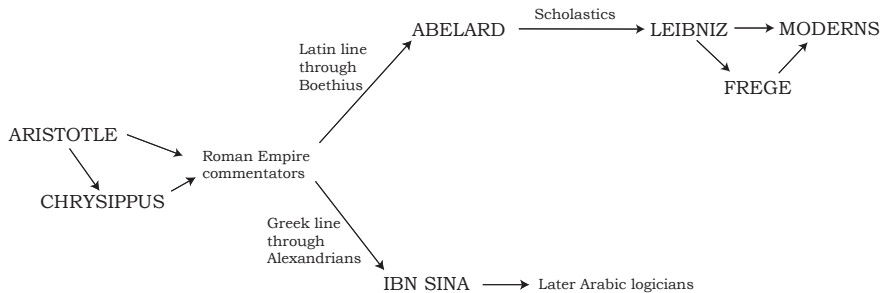


# **Ibn Sina's explanation of reductio ad absurdum.**

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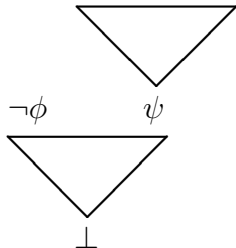
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## WESTERN LOGIC — THE BIG NAMES



**Reductio ad absurdum (RAA):**

to prove  $\phi$  from known facts  $\psi$  we argue



How to justify using this derivation as a demonstration of  $\phi$  given  $\psi$ ?

Ibn Sīnā's answer:

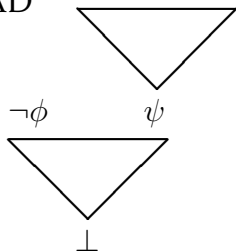
Here as often, we mean more than we say.

Common practice is to introduce the false assumption  $\neg\phi$  with 'if', but not repeat 'If  $\neg\phi$ ' when we derive things from it, although in strict logic the condition is needed.

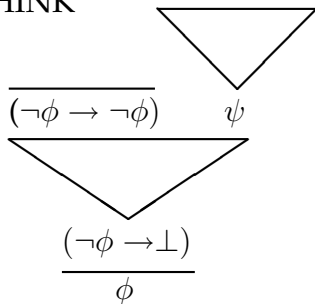
Restoring the implied condition gives a richer derivation as follows.

We restore the implied 'If  $\neg\phi$  then ...' on  $\neg\phi$  itself and every formula that depends on it:

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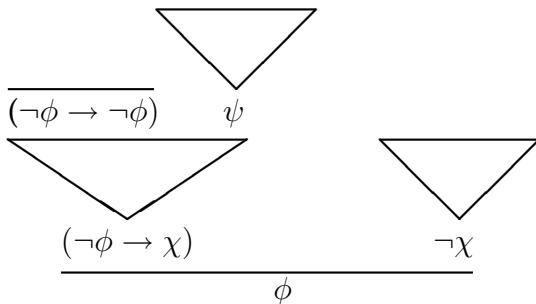


THINK



NB the richer derivation *is still valid*,  
and  $(\neg\phi \rightarrow \neg\phi)$  is a self-evident axiom.

Strictly Ibn Sīnā describes a more complicated derivation:



Two probable reasons why he adds  $\chi$ :

1. To express that the reasoner must not just deduce a contradiction, but also know that the contradiction is a contradiction.
2. To separate the 'assumption' part of RAA from the 'constructively dubious' part.

2 interprets Aristotle. Ibn Sīnā thinks the 'constructively dubious' part is a minor problem, and the challenge is to make sense of an assumption made and later discharged. His explanation covers this, not just for RAA.

In what follows we consider:

- (I) what Ibn Sīnā counted as ‘justifying’ RAA;
- (II) his evidence for the missing conditions;
- (III) his reason for thinking that the derivation remains valid when the conditions are added.



## (I) What counts as justifying RAA?

Today we require explanations to be precise and unambiguous.

Any general principles invoked should be universally true.

Traditional justifications were more like answers to a child's question 'Why?'

Hard to find anything universally true about natural language.

For Ibn Sīnā, medical reasoning was one paradigm, and all general principles in medicine are 'true in most cases' (*aktarī*).

For Ibn Sīnā and many traditional logicians, explaining a rule of reasoning means showing how someone using it could be rightly convinced of what it proves.

So the explanation should use only concepts and principles already available to people who use the rule.

Thus Ibn Sīnā rejects justifications that

- (1) are unreasonably complicated, or that
- (2) bring in metatheory or
- (3) use an artificial language.

“[One commentator makes various alterations and additions which] lengthen the discussion but give us no new information. [By contrast] the account we have given is exactly the RAA syllogism itself, no more and no less.” (Ibn Sīnā, *Šifā'* viii.3)

This is an exaggeration, obviously.

To repeat the original argument, no more and no less, would hardly be a justification of it.

But Ibn Sīnā can claim that his justification uses no new concepts and no added steps.

Ibn Sīnā also rejects justifications based on dialogue, as for example:

The form in which Euclid argues, supposes an opponent; and the whole argument then stands as follows. “When X is Y, you grant that P is Q; but you grant that P is not Q. I say that X is not Y. If you deny this you must affirm that X is Y, of which you admit it to be a consequence that P is Q. But you grant that P is not Q; therefore” (etc. etc.) (De Morgan, *Elements of Trigonometry* p. 5)

(This is a figment of De Morgan’s imagination. It draws on Aristotle, not on Euclid.)

Ibn Sīnā often refers to dialogues where we show our opponent that she is wrong to believe  $\phi$ , by *taslīm*, i.e. ‘granting’  $\phi$  and deducing something clearly false.

For Ibn Sīnā, talking about *taslīm* is a way of talking about inference from assumptions (*Ableitung*), as opposed to demonstration that something is true (*Schluss*).

He doesn’t use it for RAA, presumably because in scientific proofs there simply is no opponent.

Ibn Sīnā also rejects explanations of RAA based on counterfactual reasoning.

Reason: according to Ibn Sīnā, counterfactual inference is non-monotonic.

When we adopt a counterfactual assumption, we overrule some previous inferences by kicking out of reach the premises on which they were based.

In scientific discourse this is not allowed.

“It’s not true that if we stipulate that  $m = 2n$  then it follows that  $m$  is even. One can stipulate an impossibility that prevents that. ...

But what we do [in the sciences] is to add a condition (*šart*) which kicks out any conditions (*šurūt*) that prevent the inference.

For example ‘If  $m = 2n$  and nothing impossible is attributed to  $m$ , then  $m$  is even’.” (*Qiyās* 274.5–15)

## Background on Ibn Sīnā's view of language

For Ibn Sīnā, sentences have a basic structure and (usually) pieces added on.

He variously calls the added pieces

- ▶ condition (*šart*, plural *šurūṭ*)
- ▶ attachment (*iḍāfa*)
- ▶ addition (*ziyāda*)



**Example** (mine but based on many in Ibn Sīnā)

The sentence 'He is eating' contains no reference to the present. Proof: we can say 'If a person chews and swallows, he is eating'. A reference to *all times* is implied.

So the sentence 'He is eating' has an unspoken attachment: 'He is eating *at time t*'.

Then the longer sentence contains an unspoken condition: '*For all times t*, if a person chews and swallows at time *t*, he is eating at time *t*'.

Another unspoken condition: '*If t is the present*, he is eating at time *t*'.

## (II) Ibn Sīnā's evidence for the missing conditions

For Ibn Sīnā, a major part of logical analysis is to uncover (*rā<sup>c</sup>ay*) additions which are implied but suppressed (*maḥḍūf*) in common usage. E.g.

“In common acceptance things are taken unquantified, while in the sciences they have to be taken quantified. When you pay attention to (*rā<sup>c</sup>ayta*) this in the examples above, ... what is excellent in a thing has to be excellent absolutely, i.e. excellent without any addition (*ziyāda*).

Being excellent in every respect ... is being excellent with an addition (*ziyāda*), namely ‘in every respect’.”  
(*Jadal* 143.1–5)

Or from Ibn Sīnā's *Autobiography*:

“The next year and a half I devoted myself entirely to reading Philosophy: I read Logic and all the parts of philosophy once again. . . . I compiled a set of files for myself, and for each argument that I examined, I recorded the syllogistic premises it contained, the way in which they were composed, and the conclusions which they might yield, and *I would also pay attention to the conditions ('urā'ī šurūt) of its premises,* until I had checked out that particular problem.”

Ibn Sīnā claims that in common practice, the false assumption  $\theta$  in RAA is introduced by ‘If  $\theta$ ’, but ‘If  $\theta$ ’ is not repeated when things are derived from  $\theta$ .

We can check this from the Arabic of Euclid’s *Elements* 1 (see the handout). Thus:

1. Ibn Sīnā is right that the false assumption is always introduced with ‘If’, not with ‘Let’ or ‘Suppose’.
2. In many cases the ‘If’ is immediately followed by a ‘Suppose  $\zeta$ ’, where  $\zeta$  includes everything in  $\theta$  that will be used. Then ‘If  $\zeta$ ’ is not repeated, but we shouldn’t expect it to be. Ibn Sīnā never gives his views on ‘Suppose’.

3. In a significant number of cases  $\zeta$  is missing or doesn't cover everything used from  $\theta$ . In these cases Ibn Sīnā is confirmed; 'If  $\theta$ ' is assumed silently.
4. There is an example where 'If  $\theta$ ' is used silently, after an assumption that was not made for RAA. (In modern terms it was made for  $\exists E$ .)
5. There is also an example where during the derivation,  $\theta$  is stated as a fact, not as a condition. If the condition 'If  $\theta$ ' was added, we would get the 'If  $\theta$  then  $\theta$ ' which appears in Ibn Sīnā's account.

### (III) How does Ibn Sīnā know that the derivation remains valid when the implied conditional clauses are added?

There is no syllogistic rule that would guarantee it. My answer is a tad speculative, based partly on what Ibn Sīnā *doesn't* say.

Note first that if  $\chi$  consists of a proposition  $\phi$  plus additions (conditions etc.) then  $\phi$  normally occurs positively in  $\chi$ .

For example  $\psi$  is a condition in  $(\psi \rightarrow \phi)$ , but presumably not in  $(\phi \rightarrow \psi)$ .

**Fact** (in any standard natural deduction system):

Let  $\Gamma$  be a set of formulas and  $\phi, \psi$  formulas. Let  $\delta(p)$  be a formula in which  $p$  occurs only positively, and  $p$  is not in the scope of any quantifier on a variable free in some formula of  $\Gamma$ . Suppose

$$\Gamma, \phi \vdash \psi.$$

Then

$$\Gamma, \delta(\phi) \vdash \delta(\psi).$$

Call this *Ibn Sīnā's Lemma*. It gives exactly what he needs for the preservation of validity.

Ibn Sīnā is almost obsessively interested in the difference between affirmative and negative, but he normally shows no awareness of the notion of a positive occurrence.

For example he has no notion of scope, and no glimmering of the laws of distribution.

Nevertheless his notion of 'condition' seems to have positivity hidden in it.

In those terms, Ibn Sīnā's Lemma is a kind of statement of upwards monotonicity.



For Ibn Sīnā, the logical properties of propositions are mainly the result of their basic structure.

For example he has no examples of valid entailments which depend on additions.

He writes as if, other things being equal, making additions in a uniform way to propositions in a valid syllogism leaves the syllogism valid.

For example modal syllogisms are predicative syllogisms with modalities added.

His discussion of them is almost entirely about what additions of modalities will cancel the validity of the underlying predicative syllogism.

Ibn Sīnā often talks about the mental processing of syllogisms.

He emphasises that in predicative sentences, the analysis needed for logic never goes inside the subject and the predicate.

In this sense, logical rules apply *only at the top syntactic level*.

It was always a problem for traditional logicians to see how we can make inferences that depend on deeper analysis of the propositions. Burley, Buridan, Leibniz, De Morgan all worried at this problem.

Burley had the idea of induction on complexity.

The others relied on paraphrases, e.g. to bring the deeper structures to the top level.

It seems Ibn Sīnā has a solution:

Treat the upper layers of the proposition as ‘additions’, and apply the logical rules directly at the deeper level.

The solution is remarkably close to Frege’s.

See for example Frege’s construction of *Begriffsschrift* so that modus ponens can be applied directly to positively occurring subformulas at any depth.

Frege also has signs of Ibn Sīnā’s pretence that the upper syntactic levels can be ignored.