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The grammar of meanings underlying Ibn Sīnā's logic

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One might ask about Ibn Sīnā's logic:

Q1. What were the historical influences on Ibn Sīnā's logic?

E.g. Miklós Maróth, *Ibn Sīnā und die Peripatetische "Aussagenlogik"* (1989).

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A good question but not mine, because:

- Not my expertise.

Also not as helpful for understanding Ibn Sīnā as one might expect, because

- Most of the evidence is missing anyway.
- Even when he borrowed ideas,
  he often developed them in new ways.

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Q2. What is the formal content of Ibn Sīnā's logic?

E.g. Tony Street, 'An outline of Avicenna's syllogistic', *Archiv für Geschichte der Philosophie* 84 (2002) 129–160.

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A good question but not mine, because

- I don't believe Ibn Sīnā ever intended to produce a formal system of logic.

Also it leaves out things of key importance to Ibn Sīnā, such as

- The activities of a logician.
- The role of language in logic.

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Questions that interest me about Ibn Sīnā:

- What does he think logicians do?
- What methods does he use, for example to prove validity, or to prove invalidity, or to explain or distinguish meanings?
- What are his starting assumptions?
- What things excite him?

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The subject (*mawḍūᶜ*) of the science of logic

For Ibn Sīnā the subject of a science is the idea which appears in the subject terms (*mawḍūᶜ*) of the main theorems or questions of the science.
Equivalently, it is the idea over which the main quantifiers of the science range.

Example: the subject of arithmetic is [NUMBER].

- *Every number* is either even or odd.
- *Every pair of numbers* > 1 has a greatest common factor.

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Ibn Sīnā discusses the subject of logic in several places, most explicitly in *Mašriqiyyūn* 10.15. Different formulations in different places, but they converge on:

> 'idea that we come to know secondarily,
> i.e. through composition from primary ideas,
> either as in concept formation or as in inference.'

This should be understood as a relational term: 'x derived from $y, z \ldots$ by composition of ideas or by inference'.

Khūnajī later boiled it down to 'things known by way of concept formation or reasoned assent' (*al-maᶜlūmātu l-taṣawwuriyya wal-taṣdīqiyya*, *Kašf* p. 8).

Ibn Sīnā gives no examples of ‘secondary ideas’, except in his *Ta*^{c}*līqāt* 167.14–17, where we find:

Primary idea	Secondary ideas
[ANIMAL]	[EVERY ANIMAL]
	[SOME ANIMAL]
	[THIS/THAT ANIMAL]

Very interesting examples, for at least two reasons.

Reason 1

It was an aristotelian commonplace that composition of words corresponds to composition of ideas or meanings.

> “[We] compose sentences of expressions signifying parts of the compound affair signified by the sentence. … The imitation of the composition of meanings by the composition of expressions is by [linguistic] convention.”
> (Al-Fārābī, trans. Zimmermann with adjustment)

But before Ibn Sīnā, and in the West before Leibniz, logicians hardly went beyond the compositions described by Aristotle.

No logicians asked what compositions of words do occur, and what is the corresponding composition of meanings. For example genitive constructions:

- herder of sheep, thrower of stones (Ibn Sīnā)
- reading of poets (Leibniz)

Ibn Sīnā is relying here on a dependency analysis of the meanings of simple sentences.

$$\begin{tikzcd} & \mathit{copula}(+) & \\ \text{[ANIMAL]} \arrow[ur, "s"] & & \text{[MOVES]} \arrow[ul, "p"'] \\ \text{[EVERY]} \arrow[u] & & \text{[VOLUNTARILY]} \arrow[u] \end{tikzcd}$$

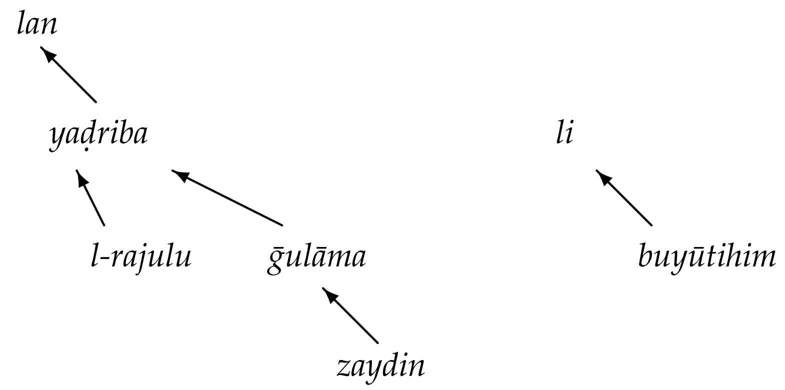
In grammar [ANIMAL] and [MOVES] are called the *heads* of the left and right terms or phrases.

Ibn Sīnā has several words for the dependency relation:

- *lāḥiq* ‘adjoined’
- *ziyāda* ‘addition’
- *’iḍāfa* ‘relation’ (used particularly for addition of items that increase the number of arguments)
- *šarṭ* ‘condition’ (used particularly but not exclusively for added clauses)

He often asks: Is this addition attached to the subject term or the predicate term?

Jonathan Owens, *The Foundations of Grammar* (1988), finds dependency analyses in classical Arabic linguists from Sarrāj onwards.
‘$a \longrightarrow b$’ means b controls the inflection (case etc.) of a.



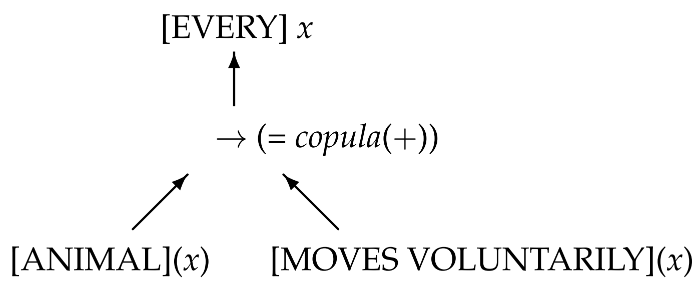
Among several differences between Ibn Sīnā’s dependency diagrams and the linguists’, Ibn Sīnā’s are *not linearly ordered*.

> “Speech doesn’t call for a natural ordering.” (*Maqūlāt* 130.1)

He notes that different languages express the same meaning in different orders. Contrast Ibn Sīnā’s near-contemporary the linguist ᶜAbd al-Qāhir al-Jurjānī:

> “The arrangement of the words in a particular construction is . . . an inevitable result of the construction of meanings.”
> (*Dalā’īl* p. 43 trans. Abu Deeb)

Frege’s *Begriffsschrift* (1879).
‘$a \longrightarrow b$’ means a fills an argument position in b.
Since [EVERY] clearly doesn’t fill an argument slot in [ANIMAL], Frege rearranges:



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Reason 2

Ibn Sīnā keeps coming back to the question: How should one distinguish between [ANIMAL] and [EVERY ANIMAL]? (They are both true of every animal.)

The standard aristotelian method for clarifying meanings was to add further text. E.g. 'Ajax the son of Oeleus of Locris' versus 'Ajax the son of Telamon of Salamis' (example from Porphyry *Categories*).

This doesn't work well with [EVERY ANIMAL].

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ᶜIbāra 66.5f, after a section on quantified predicates (for which see translation and comments by Hasnaoui),

> "The reason why we dealt with propositions with quantified predicates was in order to clarify the difference between a proposition being universally quantified and its having a universal as its subject."

The horse is an animal.

⋆The horse is an every animal.

Here Ibn Sīnā moves towards standard modern methods, asking what frames a meaning can fit into.

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The places where Ibn Sīnā distinguishes between [ANIMAL] and [EVERY ANIMAL] include *Madḵal* 65.12ff, *ᶜIbāra* i.7, *Metaphysics* 196.6ff and *'Išārāt* iii.4.

Some of these passages have been read — apparently since Ibn Dawd in 12th century — as expressing a paradoxical ontological thesis.

I plead incompetence in ontology.
But I remark that as discussions of *semantics* they are not at all paradoxical.
Methodological issues that they raise are still discussed in modern texts of natural language semantics.

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Three applications

A. Scope and non-linearity

B. Inhibitory additions

C. Procedure for validating syllogisms

A. Scope and nonlinearity

In *Qiyās* i.5, Ibn Sīnā examines some sentences of the form 'For every x there is a y ...':

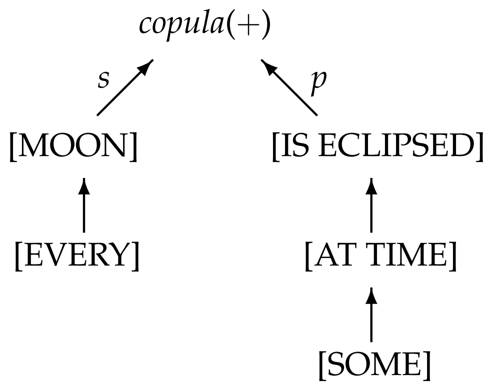
- Every human sometimes moves.
- Every moon is sometimes eclipsed.

He asks how we negate these sentences.
(For him this means negating the whole sentence and then working the negation inwards.)

Today we note that the second quantifier is 'in the scope of' the first.
In Ibn Sīnā's grammar of meanings this notion would need to appear somehow in the dependency diagram.

In some languages we can define the scope of a symbol (quantifier or negation) to be the subclause *immediately following* the symbol.
For Ibn Sīnā this would make no sense, because (as we saw) compound meanings are not linearly ordered.

??!



Ibn Sīnā's solution: explain the quantifier pair in a way which doesn't depend on an ordering between them.
Precisely, we universally quantify over a class C of *pairs* (m, t), m a moon and t a time.
The class expresses the relation of the quantifiers, viz. for each m there is a t so that (m, t) is in C.

This is close to Henkin's Skolem function semantics for partially ordered sets of quantifiers.
Ibn Sīnā misses the function quantifier 'there is a class C such that ...'.

He doesn't know how to adapt this explanation to the negated proposition, and he says so.

The problem is that quantifiers in the scope of a negation switch between universal and existential.
But Ibn Sīnā has no notion of scope of a negation. *Again there is no way of expressing it in his dependency diagrams.*

Note again Frege's solution.
He puts the negation of the whole proposition at the top, not attached below any word.

B. Inhibitory additions

ᶜIbāra 14.11ff: By adding the nominative inflection '-un' to 'Zayd' we prevent the preposition 'in' (*fī*) from being attached to 'Zayd'.

This example is purely syntactic,
but Ibn Sīnā adapts the idea to meanings.

For example (*Qiyās* 480.11) we can attach [UNQUANTIFIED] to a noun meaning,
which prevents it from accepting a quantifier.

Curious but it makes sense.
If we mean an idea to be unquantified,
[UNQUANTIFIED] has to be there in the meaning somehow.

Application: counterfactual reasoning

Ibn Sīnā notes that counterfactual reasoning is in general non-monotonic.
By making the false assumption

Suppose *not*-ϕ.

we deprive ourselves of the right to continue using ϕ as a premise, even if ϕ was proved earlier.

But in the exact sciences this non-monotonicity would be disastrous.
For example in proving ϕ by reductio ad absurdum we deduce a contradiction from *not*-ϕ and facts already established, *so we need to be able to use those facts*.

Ibn Sīnā's solution: In the exact sciences we *add a meaning* that protects proved propositions from being overruled by false assumptions.

Not a solution that commends itself today.

But note that Ibn Sīnā uses the grammar of meanings to pinpoint a difference between
(subjunctive) counterfactual reasoning and
(indicative) reasoning from false premises.

The confusion of these two is still a common source of mistakes.

C. Procedure for validating syllogisms

"I read Logic and all the parts of philosophy once again. . . . I compiled a set of files for myself, and

- for each **argument** that I examined, I recorded
- the **syllogistic premises** it contained,
- the **way in which they were composed**, and
- the **conclusions** which they might yield,
- and I would also **take into account the conditions** of its premises

until I had validated the thesis."
(Ibn Sīnā *Autobiography*, trans. Gutas)

Ibn Sīnā checks validity of syllogistic arguments thus.

First find the head terms *without additions*,
and whether the copula is affirmative or negative.
This gives preliminary information on whether the argument can be valid.
E.g. in second figure one premise must be negative.

If OK so far, add the quantifiers. Does validity survive?

> "The terms may look like those of a syllogism, but there can be a violation through some condition attached to the terms, e.g. quantifiers." (*Qiyās* 472.7)

If still OK, check the other conditions and additions.
Do they destroy validity?

This checking of the conditions is the "taking into account the conditions of the premises" referred to in Ibn Sīnā's Autobiography.

This includes modalities. Ibn Sīnā never recognises a modal argument as valid unless it is still valid without the modalities.

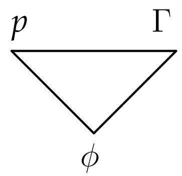
It includes some other important cases too.

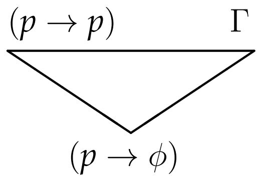
An important case is where 'If p' is added to the conclusion and one premise.
In *Qiyās* Chapter vi Ibn Sīnā checks case by case that this operation takes valid syllogisms to valid syllogisms.

In his treatment of reductio ad absurdum, *Qiyās* section viii.3, Ibn Sīnā shows how to use this operation to explain making and resolving assumptions in mathematical arguments.

He notes Euclid's usage: Say 'If p', and then make further deductions that depend on p, but without mentioning p. (This is a correct observation on Euclid *Elements* i as translated by Al-Nayrizī.)

He concludes: Throughout the relevant section of the argument, each proposition should be understood as beginning with an implicit 'If p', so for purposes of logical validation we should make this clause explicit.

Read: 

Think: 

The $(p \to p)$ at right top is an axiom and can be discarded.
In effect this is the natural deduction rule $\to$I.

In fact *Qiyās* Chapter vi witnesses to an unstated heuristic:

> A valid syllogism normally remains valid if additions are consistently made to its terms.

Ibn Sīnā even follows this heuristic to try to prove applications that don't work.
But we can formalise part of the idea behind the heuristic as a valid metatheorem of logic:

“Ibn Sīnā’s Rule” (in standard first-order logic):

Let T be a set of formulas and ϕ, ψ formulas.
Let $\delta(q)$ be a formula in which q occurs only positively, and q is not in the scope of any quantifier on a variable free in some formula of T. Suppose

$$T, \phi \vdash \psi.$$

Then

$$T, \delta(\phi) \vdash \delta(\psi).$$

This is a very strong rule, much stronger than the monotonicity rules (dictum de omni et nullo etc.) followed by the western Scholastics.

One reason for the strength of the rule is that,
unlike the standard rules of syllogism,
it allows us to carry out inferences at any syntactic depth in the sentences.

We can do this by choosing the head terms at the appropriate depth,
regarding the higher syntactic levels as additions.

The chosen head terms then become black boxes for deduction, as Ibn Sīnā often suggests they should be.

This point is hard to develop briefly,
because it leads us right up to the frontiers of our understanding of the development of logic.

In Ibn Sīnā’s logic,
such things happen too often for it to be an accident.

Ibn Sīnā