above, giving letters for the terms of the syllogism. He described the process of writing this chart as ‘setting out of terms’, in Greek *ékthesis tōn hórōn*. Finally he could use the chart to correlate *N* with some syllogistic form in the formal core. So Aristotle’s procedure was close to the modern one, with two main differences. He would analyse only single syllogisms; and instead of generating derivations within a proof calculus, he would simply point to one of the finitely many valid syllogistic forms.

Aristotle’s device of charts seems to have died with Aristotle himself. Theodorus’ Arabic translation of the *Prior Analytics* (in [1]) describes it in a way that would have obscured it completely for Arabic readers. For example he translates *ékthesis* with a term *wadʿ*^c normally associated with subject terms as opposed to predicate terms. At 48b40f Aristotle says that terms should be set out in the nominative case (*katà tās klēseis tōn onomátōn*); this remark had a lasting influence in the western tradition right down to Leibniz (as witness the examples discussed in [7]). But no reference to the nominative case survives in Theodorus’ translation of the passage ([1] *al-qiyās* p. 305).

We turn now to Ibn Sīnā. He recognises at once that there is no limit on the size of arguments that may need to be analysed. In theoretical sciences as opposed to debate, the reasoner

sees no problem about there being a goal proposition that can only be reached through a thousand middle terms and at great length ..., provided he has the spare time for it. (*Burhān* [12] 141.14–17) (5)

In the modern procedure, nothing is done to break down the argument into simple steps until after the symbolising chart has been chosen. By contrast, for Ibn Sīnā the first step of all is to break the argument down into separate natural language syllogisms and identify their premises and terms. He describes the process briefly at *Qiyās* [11] pp. 460-2.

With a natural language syllogism in his hands, Ibn Sīnā now needs to validate the syllogism by tying it to some valid form in his formal core. He has no formal language, so there is no question of translating the syllogism into a formal sequent. Instead he paraphrases the syllogism into a *normal form* where the propositions and their terms can be read off more or less automatically. This normal form is written in Arabic, without even any of the letter symbols that Aristotle used. But the Arabic is regulated so as to leave no doubt which item in the formal core it corresponds to. We will

discuss Ibn Sīnā's normal forms in a moment.

In today's logic, the final step of analysis is to generate the formal sequent by a formal derivation in the appropriate logical calculus. Perhaps most logicians see this as the single most important part of the analysis. But nothing corresponds to it in Ibn Sīnā's account. There is no need for it, because Ibn Sīnā has only finitely many syllogistic forms in his formal core, and he expects his students to have at least the predicative syllogisms by heart:

If you didn't memorise what was said earlier [about forms of syllogisms], you won't have been able to make any use of this [lesson]. (*Qiyās* [11] 466.5) (6)

He does spend some time in *Qiyās* section 9.4 explaining how separate syllogisms can be assembled into a single compound syllogism, but he regards this as synthesis rather than analysis.

3 Ibn Sīnā's normal forms

3.1 Topic-comment sentences

In analysis Ibn Sīnā mainly handles predicative syllogisms, i.e. syllogisms whose sentences take the forms

$$\left\{ \begin{array}{c} \text{Every} \\ \text{Some} \end{array} \right\} A \left\{ \begin{array}{c} \text{is} \\ \text{is not} \end{array} \right\} a B. \quad (7)$$

Western logicians after Aristotle used these forms as normal forms. The terms A and B are identifiable as the largest noun phrases in the nominative case; the earlier, i.e. A , is the subject term and the later is the predicate term. (See for example Buridan [3] pp. 238–246.) This works in Greek, Latin and German. But in Arabic it's hopeless; the term A after the quantifier normally goes into the genitive case, the standard opening *inna* sends the quantifier itself into the accusative, and in negative sentences B tends to carry a preposition.

At [1] *al-qiyās* p. 309 Theodorus — still struggling to make sense of Greek grammatical distinctions — suggests that Aristotle is talking about sentences consisting of an *ism* and a *kabar*. In Arabic sentences the verb normally comes before the subject. But Arabic also has a form of sentence often called the 'nominal sentence', where a noun or noun phrase (*ism*) comes before the verb. This noun phrase is often the subject of the verb, though it

need not be. For linguists the noun phrase forms the ‘topic’ (*mubtada’* in Arabic), i.e. it introduces into the sentence some thing or person that is already available in the context. The rest of the sentence is the ‘comment’ (*ḵabar*), which conveys some information about the thing named by the topic. So this kind of sentence is known to linguists as a ‘topic-comment’ sentence. Theodorus is suggesting, probably without any deep reasons, that the proper Arabic equivalents of Aristotle’s standard predicative forms (7) are topic-comment sentences.

There is an objection to Theodorus’ suggestion. A topic introduces into the sentence something that was already known in the context of utterance. So topics are normally linguistically definite, for example ‘The X’, ‘All Ys’. A noun phrase beginning ‘Some’ is not a convincing topic. The classical Arabic linguists make this point about the *mubtada’*. For example Ibn Sīnā’s older contemporary Ibn Jinnī in his *Ḷaṣā’iṣ* [8] i.321 says that Arabic speakers avoid ‘the ugliness of a (syntactically) indefinite *mubtada’*’ (*qabīḥu l-ibtidā’i bil-nikra*), even in sentences like ‘Money — you have it’ where the *mubtada’* is semantically definite.

So to use topic-comment form for the sentences (7) does violence to Arabic. Examples like

Some human — he is not a writer. (*ba^cḍu l-nās laysa bi-kātibin*) (8)

(Ibn Sīnā *‘Ibāra* [10] 51.4) could only reinforce the prejudices of anti-logicians who thought of logicians as Greek infiltrators with subversive intent.

3.2 Ibn Sīnā’s use of topic-comment form

Theodorus’ translation was probably the origin of Ibn Sīnā’s normal forms. Ibn Sīnā uses topic-comment sentences as his normal form: the subject term (usually with a quantifier) is the topic, and the predicate term is the comment. For example at *‘Ibāra* [10] 31.17–32.1 he introduces subject-predicate sentences as ‘composition by way of comment’ (*tarkīb alladī ‘alā sabīli l-ḵabar*); see also [10] 29.12. A typical example is at *Qiyās* [11] 478.11f, where he discusses the analysis of a syllogism containing the premise

Disease is in every human. (*al-marḍ fī kulli ‘insānin*) (9)

This is a topic-comment sentence, but the quantifier is in the comment and not in the topic. After discussing various features of the example, Ibn Sīnā opts for the following paraphrase:

Every human, there is disease in him. (*kull ‘insānin fīḥ marḍ*) (10)

([11] 479.15.) This is again a topic-comment sentence, but now the logical subject forms the topic, as it should in the topic-comment normal form. The logical subject is not the grammatical subject of the comment, but no matter.

3.3 Making hidden meanings explicit

Ibn Sīnā's logical writings are dotted with references to normal usage (*ʿāda*, *mustaʿmal*) and to how linguistic expressions are understood in practice (*mafhūm*). It's one of his most characteristic teachings that the surface forms of sentences that we hear or read may express only a part of what the utterer means; we must always be prepared to supply further information from the context of utterance or our background knowledge of the subject. For example in *Mašriqiyyūn*, in a section about conceptual problems that may hinder us from grasping an intended predicate correctly, Ibn Sīnā reminds us that we may miss an implied reference to

part versus whole, or a time, or a place, or a qualitative comparison, or reaching a certain degree, or something done or experienced ... ([14] 48.6f) (11)

Even when normal usage conflicts with strict use of language, says Ibn Sīnā,

since they are possible and permissible forms of predication, all these phenomena do occur, and it would be a great mistake to ignore them. (*ʿIbāra* [10] 101.2f) (12)

He goes on to accuse Aristotle of making this great mistake.

Hence an important ingredient of analysis is to make explicit things that were intended in the raw argument but were not expressed. Ibn Sīnā studies an example at *Qiyās* [11] 484.4–486.5. Suppose our raw argument contains the sentence

We don't know what infinite things are. (*inna ġayra l-mutanāhiy lā yuʿlam*, [11] 484.7) (13)

This is not strictly true, he says. We do know something about what infinite numbers or infinite lines are, since we know that they are numbers, or that they are lines. What we don't know is what, for a number or a line, constitutes being infinite. So presumably the person who proposed the raw

argument didn't intend (13) in its natural meaning.

[We would agree if someone said:] The normal meaning (*mafḥūm*) of the expression 'It is not known' has greater extension than the normal meaning (*mafḥūm*) of the expression 'It is not known in respect of what it is to be infinite'. ([11] 485.8–10) (14)

In this case analysis of the raw argument includes replacing expressions that the speaker didn't intend in their normal meaning by expressions whose normal meaning agrees with the speaker's intentions. There is something uncomfortable about the notion of the 'normal meaning' of sentences that are not normal Arabic — for example sentences with an indefinite *mubtada'* — but I think this is not a serious problem.

In *Qiyās* [11] Section 1.3 Ibn Sīnā presents a number of important examples of sentences taken from standard scientific discourse, where if we think carefully about what we understand them to mean, we can see a layer of extra structure that is not visible on the surface. They all have the common feature that their meaning — though not their syntax — is *two-dimensional* in the sense defined in 1883 by Oscar Mitchell [17]. Namely, besides the quantifier over the subject term, they also carry an implied quantifier over time. For example

Everybody who travels from Rayy to Baghdad passes through Kermanshah. ([11] 22.12f) (15)

The clear implication is that each of these travellers passes through Kermanshah *during the journey from Rayy to Baghdad*. In other words, for every person p and every interval of time t , if p is in Rayy at the beginning of t and in Baghdad at the end of t , then p passes through Kermanshah within t .

Incidentally this is not an anachronistic description. Although you may have read that De Morgan introduced the word 'quantifier' in the nineteenth century, Ibn Sīnā has two words for 'quantifier' (*sūr* and *ḥaṣr*), and in his discussion of (15) and its fellow two-dimensional statements he explicitly refers to the two kinds of quantifier that they contain. Also incidentally, Ibn Sīnā differs from Mitchell in details of the second dimension. For Mitchell the second quantifier is over points of time, whereas Ibn Sīnā is happy for it to be over intervals of time, or even over occurrences of situations. For example one of his analyses in *Qiyās* involves a quantifier over continuous periods of someone being rich ([11] 476.7–12). In general Ibn Sīnā has a sophisticated technical toolkit for his analyses. He knows for

example that a pair of universal quantifiers can be combined into a single universal quantifier over ordered pairs, and that a universal and an existential quantifier can't be combined in this way. He gives necessary and sufficient conditions for two ordered pairs to be equal, just the same as in modern texts ([11] 476.11f). He knows that in some cases these technical tools, together with other devices like putting a quantifier inside a term, allow a two-dimensional sentence to be expressed in standard predicative form even after the hidden but intended quantifier has been made explicit.

4 Analysis and proof search

There is another interesting technical aspect of Ibn Sīnā's discussion of analysis that we won't have occasion to mention elsewhere, so let me record it here. In *Qiyās* [11] pp. 462-8 Ibn Sīnā trains his reader in a procedure for finding what premises need to be added in order to make an argument valid. In this procedure Ibn Sīnā gives what is probably the world's first proof search algorithm, and very likely the first search algorithm of any kind in Arabic mathematics. These historical claims are defended in [6].

5 Ibn Sīnā versus Fakhr al-Dīn al-Rāzī

It should help to illuminate Ibn Sīnā's approach to analysis if we contrast it with that of Fakhr al-Dīn al-Rāzī nearly two hundred years later. The following example comes from Rāzī's *Manṭiq al-Mulakkaṣ*, and Khaled El-Rouayheb ([4] p. 45f) recently called attention to its influence on later Arabic logicians. Rāzī writes ([19] p. 319):

The body has a blackness (*al-jism fih sawād*).
 Every blackness is a colour (*kull sawādin lawn*). (16)
 (It follows that) the body has a colour (*al-jism fih lawn*).

Rāzī's comment on this argument is contained in the title of this section of his book:

The middle term is not fully repeated. (17)

Presumably he has identified the middle term as 'has a blackness' (*fih sawād*) in the first premise, and noted that the 'has' (*fih*, literally 'in it') is missing from the second premise.

How would Ibn Sīnā deal with (16)? The premises and conclusion have been identified, so the next step is to find the terms. The middle term certainly contains ‘blackness’ (*sawād*) since this is the only common feature between the two premises. But now the particle ‘in’ (*fī* in *fīh*) would catch Ibn Sīnā’s eye:

You need to take a close look at the behaviour of particles. ...
 When you find [a preposition] in the terms, check whether it is a part [of a term]. [If it is one,] then keep it as a part and add it to the whole, making a single term out of the two parts. (*Qiyās* [11] 482.6–13) (18)

So there are three possibilities. One is that the ‘in’ is not a part of a term at all — for example it might be a disguised copula. This seems not to work here. A second possibility is that ‘in’ is part of the same term as ‘blackness’; but this won’t work either, since ‘in’ is missing from the second premise. That leaves the third possibility, that ‘in’ is part of the same term as ‘the body’. Success! The phrase ‘the body has’ (*al-jism fīh*) appears in both the first premise and the conclusion, so it must be a term. Then we can read off the other terms as ‘blackness’ and ‘colour’. It remains to paraphrase the original argument into a form where these terms can be read off automatically:

Every **blackness** is a **colour**,
 Some **blackness** is **in the body**. (19)
 (It follows that) some **colour** is **in the body**).

In this revised form the three sentences are in topic-comment form, and in each case the terms (which are shown in bold) can be read off as the topic and the comment. The argument has the form of a syllogism in mood Disamis, or as Ibn Sīnā would say, the fourth mood of the third figure. So the analysis is successful and the argument is validated.

Rāzī says none of this. In fact he makes no attempt at an analysis of the argument. Curiously Rāzī does have a brief section on analysis (*taḥlīl*) a few pages later ([19] p. 336). In it he assumes that the syllogism to be analysed is already set out so that the terms and propositions can be read off, and he tells us how to tell from this information what kind of syllogism (recombinant or duplicative, simple or compound) we are looking at, and which are the major, minor and middle terms. This corresponds to just sixteen lines (461.12 to 462.10) of Ibn Sīnā’s own thirty-seven pages on analysis in *Qiyās*. Evidently Rāzī lacks almost all of Ibn Sīnā’s notion of analysis.

Is Rāzī just ignorant and foolish, or does he have a reason for rejecting Ibn Sīnā's procedure of analysis? I know too little of Rāzī's writings to make an informed judgement on this.

Rāzī seems to believe that we can take a natural language argument, extract its terms by purely syntactic criteria without regard for the sense, and then use logic to check the validity of the argument on the basis of these terms. But Rāzī's own example shows that in general we can't validate an argument by correlating its *syntactic* terms with the terms of a *syllogistic* form. So, one might argue, we need a larger supply of argument forms to cope with the vicissitudes of natural language. Read this way, Rāzī is pointing to the possibility of a *natural language logic* that works directly with the sentence forms that we meet in informal usage.

Did Rāzī himself say any of this? Not as far as I know. But in the history of Arabic logic he does mark the beginning of a period where the tendency was to introduce new argument forms. There was a parallel development in the West a hundred or so years later. Rāzī's closest counterpart in the West was probably William of Ockham, who experimented with the idea that we can work with arguments where the terms appear in oblique cases ([18] iii.1.9, p. 385ff). A curious fact in both Rāzī's case and Ockham's is that their examples do nothing to validate any arguments that couldn't already be brought into standard form by obvious paraphrases; so the immediate effect was to redistribute the effort in logic without achieving any extra strength. It seems to me rash for any historian to describe this development in Rāzī and Ockham as progress, though it certainly marks a change of direction.

Ibn Sīnā himself would almost certainly not have accepted this change of direction. From Ibn Sīnā's point of view the project that we are ascribing to Rāzī and his successors could never work, because in general the explicit syntax of sentences simply doesn't contain the required information about the intended meaning.

6 What was in Ibn Sīnā's formal core?

The preceding sections of this paper, if they interpret Ibn Sīnā correctly, show that his logic did have a formal core in the same sense as Aristotle's. But they leave open the question what was in this formal core. I think the answer is not entirely clear, though I will give some preliminary views and some evidence for them.

Ibn Sīnā describes several types of argument form, certainly more than

we find in Aristotle. They include:

- (1) predicative syllogisms,
- (2) argument forms involving non-aristotelian absolute propositions,
- (3) recombinant (*iqtirānī*) syllogisms,
- (4) duplicative (*istitnā'ī*) propositional syllogisms,
- (5) modal syllogisms,
- (6) other valid argument schemes such as *reductio ad absurdum*, conditionalisation and proof by cases.

For lack of space I leave (4) and (6) on one side in this paper.

6.1 Predicative versus absolute

I claim first that (1) and (2) are quite distinct, though Ibn Sīnā's terminology obscures this. Moreover (1) is the heart of the formal core, but (2) is not a part of the formal core at all.

Ibn Sīnā's use of the term 'absolute' (*muṭlaq*) is chaotic. Theodorus in his translation of the *Prior Analytics* had used the word to distinguish those syllogistic forms in which no modal words ('possible', 'necessary') occur. The two-dimensional sentences that Ibn Sīnā presents in *Qiyās* [11] Section 1.3 are absolute in this sense. But some of these 'absolute' sentences contain universal quantification over time, in a way that can be read as a kind of necessity. Ibn Sīnā's view is that the relationship between permanence and other kinds of necessity is not a proper question for logicians to study, though logicians need to take note of the fact that some people do regard permanence as a kind of necessity. ([11] 48.16ff.) The existence of sentences that contain no modal words but nevertheless behave like modal sentences is very confusing for Aristotle's distinction between modal and non-modal syllogisms. Ibn Sīnā is evidently aware of the problems caused by his examples, but he makes very little attempt to resolve them. He has several discussions of the meaning of 'absolute', but they are obscure and may not be mutually compatible. He makes the telling comment that Aristotle's failure to recognise two-dimensional sentences led to *maḥālāt* ([11] 30.6), the plural of an uncommon word glossed by Jawharī [15] p. 487 as *ḥīla*, i.e. artifice, subterfuge, trick.

The upshot is that when we read Ibn Sīnā, we have to distinguish between the sentences that are 'absolute' in the sense that they have the forms

as in Aristotle's predicative syllogisms, i.e. the forms (7), and a much broader class of 'absolute' sentences that includes some of the two-dimensional sentences that Ibn Sīnā displays in *Qiyās* Section 1.3.

In fact Ibn Sīnā himself has to distinguish these two classes when he deals with situations where they behave differently. One such case is e-conversion:

From 'No *A* is a *B*' infer 'No *B* is an *A*'. (20)

Ibn Sīnā has no misgivings about using e-conversion for predicative sentences as in (7). For example he uses it to justify the syllogistic moods Cesare, Camestres and Festino ([11] 114.6, 115.17f, 116.4). Of the fourteen valid predicative moods that Ibn Sīnā discusses, these are exactly the ones where Aristotle invoked e-conversion. But speaking about a two-dimensional absolute sentence that 'would naturally be understood' (*yufham*, [11] 81.7) as expressing the true sentence

Every human is at some time not laughing. (21)

Ibn Sīnā notes that a syntactic conversion turns the sentence into something to the effect that everything that laughs is non-human, which is clearly false. So he warns that there are absolute universally quantified negative sentences that don't obey e-conversion ([11] 81.7–82.9). In order to distinguish this broader class of absolute universally quantified negative sentences from the well-behaved predicative sentences, he calls the latter 'standard' (*mašhūr*, e.g. at [11] 113.9 and 114.1).

While we are on this example, let me remark that the English sentence 'No human is laughing' is clearly not one that 'would naturally be understood' as expressing (21). So whatever sentence of Arabic Ibn Sīnā has in mind, this English sentence is certainly not a correct translation of it. In fact Ibn Sīnā gives different Arabic formulations in different places, and some of them are hard to interpret because of peculiarities in the combination of a quantifier and the Arabic negating verb *laysa*. I did consult a well-known Arabic-speaking linguist who is an expert on negation in Arabic, and he was unwilling to commit himself to any general rules. But he did make the interesting remark that the interpretation of the kind of sentences we are looking at depends strongly on linguistic focus. Clearly there is a lot more work to be done here before we can fully understand what Ibn Sīnā intended.

Does Ibn Sīnā's formal core include any valid argument forms that involve two-dimensional absolute propositions? It seems to me the answer

is definitely No. First, Ibn Sīnā nowhere validates an argument by reduction to two-dimensional forms. Second, Ibn Sīnā never suggests any valid argument forms using these absolutes. (Later Arabic logicians did, but not Ibn Sīnā himself.) I think these two points together are decisive.

Ibn Sīnā's two-dimensional absolute propositions are a vivid illustration of the need for analysis, and this is probably the main reason why he included them. Some steps towards analysing them are already contained in *Qiyās*, as we saw at the end of Section 3.3 above. But their proof theory demands resources that Ibn Sīnā didn't have. Mitchell [17], writing in the very different climate of a late 19th century New England university mathematics department, was able to write down some proof rules for two-dimensional sentences. But I doubt that Ibn Sīnā would have been happy with Mitchell's rules, because they were mathematical devices rather than descriptions of how the mind reaches certainty.

Throughout his discussion of 'absolute' syllogisms in *Qiyās*, Ibn Sīnā passes freely between 'standard' predicative sentences and two-dimensional ones. He is reasonably careful to explain what kind of sentence he is talking about at any one time, but he makes no attempt to segregate the two types into different sections. Personally I would have segregated them, so I wonder why he didn't. Possibly he felt obliged to follow the common schedule of the commentary tradition in which he was writing. One regular phrase in *Qiyās* is 'Now the usual custom is to mention ...'.

6.2 Recombinant syllogisms

When Ibn Sīnā presents Aristotle's predicative syllogisms in *Qiyās* [11] Section 2.4, he introduces them as examples of 'recombinant' (*iqtirānī*) syllogisms. The mark of a recombinant syllogism is that each of the two premises has two terms, the premises share one of these terms, and the conclusion is built up by 'recombining' the two remaining terms. ([11] 106.5–13.) In Books 6 and 7 of *Qiyās* Ibn Sīnā discusses in detail some propositional syllogistic forms that fit the same description, except that the 'terms' are now propositions and the copula is replaced by a binary sentential operator. He believes ([11] 397.5) that Aristotle was well aware of the class of propositional recombinant syllogisms, but that this work of Aristotle went missing already in antiquity.

At the end of one of his essays on analysis in *Qiyās*, Ibn Sīnā tells his readers to 'apply exactly the same considerations to propositional compounds' ([11] 468.7). He must mean recombinant propositional syllogisms, and his point is clear. Virtually the whole theory of predicative syllogisms

translates mechanically to recombinant propositional syllogisms. The only serious glitch that I know is that the classification of propositional compounds as ‘affirmative’ or ‘negative’ is troublesome — the linguistic and the logical criteria don’t match.

Ibn Sīnā’s reading of the sentences in recombinant propositional syllogisms is very close to those of Wallis [23] p. 280ff and Boole [2] Chapter xi, except that unlike those authors he makes no attempt to reduce the propositional case to the predicative one. There is a place (*Qiyās* [11] 470.8–472.5) where Ibn Sīnā points out that a certain recombinant propositional argument can be translated into a predicative syllogism. But he adds that if one does that, an extra non-syllogistic step is needed in order to paraphrase the predicative conclusion back to the original propositional one. There is no suggestion that the translation from propositional to predicative is in any sense a reduction to something more basic.

Probably Ibn Sīnā regarded predicative syllogisms and recombinant propositional syllogisms as examples of the same phenomenon; the mental processing in both cases would be more or less the same. So everything points to the forms of recombinant propositional syllogisms being part of his formal core.

6.3 Modal syllogisms

The hardest case to settle is that of modal syllogisms. Łukasiewicz himself made only a perfunctory attempt to extend his methodology to Aristotle’s modal syllogisms, because he regarded these syllogisms as

full of careless mistakes and [without] any useful application to scientific problems. ([16] p. 181) (22)

Paul Thom [22] proposed a way of interpreting them as a coherent formal system. But our problem is their place in Ibn Sīnā’s scheme of logic, not Aristotle’s. There are two natural questions here. First, do the modal syllogistic forms that Ibn Sīnā accepts form a coherent formal system? And second, does he analyse down to modal syllogistic forms?

We begin with the first question. Ibn Sīnā puts his modal syllogisms in a sequence after the predicative syllogisms and before the recombinant propositional syllogisms. It could be argued that both these other kinds belong in the formal core, so we should at least start with a presumption that the modal syllogistic forms belong in the formal core too. I tend to the opposite view. We have already seen that in his treatment of absolute syllogisms, Ibn Sīnā moves freely between ‘standard’ core forms that don’t

allow any further analysis, and syllogisms with two-dimensional sentences that definitely do require analysis. So it makes just as good sense to start with a presumption that Ibn Sīnā's modal syllogisms are mixed bag too.

In fact they clearly are a mixed bag. We have already seen that Ibn Sīnā uses statements that are explicitly modal and statements that have universal quantification over time but no modal notions; in practice he moves between these two cases and it isn't always clear which he means. This is not the only dichotomy; at *ʿIbāra* [10] 113.2f he introduces another distinction:

If there is a quantifier there are two places [for the modality],
namely [adjacent to] the copula and [adjacent to] the quantifier. (23)

His examples show that there is some similarity to the western distinction between modality *de re* and modality *de dicto*, but there is more work to be done on the relationship. The logical properties of Ibn Sīnā's two types of modality are quite different, and Ibn Sīnā constantly calls attention to this fact in his discussion of modal syllogisms in *Qiyās*. Thus:

'Every moving thing can be not human' is false in Aristotle's usage, but it is true when [the modality] is interpreted as being on
the quantifier. ([11] 207.5f) (24)

Here he is telling us that Aristotle always put the modality on the copula; in that reading the sentence says of each moving thing that it has a possibility of being non-human, which is false. But on the quantifier reading it says (at least as I now interpret that reading) that there is no contradiction in assuming that there are no human moving things — for example because there is no contradiction in assuming there are no humans. Quite often Ibn Sīnā doesn't tell us which reading he has in mind, even when it makes a difference. This to-and-fro between readings doesn't lie well with the suggestion that Ibn Sīnā is intending to produce a coherent system of modal argument forms.

We turn to the second question. In his four sections on analysis in *Qiyās*, Ibn Sīnā does discuss two examples that involve modal notions. In neither case does he give a successful analysis down to a modal syllogism, but the reasons are different in the two cases. The first is a fairly lengthy discussion of an argument which involves the predicate 'has the possibility of being eternal' (*yumkin 'an yakūna 'azliyyan*, [11] 472.5–476.1). Here the modality is part of the predicate, and nothing that Ibn Sīnā says about the argument requires us to take it out of the predicate. So no modal syllogistic form is needed, and he mentions none. One should add here that in his *Letter to the*

Vizir Abu Sa'ad ([9] p. 37f) Ibn Sīnā analyses some arguments of al-Kirmānī by reducing them to syllogisms, and two of these syllogisms contain modal notions; but again the modal notions are inside terms, and the validity or otherwise of the syllogisms doesn't depend on bringing the modalities out of the terms.

In the second of the examples in *Qiyās* ([11] 478.7–480.10) the modality is essential to the argument, and Ibn Sīnā does at one point invoke modal Celarent with possible minor premise and necessary major. But his main claim about the argument is that it doesn't analyse successfully, because of a fatal confusion between 'health' and 'healthy'. So the modalities emerge in the course of casting around for ways of making the argument work. For example one premise was (9) above, which we saw that Ibn Sīnā analysed into (10). The snag is that (10) is false; some humans are perfectly healthy. So Ibn Sīnā suggests adding 'possibly' in hopes of reaching a convincing argument from true premises, but the hopes are never fulfilled. We simply don't know whether Ibn Sīnā would have regarded a modal syllogistic form as an acceptable stopping point for the analysis.

In the thousands of pages of Ibn Sīnā's available works there could well be examples of successful analyses down to modal syllogistic forms. When these are found, they may throw more light in this dark corner.

The jury hasn't yet returned an answer on the question whether Ibn Sīnā had a 'modal syllogistic'. In fact the question hasn't yet been properly put to the jury together with the relevant evidence. Certainly Ibn Sīnā had plenty to say about modal syllogisms. But the jury need to be asked whether these discussions of his were aimed at producing a coherent formal system of modal syllogisms. They could have had a quite different aim, namely to clarify the logical relationships between statements in relation to how those sentences are used in scientific discourse. On this view, Ibn Sīnā's discussions of modal logic are in aid of setting up a methodology for assessing the meanings of arguments in concrete situations. As Ibn Sīnā himself is reported to have told his students:

In analysis, do not spend too much time taking into account the forms of syllogisms for that's one of the easy parts and a sound instinct rarely makes a mistake about it; you should rather practice examining in detail the matters [of syllogisms]. (From *Memoirs of a Disciple from Rayy*, trans. Gutas [5] p. 70) (25)

It is of course an interesting and perfectly legitimate question how far one can assemble Ibn Sīnā's modal syllogisms into a convincing formal system.

I suggest only that this question sidesteps Ibn Sīnā's main aims in logic, which were more linguistic and more severely practical.

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