

Affirmative and negative in Ibn Sīnā

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1 Dedication

[Omitted]

I thank Manuela Giolfo, Ruth Kempson, Amirouche Moktefi, Paul Thom and Kees Versteegh for helpful information and comments. There is a different take on some of this material in Baeck [4].

2 Aristotle on affirmative and negative

Aristotle in his *Categories* 10, 13a37ff, *De Interpretatione* 4f, 17a8–37; 10, 19b5–20b12 and *Prior Analytics* i.2, 25a1–13; i.46, 51b5–52b35 introduced a distinction between affirmative and negative (*kataphátikos* and *apophátikos*, or in noun form *katáphasis* and *apóphasis*). Aristotle's views are not my concern here. But already Aristotle's treatment of the notions raises some fundamental questions:

- (a) What is the point of the distinction? (It is built into Aristotle's predicative syllogisms through the classification of propositions, and hence into his modal syllogisms. But the notions appear in *Categories* and *De Interpretatione* without any reference to syllogisms.)
- (b) Is it just propositions that can be affirmative or negative, or can we also classify predicates as affirmative or negative? (The treatments in *Categories* and *De Interpretatione* seem to be about propositions, but the discussion in *Prior Analytics* i.46 extends the notions to predicates.)

- (c) Among propositions, is it only predicative ones that can properly be called affirmative or negative, or does the classification also apply to compound sentences? (*De Interpretatione* 5 can be read either way.)
- (d) What is the basis for the distinction that Aristotle makes in *De Interpretatione* 10 between, for example, denying that a man is just and affirming that a man is not just? (The question seems to be about which sentences containing a negation are affirmative and which are negative; so an adequate answer to (a) should go some way towards answering (d) too. But to complicate matters, Aristotle comes at similar examples from another angle when he introduces negated nouns like ‘not human’ under the name of ‘indefinite names’, as in *De Interpretatione* 2.)

Here ‘predicative’ propositions are the standard syllogistic ones, for example the affirmative ‘Every horse is an animal’, ‘Some horse is an animal’ and the negative ‘No horse is an animal’, ‘Not every horse is an animal’. Aristotle sometimes includes singular sentences along the lines of ‘Cleon is an animal’ (which is affirmative) and ‘Cleon is not a horse’ (which might be either affirmative or negative depending on the answer you give to (d)). A ‘predicative’ syllogism is one consisting of predicative sentences. Today we distinguish predicative propositions and syllogisms from modal ones, which have ‘necessarily’, ‘possibly’ or similar modalities added. Predicative syllogisms are often referred to as categorical syllogisms; it was Stephen Read who persuaded me that the same word should be used for the propositions as for the syllogisms.

One of the jobs of a commentator should be to answer questions like (a)–(d) above, rather than simply repeating what Aristotle says. But it does seem that Ibn Sīnā is the first commentator who is on record as taking these or related questions seriously. Before him, Aristotle’s pupil Theophrastus already felt that (d) needed further attention. Theophrastus introduced the term ‘transposed’ (*ek metathéseōs*) in connection with Aristotle’s examples of propositions that contain negation but are affirmative. But we don’t know whether he had a general definition of *ek metathéseōs*, or even what he thought was the purpose of the notion. (See [5] pp. 148–153 for the sources.) Theophrastus’ term gives the modern term ‘metathetic’. The Arabic logicians rendered it as *ma^cdūl*, which literally means ‘deflected’. The term seems not to have come through to the Latin West.

Here is a brief summary of some of Ibn Sīnā’s contributions to (a)–(d). On (a) he provides a new distinction between affirmative and negative sentences, namely the following two principles:

Affirmative Principle. In every true affirmative predicative sentence the subject term is satisfied (i.e. non-empty). (*ʿIbāra* 79.13)

Negative Principle. A negative predicative sentence is true when its subject term is not satisfied. (*ʿIbāra* 81.3f)

Together these two principles — if they are true — make it essential in logic to distinguish between affirmative and negative sentences. Below we will ask whether these principles are really new with Ibn Sīnā, what he meant by them, and why he regarded them as true.

As to (b), Ibn Sīnā’s views on Aristotle’s discussion in *Prior Analytics* i.46 are withering. Ignoring Aristotle’s own text, he quotes at length a very garbled discussion by an unknown commentator, and concludes

- (1) It’s fair to say that any explanation of all this is going to come from somebody other than me. At any rate there is nothing convincing about it in the commentaries. They just charge around randomly. (*Qiyās* 195.14–16)

We will note below that Ibn Sīnā normally regards ‘affirmative’ and ‘negative’ as a classification of *sentences* (spoken or written).

Ibn Sīnā devotes *ʿIbāra* 41.15–42.15, part of his commentary on *De Interpretatione* 5, to question (c). He begins by noting that when Aristotle restricts affirmation and denial to ‘simple’ as opposed to ‘compound’ statements, he could be using ‘simple statement’ to mean predicative statement, or he could be using it to mean propositions as opposed to syllogisms (which are compounded from propositions). In the second case, propositional compounds have to be classifiable as affirmations or denials. Rather surprisingly, Ibn Sīnā suggests that ‘Either *S* or *T*’, read as an *exclusive* disjunction, should be counted as an affirmation. To us there is something negative about exclusive disjunction; but we saw in (d) that the presence of a negation doesn’t automatically make a sentence a denial.

Ibn Sīnā’s response to (d) is perhaps his most interesting. For comparison, consider the views of Ibn Sīnā’s predecessor Al-Fārābī, a philosopher for whom Ibn Sīnā had great respect — though for Ibn Sīnā, respecting someone and agreeing with them were never the same thing. Al-Fārābī notes that Aristotle’s own examples of metathetic propositions all involve a phenomenon which is found in Greek but

- (2) hardly exists in the Arabic language, except as an imported irregularity:

we can attach a negative particle to a noun or adjective so that the compound behaves as a single word, which (following Aristotle) he calls an ‘indefinite noun’. He remarks that the phenomenon is found also in Persian and Syriac.

- ... the communities that use them do not count them as phrases; indeed, to them their shapes are the same as those of single expressions: they behave like single expressions and they inflect like single expressions. (*Shorter Treatise on ^cIbāra*, trans. Zimmermann [17] p. 222)

He goes on to suggest that these indefinite nouns should be regarded as ‘affirmations’ (note the implied answer to (b) above) because they translate into Arabic expressions that don’t contain a negative particle at all.

Al-Fārābī has a good deal to say about indefinite nouns and metathetic propositions; they are the meat of pages 105–141 of his *Commentary on ^cIbāra* (pp. 100–137 of Zimmermann [17], see also Thom [16]). But his explanations cited above make it a mystery what he thinks he is talking about. Since the relevant phenomenon ‘hardly exists in Arabic’, is he talking about doing logic in Greek (or perhaps also in Persian or Syriac)? Or is he recommending a reform of Arabic that would introduce these indefinite nouns? (Al-Fārābī’s failure to detach himself from Greek can be put in a wider context; see the discussion in Zimmermann [17] pp. cv–cxxxix.)

Contrast this with Ibn Sīnā’s approach. For Ibn Sīnā the distinction between affirmative and negative should apply across all languages, including ‘possible’ ones (*^cIbāra* 79.2f). Since each language handles negations in its own way, the users of a language will need to use their native intuition (*šū^cūr*) to determine which constructions give affirmative sentences and which give negative (*^cIbāra* 79.8). We infer that for a particular language, the dividing line between affirmative and negative is an empirical fact. But then which empirical fact is it? What is the language user’s intuition supposed to be detecting? Below I will argue that there is a reasonable candidate, with due deference to the dangers of anachronism. It’s worth remarking that Ibn Sīnā’s appeal to user’s intuition, though quite natural for a modern linguist, was unusual for the Arabic linguistics of his day, which was heavily corpus-based.

3 Definitions

We have four words to understand: *’ijāb* ‘affirmation’, *salb* ‘denial’, *mūjjib* ‘affirmative’ and *sālib* ‘negative’. As a rough estimate of the importance

of their importance, here is a tally of the number of times each of them appears per ten pages of the Arabic of *ʿIbāra* and *Qiyās*:

	<i>ʿijāb</i>	<i>mūjib</i>	<i>salb</i>	<i>sālib</i>
<i>ʿIbāra</i>	9	7	14	5
<i>Qiyās</i>	3	8	5	9

In this section I sketch how Ibn Sīnā seems to understand the relations between the four words. I beg leave to be rather dogmatic; the full reasoning would multiply the length of this paper by ten.

From Ibn Sīnā’s discussion at *Maqūlāt* 255.11–14, read with some charity, it appears that the basic notions are ‘ x is affirmed of y in z ’ and ‘ x is denied of y in z ’, where z is a sentence. Translating Ibn Sīnā’s own clunky definitions into modern formats, we have:

- (4) x is affirmed of y in $z \Leftrightarrow z$ is a sentence which asserts of y that it satisfies the description x .

In moving from ‘is affirmed’ to ‘affirming’ or ‘affirmative’, we suppress the first and second arguments, in effect by quantifying them out:

- (5) z is affirmative $\Leftrightarrow z$ is a sentence which asserts, for some x and y , that y satisfies the description x .

At *Maqūlāt* loc. cit. Ibn Sīnā suggests that ‘denied’ and ‘negative’ can be treated in the same way, I presume by putting ‘doesn’t satisfy’ in place of ‘satisfies’.

The two nouns ‘affirmation’ and ‘denial’ are harder to pin down, not least because Ibn Sīnā himself is not wholly consistent in his use of abstract nouns. There are some disheartening contradictions, such as

- (6) The two mutual opposites in affirmation and denial are not the affirmation and the denial [themselves]. (*Maqūlāt* 256.1f)
I mean that affirmation and denial are genuinely mutual opposites. (*ʿIbāra* 43.8f)

I think that in the sentence from *Maqūlāt*, Ibn Sīnā is using ‘affirmation’ and ‘denial’ as names for the *meanings* of ‘affirmative’ and ‘negative’ respectively. In the sentence from *ʿIbāra* he is probably using ‘an affirmation’ to mean ‘an affirmative sentence’, and likewise with ‘a denial’. But there may be better explanations.

It might seem more sensible to translate *salb* as ‘negation’ rather than ‘denial’, to match ‘negative’. But in practice this translation could mislead.

For example at *ʿIbāra* 78.12 we have to read ‘the predicate is denied’ (i.e. denied of the subject). To say that the predicate is ‘negated’ would suggest that it has a negation at the beginning of it, which is precisely not what Ibn Sīnā wants to say here. But in some other contexts we do have to translate *salb* as ‘negation’, for example in ‘particle of negation’.

Ibn Sīnā is clear throughout this discussion in *Maqūlāt* that affirmation and denial have to do with *sentences*, and a sentence (*qawl*) is for him a linguistic object. The fuller discussion of affirmation etc. in *ʿIbāra* is also in terms of sentences. (He does say ‘proposition’ (*qaḍīya*) at *Maqūlāt* 255.17 and twice at 256.2, and indeed many times in *ʿIbāra*. But reading *qaḍīya* as a linguistic object is a lot easier than reading *qawl* as a kind of meaning; for example at *ʿIbāra* 33.5 he gives a sentence and calls it both *qawl* and *qaḍīya*.) It should follow that in spite of their basic status in Ibn Sīnā’s logic, neither ‘affirmative’ nor ‘negative’ is a notion inherited by logic from First Philosophy, the part of metaphysics that establishes basic notions and facts about existence and ideas. If ‘affirmative’ and ‘negative’ were such notions, we would expect to find them discussed in the *Ilāhiyyāt*, the volume of Ibn Sīnā’s *Šifā’* dealing with metaphysics; but they are not mentioned there, as Goichon’s lexicon [6] confirms.

Nevertheless the definitions given above rest on the notion of an idea satisfying the description given by another idea, and this is undoubtedly a notion from First Philosophy. If I interpret Ibn Sīnā correctly, he intends that the difference between ‘affirmative’ and ‘negative’ lies at the level of meanings; but for any particular language, the question which sentences meet the criterion for being affirmative or negative is a matter of linguistics.

At *ʿIbāra* 42.16f Ibn Sīnā offers separate and parallel definitions of affirmation and denial for predicative sentences. Nevertheless at *ʿIbāra* 34.11f he has already explained that denial has to be defined in terms of affirmation and not vice versa. I think his choice for the order of definition is unfortunate. Some sentences are marked out as having negation in a certain dominant position in them; these sentences are called negative. The other sentences, the unmarked ones, don’t need a special name at all.

4 The distinguishing criterion

Ibn Sīnā’s discussion of the difference between denial and metathesis in *ʿIbāra* 78.13–19.10 should be read in the light of his general views about the structure of meanings. For predicative sentences he adopted a sort of crude X-bar theory at the level of meanings. The sentence has two descriptive

components: a subject (today we say Noun Phrase, NP) and a predicate (Verb Phrase, VP). Each of NP and VP consists of a head descriptive term together with various adjuncts or attachments (*lawāḥīq, ziyādāt, šurūṭ*). In a metathetic sentence there is a negation, but it is an adjunct of VP. (At *ʿIbāra* 81.12 he briefly mentions another kind of metathesis where the negation is an adjunct of NP.) In a true denial the negation is not incorporated into NP or VP; instead it controls the relationship between the two.

This is one of a number of places where Ibn Sīnā’s language of adjuncts and attachments seems to cover material that today we would handle in terms of scope. Could it be that a negative sentence is one which has a negation whose scope is the whole sentence, whereas a metathetic affirmative sentence is one where a negation occurs but has its scope confined within one of NP and VP? Thus at *ʿIbāra* 78,15 Ibn Sīnā contrasts the two cases where

- (7) the particle of negation is included so as to deny the predicate [of the subject], or . . . it is included so as to be a part of the predicate, making the predicate the [combined] whole.

At *ʿIbāra* 79.8f he talks of the copula (which joins NP to VP) ‘taking precedence over’ (*tuqaddamu*) the negation, with the effect that the sentence is affirmative. These feel to me like scope distinctions.

Can we make sense of this in terms of logical scope of negation? I think not. Put briefly, the chief criterion for an occurrence of a descriptive term X to lie in the logical scope of an occurrence P of a negation in a sentence $\phi(X)$ is that removing P reverses the monotonicity of the occurrence, in the following sense. We say that the occurrence has upward monotonicity if $X \subseteq Y$ and $\phi(X)$ together imply $\phi(Y)$; it has downward monotonicity if $Y \subseteq X$ and $\phi(X)$ together imply $\phi(Y)$. Ideas of this kind played a major role in Scholastic thinking about syllogisms and were sometimes taken as a general justification for syllogistic reasoning [15]. Logical scope in this sense is unlikely to have been what Ibn Sīnā intended, because he barely ever shows the slightest awareness of monotonicity. He makes no distinctions along the lines of distributed/undistributed. Quite generally he seems blind to the logical scopes of negations. In [9] I give some evidence on this and suggest some reasons for this blind spot.

On the other hand there is a distinction in modern syntactic literature that seems to match Ibn Sīnā’s distinction rather closely. This is the distinction introduced by Klima [14] between sentence negation and constituent negation. In sentence negation the negating expression includes the whole sentence within its syntactic scope; in constituent negation it includes only

a constituent of the sentence. It's at first sight plausible that Ibn Sīnā's negative sentences are Klima's sentence negations and Ibn Sīnā's affirmative metatheses are Klima's constituent negations.

Klima suggested some criteria for this distinction, in terms of where the particle of negation allows us to add expressions of 'negative polarity' without violating grammaticality. Later authors proposed other criteria. The criteria don't always agree, and the differences are interesting for us. Take for example the two sentences

- (8) (i) Some birds don't fly.
(ii) Not all birds fly.

Adger offers the criterion that a negation is 'sentential' if it 'simply denies the truth of the non-negated version of the sentence' ([2] p. 176). We note that (i) doesn't simply deny the truth of (i') below, but (ii) does simply deny the truth of (ii'):

- (9) (i') Some birds fly.
(ii') All birds fly.

So by Adger's criterion (ii) is an instance of sentence negation but (i) is not. By contrast one of Klima's main criteria is that in cases of sentence negation one can grammatically add an affirmative tag question, but with constituent negation one can't. Thus:

- (10) (i'') Some birds don't fly, do they?
(ii'') Not all birds fly, do they?

By Klima's criterion, (i) and (ii) are both instances of sentence negation. It's clear why these examples are interesting for us: both (i) and (ii) illustrate standard English translations for Aristotle's negative existentially quantified predicative statements. Should we infer from Adger that (i) is really affirmative, or is Klima's criterion enough to certify it as a denial? The reader might like to run (i) and (ii) past the further criteria proposed in Haegeman [7].

I think I can see what is happening here. Throughout the 78 pages of Klima's seminal paper, he limits his examples almost entirely to predicative sentences (today we would say single-clause sentences), and moreover very few of his NPs carry explicit quantifiers. So he treats the NPs as referring to fixed individuals or sets, and as a result his criteria are not designed for determining whether the NPs lie within the scope of the negation. Both Haegeman's and Adger's criteria do a better job in this direction. (I think

that as stated, Adger's criterion will run into trouble with Klima's negative polarity items, but that's irrelevant for us.)

However, both Haegeman and Adger generally continue the restriction to single-clause sentences. Consider another logically relevant sentence:

(11) It's possible that no green things conduct electricity.

This is not the denial of 'It's possible that green things conduct electricity', so by Adger's criterion it is an example of constituent negation. Would Ibn Sīnā count it as negative? The evidence here is interestingly complex. Aristotle lifted the classification of predicative sentences to their modalised versions, so he would have had to regard this sentence as negative. But Ibn Sīnā normally — though not universally — tacks the modality onto the end of the sentence, as it were

(12) No green things conduct electricity, with possibility.

By Adger's criterion I still make this a constituent negation, but the crucial difference is that in (12) the main clause is undoubtedly negative, whereas in (11) it surely has to count as affirmative.

It does seem that the distinction between sentence negation and constituent negation is problematic for compound sentences. In English the only way of making sure that a negation encloses both clauses of a compound sentence, for example a conjunction '*S* and *T*', is to preface it with 'It is not the case that ...', which shrinks the compound sentence down to a single nominal 'that ...'. I don't want to put words into the mouths of the syntacticians, but it seems that this could be grounds for arguing that in English any genuine compound proposition has to be reckoned affirmative. If the same applies to Arabic, then we have a confirmation of Ibn Sīnā's treatment of exclusive disjunctions as affirmations, which we mentioned in §2.

In sum: Ibn Sīnā's classification of some sentences as affirmative and others as negative makes reasonable sense if we take his 'negative sentences' to be sentences containing a negation whose syntactic (rather than logical) scope is the entire sentence. His metathetic affirmations then fall into place as sentences where a negation occurs but has a more limited syntactic scope. This notion of syntactic scope of negations is not unproblematic even for modern syntactic theory, but there is an overlap between the cases that are difficult today and those where Ibn Sīnā's views are unclear.

There remains the question why Ibn Sīnā believed the Affirmative Principle and the Negative Principle. Although the earliest statement that we

have of the Affirmative Principle seems to be Ibn Sīnā's own, he signals that it had been discussed earlier, though it was denied only by some deranged individuals who 'worked themselves up into a state' about it (*ʿIbāra* 80.3f). So evidently he thought it was the orthodox position. How could he come to think this?

Ibn Sīnā will have had in front of him the statements of Aristotle and Al-Fārābī of the Affirmative Principle *for singular sentences*:

- When Socrates doesn't exist at all, it is not true either that Socrates is ill or that he is healthy. (Aristotle *Categories* 10, 13b18f; I translate from Iṣḥāq's Arabic version)
- (13) But if the answer 'no' is given to the question whether Socrates is wise while Socrates does not exist, we may not turn it into the metathetic affirmation 'Socrates is not-wise', but must put it negatively as 'Socrates is not wise'. (Al-Fārābī *Shorter Treatise*, trans. Zimmermann [17] p. 240)

So Ibn Sīnā will have been able to quote the authority of both Aristotle and Al-Fārābī for the Affirmative Principle in the case of singular sentences. Not only that, but the claim made by Aristotle and Al-Fārābī is very plausible: if Socrates is wise then surely Socrates does exist. Ibn Sīnā could reasonably suppose that for singular sentences the Affirmation Principle was an obvious truth which was widely accepted.

So the step that needs explanation is the generalisation to quantified sentences. There are passages, for example *ʿIbāra* 80.1f, where it seems that Ibn Sīnā is trying to infer the truth of the Affirmative Principle in general from its truth in the singular case. I won't try to defend his argument, but my guess is that Ibn Sīnā felt that there is no relevant difference between singular sentences and quantified ones. He might reason that (by definition) the only difference between a particular and a universal is that a particular carries a restriction on its meaning, to the effect that it can't legitimately be used if it describes more than one thing. How could this difference affect the Affirmative Principle?

The Negative Principle should probably be inferred from the Affirmation Principle and the thesis that every affirmative sentence has a negative contradictory negation and vice versa. The contradictory negation of a sentence is true if and only if the sentence is false.

We should say something about Ibn Sīnā's remark that the subject term of an affirmative predicative sentence must be satisfied 'either in the world or in the mind (*dīhn*)' (*ʿIbāra* 79.13f). The example that he gives immediately after stating the principle (the regular icosahedron, an example he uses for

the same purpose at *Qiyās* 21.7f, *Mašriqiyyūn* 12.5) shows at once why he adds the phrase ‘in the mind’. We can and do reason logically about regular icosahedra, as for example in Book 13 of Euclid’s *Elements*, and surely we are justified in doing this. But nobody knows for certain that there are any exact regular icosahedra anywhere in the world. There is more to be said about what he intends by ‘in the mind’, but it is not closely related to the distinction between affirmative and negative — and so I leave it unsaid here.

5 Affirmative and negative in syllogisms

The words ‘affirmative’ and ‘negative’ are very frequent in Ibn Sīnā’s theory of predicative syllogisms. Often these words are purely for purposes of classification. Each predicative sentence is identified by its subject term, its predicate term and whether it is (1) affirmative or negative and (2) universally or existentially quantified. Ibn Sīnā, following Aristotle, classifies modal syllogisms on the basis of their underlying predicative syllogisms, so this classificatory use extends to modal syllogisms too.

There are two uses of the words ‘affirmative’ and ‘negative’ that are not purely classificatory. One is their use in the conditions of productivity, and the other is their use for justifying conversion rules.

To begin with productivity: for Ibn Sīnā a predicative syllogism is a pair of predicative sentences (the ‘premises’) which is ‘productive’ in the sense that there is a third predicative sentence which takes a term from each premise, and which follows from the pair of premises. One of the skills of a logician is to know when a pair of predicative sentences is productive. Ibn Sīnā approaches the question as follows. The first step is to determine the terms of the two premises and label each of them as subject term or predicate term. This information determines the figure of the resulting syllogism if the pair is productive. Next, the logician must check which of the premises are affirmative and which are negative. With this information the logician can rule out some premise pairs as unproductive, by using the following ‘conditions of productivity’:

- (a) No productive premise pair consists of two negative sentences. (*Qiyās* 108.8)
- (b) In second figure every productive premise pair includes a negative sentence. (*Qiyās* 111.10)

- (c) In third figure every productive premise pair has affirmative minor premise. (*Qiyās* 116.15)

Then one can add the quantifiers, and some further conditions of productivity apply; some of these mention affirmative and negative.

The conditions (a)–(c) are a mixed bunch. Item (b) reflects a basic requirement on valid first-order entailments. The requirement in this case is that the predicate term which is common to the two premises should be negated in one premise and not in the other; so one need not mention affirmative and negative, though the formulation in (b) is convenient. (The underlying metatheorem is the Lyndon interpolation theorem, cf. [8] for more details.) By contrast item (c) is not so much a logical truth as a blemish in Aristotle’s system: the premise pair

- (14) Not every A is a B . No A is a C .

violates (c) but does in fact entail that not every non- B is a C . Aristotle’s system ignores this sentence-form.

We turn to the use of the notions of affirmative and negative to justify the conversion rules. For Ibn Sīnā the inference relations between sentences should be read off from the meanings of the sentences. So a logician must determine the meanings of sentences before describing their inference relations. The section of *‘Ibāra* partially translated below is aimed in this direction: Ibn Sīnā sets out some general principles and gives precise conditions for sentences with singular subject to be true or false.

One inference relation is A-conversion:

- (15) ‘Every A is a B ’ entails ‘Some B is an A ’.

Assuming the Affirmative Principle, ‘Every A is a B ’ gives ‘Some A is a B ’, which self-evidently entails ‘Some B is an A ’. Another inference relation is that

- (16) ‘Every A is a B ’ and ‘Not every A is a B ’ are mutually contradictory.

The tricky case is where there are no A s; in this case the first sentence is false by the Affirmative Principle and the second is true by the Negative Principle. (This assumes we are correct in counting ‘Not every A is a B ’ as a denial.)

The two inference relations (15) and (16) are fundamental to Aristotle’s system of predicative syllogisms. So it’s surprising that we have no evidence for the Affirmative and Negative Principles, or any other basis for

these two inference relations, before Ibn Sīnā. Perhaps if Al-Fārābī's lost longer commentary on the *Prior Analytics* is recovered it will throw some light on the matter.

We noted earlier that the distinction between affirmative and negative seems to be geared towards predicative sentences. But Ibn Sīnā didn't believe that syllogistic reasoning is in any way limited to predicative sentences. In fact he defined a larger class of syllogisms, called 'recombinant' (*iqtirānī*), which include the predicative syllogisms and depend on the same mental procedures as predicative syllogisms, but also include some propositional compounds. If the rules for recombinant syllogisms directly generalise those for predicative syllogisms, then we would expect that the Affirmative and Negative Principles generalise to the relevant propositional compounds. Does Ibn Sīnā think that they do? The evidence is conflicting.

In favour of the Affirmative Principle for some propositional compounds: at *Qiyās* 302.15–303.1 Ibn Sīnā cites the valid recombinant syllogism

- (17) In all cases, if q then r ; and in all cases, if q then not p . Therefore it is not always the case that whenever r then p .

(I believe these are correct translations, but I can't guarantee that they preserve Ibn Sīnā's intuitions about affirmative and negative.) He claims that (17) can be proved by 'conversion' of the first premise. He must have in mind here the proof of the analogous predicative syllogism Felapton; so the reasoning is that the first premise entails 'Sometimes both q and r '. If the analogy really holds, then he is relying on an Affirmative Principle of the form

- (18) If the sentence 'In all cases, if q then r ' is true, then there is at least one case in which q is true.

At *Qiyās* 303.3–5 there is another example of the same point; it is a recombinant analogue of Darapti.

As evidence in the other direction, Ibn Sīnā remarks at *ʿIbāra* 42.9 that a negated exclusive disjunction can be paraphrased as an affirmative predicative proposition. This is hard to reconcile with the dichotomy expressed by the Affirmative and Negative Principles.

It does seem that Ibn Sīnā says seriously incompatible things in different places. But there are other grounds for believing that Ibn Sīnā didn't intend to produce a monolithic system of logic. His attitude was more along the lines 'If you mean X and accept principle Y then you can deduce Z ', and he was not too concerned to tell us what principles we should accept. The one exception to this is the system of predicative syllogisms, which he regarded as sacrosanct and definitive.

6 Translation

What follows is a translation of Ibn Sīnā's *ʿIbāra*, pages 76 to 87 in the Cairo edition [11]; the division into paragraphs is mine. I thank Amirouche Moktefi for his invaluable advice and help; blame me for any errors of language or judgment. [For reasons of space the translation in the Festschrift paper will go only up to the end of [2.1.18].

/76/ Second part
Section ii.1

Two-part and three-part propositions; metathetic, simple and privative propositions and the connection between the contradictories of these three kinds of proposition in the singular and unquantified cases 76.5

[2.1.1] Propositions are of two kinds. Either they contain an explicit copula (in the sense of 39.5 above) — and in some cases this copula mentions a time, but in others it doesn't — or they don't contain an explicit copula. If the copula is explicit, the proposition is said to be three-part, and if it is not explicit, the proposition is said to be two-part. 76.8

[2.1.2] Two-part propositions are shorter than they ought to be, except when their predicates are verbs. This is because it's reasonable to say that verbs contain their own copula, since verbs have a syntactic part that signifies their subject. So the only case where a copula is needed for signifying the connection of the predicate to the subject is the case where the predicate is a separate and self-contained noun. When the verb contains something that signifies that there is a subject, the verb doesn't require a copula in the same way as basic nouns do. /77/ In this respect, participles behave like verbs. But verbs themselves don't express declarative propositions, because even if they signify that there is a subject, they don't contain anything to specify which subject. The need that they do satisfy is just the need for something that couples up [the predicate] with a specified subject by a copular expression that points in its direction. 76.10

[2.1.3] Arabic has this copula in the form of a noun-like particle, and it also has it in the form of a verb-like particle. In the sentence 77.3

(19) Zayd $\left\{ \begin{array}{c} huwa \\ \text{is} \end{array} \right\}$ alive.

the word *huwa* refers to Zayd, though the only thing that it expresses about him is that he is indicated. But in the sentence 77.5

(20) Zayd $\left\{ \begin{array}{c} kāna \\ \text{was} \end{array} \right\}$ alive.

the word *kāna* doesn't contain anything to specify Zayd. For this reason the experts in their language say that we have an ellipsis, and the verb means

(21) $\left\{ \begin{array}{c} huwa \\ he \end{array} \right\}$ was (alive).

Other languages handle it differently.

[2.1.4] So propositions have one of three ranks. Those of the first rank have an element that signifies and specifies the connection, those of the second rank have an element that signifies the connection but doesn't specify it, and those of the third rank don't have an element that signifies a connection at all. The propositions of the third subdivision are the perfect two-part propositions. The propositions of the other two subdivisions are the three-part propositions, but those of the first rank are the perfect three-part propositions, and those of the second are the propositions that are three-part but not perfected three-part. In general a three-part proposition is one in which the copula is stated explicitly, as in the sentence

(22) The human $\left\{ \begin{array}{c} yūjadu \\ is \end{array} \right\}$ just.

or the sentence

(23) The human $\left\{ \begin{array}{c} huwa \\ is \end{array} \right\}$ just.

The expressions *yūjadu* and *huwa* are not included as predicates themselves, but rather to signify that the subject 'has' the predicate. The expression *yūjadu* signifies that the subject 'has' the predicate at a time in the future. The expression *huwa* signifies without further qualification that the subject 'has' the predicate. The copula ['is', in either form,] signifies the connection of the predicate, and the quantifier ['the'] signifies the quantity of the subject. Moreover the part that forms the copula counts as being on the predicate side of the proposition, and the quantifier /78/ counts as being on the subject side.

[2.1.5] If the proposition is three-part and the particle of negation is linked to it, then one of two cases holds: either the particle of negation is put in front of the copula, or the copula is put in front of the particle of negation. An example of the first is the sentence:

(24) Zayd $\left\{ \begin{array}{c} laysa \\ is not \end{array} \right\} \left\{ \begin{array}{c} yūjadu \\ is \end{array} \right\}$ just.

and an example of the second is the sentence

(25) Zayd himself $\left\{ \begin{array}{c} yūjadu \\ is \end{array} \right\}$ unjust.

[2.1.6] Putting the particle of negation before the copula negates its coupling, 78.3

which makes the proposition a genuine denial. But if the copula is put before the particle of negation, this makes the particle of negation a part of the predicate, so that the predicate is not 'just' by itself, but the whole expression 'un-' and 'just'. Then the expression 'is' causes the whole of 'unjust' to be predicated of 'Zayd' affirmatively as if the proposition said 78.5

(26) Zayd fits the description 'unjust'.

So it would be legitimate to negate this proposition [in turn] by putting a second particle of negation in front of the copula, to get the proposition

(27) Zayd $\left\{ \begin{array}{l} \textit{laysa} \\ \text{is not} \end{array} \right\} \left\{ \begin{array}{l} \textit{yūjadu} \\ \text{is} \end{array} \right\}$ unjust.

[2.1.7] [Aristotle] stipulates at this point [(*De Interpretatione* 10, 19b26)] that there are two kinds of affirmative proposition and two kinds of negative proposition. The sentence 78.8

(28) Zayd is just.

is the opposite of the sentence

(29) Zayd is not just.

and these two propositions are [known as] simple affirmative and simple negative. The sentence 78.10

(30) Zayd is unjust.

is opposite to the sentence:

(31) Zayd is not unjust.

and these two are [known as] affirmative metathetic and negative metathetic. A proposition is described as metathetic, or transposed, when its predicate is an indeterminate noun or an indeterminate verb. If that predicate is affirmed [of the subject], the proposition is affirmative metathetic; if the predicate is denied, the proposition is negative metathetic.

[2.1.8] If the proposition has no copula, making it two-part, then the negation sign is linked to its predicate. This in itself is not an indication either that the 78.13

particle of negation is included so as to deny the predicate [of the subject], or that it is included so as to be a part of the predicate, making the predicate the [combined] whole. But with /79/ some particles of negation, and at least in our languages particularly those put in front of a predicate which is a verb, the thought that prevails is that the particle of negation denies the connection. We don't know how it goes in other actual or possible languages. Maybe some negation expressions that are attached to verbs in these languages carry an inflection or some other kind of marker to show the intended meaning. 78.15

[2.1.9] Likewise of those Arabic particles of negation which go in front of nouns, some are more suggestive of denial and some of metathesis. Thus it's reasonable to say that *laysa* fits better with denial and *ḡayr* fits better with metathesis. But when either of these is preceded by *mā*, the result is an affirmative expression. For example if someone says 79.5

(32) $A \left\{ \begin{array}{l} \textit{laysa bi-} \\ \textit{it is not} \end{array} \right\} B.$

the mind can get the feeling that the meaning of this sentence is closer to

(33) $A \left\{ \begin{array}{l} \textit{huwa} \\ \textit{is} \end{array} \right\} \left\{ \begin{array}{l} \textit{mā} \\ \textit{what} \end{array} \right\} \textit{is not } B.$

Then in the mind the copula *huwa*, even though it isn't stated explicitly, takes precedence over the negation, and so the mind feels the phrase as affirmative. If there is no [explicit] indication, then the obvious reading is that the proposition is binary and has no copula. 79.10

[2.1.10] /79/ Now we must move on to settle something that really does need to be settled under this head. We say: The criterion for [the proposition to express] an affirmation is that the subject 'has' the predicate. It's impossible to reckon that an idea which is unsatisfied 'has' an[other] idea. So every subject of a [true] affirmative proposition is satisfied — either in the world or in the mind. If one says 79.11

(34) Every icosahedron is an *X*.

what is meant by this is that every icosahedron, regardless of where it is found, is an *X*. That doesn't mean that every /80/ nonexistent icosahedron is a nonexistent *X*. If the icosahedron doesn't exist then it doesn't satisfy any description, because being nonexistent it can't satisfy any description. If it doesn't exist then how could it be the case that it satisfies something? — except for those people who work themselves up into a state of willing nonexistent things to have descriptions that are well-defined but not satisfied, so that according to them an idea can be well-defined and not satisfied. We are talking about 'well-defined' in the normal sense of this word, and what we intend by 'satisfied' in the normal sense is no different from this. These people are entitled to use 'satisfied' to mean whatever they choose. 80.5

[2.1.11] But [the fact is that] the mind judges something to hold affirmatively of certain ideas, either on the basis that they themselves in their state of being satisfied satisfy the [relevant] predicate, or else on the basis that they satisfy the predicate in the mind, not just as mental entities, but rather on the basis that when they are satisfied, they satisfy this predicate. If they are satisfied only in the mind at the time when the mind judges this, then in this case it is impossible for us to say (for example) 80.6

(35) *A* is satisfied by *B*, not in the mind but in the facts themselves.

[The subject] isn't satisfied in the facts themselves, so how could an idea be satisfied by it? 'Affirmation' and 'assertion' in the usual sense are the asserting that something holds of an idea, i.e. that it is satisfied by the idea. Likewise 'denying' in the usual sense is the un-asserting that something holds of an idea, which of course amounts to the thing not being satisfied [by the idea]. It's clear from this that there is no affirmation at all unless the subject is [taken to be] as we said. 80.10

[2.1.12] In the case of ideas that aren't satisfied in any way, there is a usage in which one asserts something of them, and it looks as if the mind judges it to be the case that they are X's; [but] what is meant is that if they were satisfied /81/ in the mind, then they would be X's. For example one says in this sense that 80.13

(36) The void consists of dimensions.

[2.1.13] One can truthfully deny a thing of an idea both when the idea is satisfied and when it is unsatisfied. The distinction made earlier between the simple negative proposition and the metathetic affirmative proposition is that the subject of the [true] simple negative proposition can be either satisfied or unsatisfied. It's correct to deny something of an idea that is unsatisfied. But it's not correct to assert an affirmative metathetic proposition whose subject is unsatisfied. 81.1

[2.1.14] This being agreed, some people go on to try to distinguish between the affirmative metathetic proposition and the simple negative proposition by making the metathetic signify the absence of something that it would be natural to find in the proximate genus or the remote genus or the species, so that they say: The expression 'unjust' is true of things that lack justice but have it in their nature, or that of their genus, to be just. Thus one says about brute animals that they are irrational, and one says about the rational soul that that it is immaterial, and in both cases the meaning in question is satisfied by some things in the same genus. Also some people say that 'unjust' is parallel to 'oppressive or intermediate [between just and oppressive]', and 'non-seeing' is parallel to 'blind'. So 'non-seeing' and 'blind' amount to the same thing, so that according to them it's not correct to say of the Eternal that he is non-seeing. This is what these people say. But the correct account of the matter will be clear from the examples we are about to give. 81.5

[2.1.15] We say: in the discourse 81.10

(37) Every body is a thing not found in a subject,
and everything that is not found in a subject is a substance,
so every body is a substance. 81.12

what we deduced does follow. And it's a known fact that the two propositions are affirmative, and that the expression 'not' is taken as a part of the predicate, so that it is repeated as a part of the subject. The conclusion does follow, and that's a fact. But /82/ 'not' applied to 'found in a subject' doesn't in any way indicate the absence of something that could have existed in the genus of [SUBSTANCE], since [SUBSTANCE] doesn't have a genus at all except that [EXISTING] is a kind of genus for it. If instead of that he had taken the metathetic to signify the absence of something that one would naturally expect to find somewhere in the whole of reality, that would get somewhere nearer the truth. 81.15

[2.1.16] Really the metathetic proposition is one where the particle of negation is part of the predicate, regardless of what form the predicate takes. When we take the particle of negation, and what would be the predicate if it was taken separately and on its own, together as a single thing, and then we assert this thing of the subject through the affirmative copula, the proposition composed in this way is affirmative. The matter and its quality are something else. 82.4
82.5

[2.1.17] Also they have read, in [Aristotle's] *Categories* and the subsequent [commentaries], that the expression 'non-human' is not applied to some [kinds of] thing to the exclusion of others — [in particular] it is not applied to existing things to the exclusion of nonexistent ones; and that it can legitimately be used as a predicate. One thing that is bound to cause confusion is that the requirement that the subject of an affirmative metathetic proposition has to be satisfied is not because the expression 'unjust' itself requires this, but because the truth of the sentence requires it, regardless of whether [UNJUST] applies both to ideas that are satisfied and ideas that aren't, or applies only to ideas that are satisfied. 82.8
82.10

[2.1.18] One should know that the distinction between the sentence 82.13

(38) X is a non- Y .

and the sentence

(39) X is not a Y .

is that the simple negative proposition [(39)] is broader than the metathetic affirmative proposition [(38)], in that it is true if [the subject] is and is taken to be unsatisfied, whereas the affirmative metathetic proposition is not true in this case. It is said truthfully that 82.15

(40) The griffin is not a thing that can see.

but it is not truthfully said that

(41) The griffin is a thing that can't see.

This is because 'the griffin' is a name that signifies a meaning in the imagination and there is nothing satisfying it in the real world. /83/

[2.1.19] We move on and say: Each simple well-defined predicate either has a contrary or it doesn't. If it does have a contrary, then either there is an idea which is intermediate between the predicate and its contrary, or there isn't. Also a subject X is either satisfied or

(1) unsatisfied (taken in terms of when it is unsatisfied).

If X is satisfied and we suppose that some idea Y bears some relation to it as a predicate, then either:

(2) Y will be true of X , or

(3) the contrary of Y will be true of X , or

- (4) something intermediate between *Y* and its contrary (if there is such a thing) will be true of *X*, or
- (5) both *Y* and its contrary will be potentially true of *X* at the same time, like a puppy whose eyes haven't yet opened, so that it is both potentially blind and potentially seeing, or
- (6) *X* is not receptive of either *Y* or its contrary, as for example the soul is not receptive of either the colour white or the colour black (or indeed any colours between these).

The sentence

(42) Zayd is not just.

is false only in case (2) and true in the remaining cases. The sentence

(43) Zayd is unjust.

is true if [Zayd] is oppressive [(case (3))] or cases (4), (5) or (6) hold (though there is some disagreement about case (6)). It is false when he is just [(case (2))] and when there is no such person [(case (1))]. 83.10

[2.1.20] The custom was to refer to the worse of two opposites, both in the vernacular and with the obvious standard name for it in topics like the present one, as a 'privation', regardless of whether it is a genuine privation like blindness and mercilessness, or a contrary like oppressiveness. 83.10

[2.1.21] The affirmative privative proposition falls in between the affirmative metathetic proposition and the simple negative proposition. The relation of the two privative propositions [(i.e. affirmative and negative)] to the two metathetic propositions is that the affirmative privative proposition overlaps with the affirmative metathetic proposition, and the negative privative proposition overlaps with the negative metathetic proposition. When the affirmative privative proposition is true, that makes the affirmative metathetic proposition true too, but not conversely, because the affirmative metathetic proposition is broader than the affirmative privative. When the negative metathetic proposition is true then so is the negative privative, but not conversely. Thus when the sentence 83.12

(44) Zayd is not unjust.

is true, then so is the sentence

(45) Zayd is not oppressive.

But the converse fails. /84/ It is not the case that when the sentence (45) is true then (44) is true. In fact (45) is true when Zayd is partly just and partly not just, and in cases (5) and (6), but (44) is not true in these cases.

[2.1.22] The relation of the two privative propositions to the two metathetic 84.3

propositions is that the affirmative privative proposition corresponds to the affirmative metathetic proposition and the negative privative to the negative metathetic, and they differ as broad to narrow. The relation of the two privative propositions to the two simple propositions is that the negative privative proposition corresponds to the affirmative simple proposition and the affirmative privative to the negative simple. Here is a table of these singular propositions, showing their content: 84.5

Zayd is just (2)	Zayd is not just (1), (3), (4), (5), (6)	/85/
Zayd is not unjust (1), (2)	Zayd is unjust (3), (4), (5), (6)	
Zayd is not oppressive (1), (2), (4), (5), (6)	Zayd is oppressive (3)	

[2.1.23] This will have made clear to you a way in which the two privative propositions behave like the [simple] affirmative and negative propositions, while the other two propositions behave differently. This is that with the privatives and the simples, the affirmative proposition is true in just one case and the negative proposition false in just one case. This is one way in which the two privative propositions resemble the two simples, and it is a difference between the two privatives and the two metathetics. 85.1

[2.1.24] Know that if X has a narrower range of truth than Y , then the contradictory of X has a broader range of truth than the contradictory of Y . This is because having narrower truth conditions is the same as having broader falsehood conditions, and having broader truth conditions is the same as having narrower falsehood conditions. The two contradictory negations [of X and Y] are both true when the original overlapping propositions [X and Y] are both false. The more cases there are in which a proposition is false, the more there are in which its opposite is true. Therefore, if a proposition p follows from a narrower proposition q , and not conversely, then the contradictory negation of q follows from the contradictory negation of p , and not conversely. When the broader proposition p is false, then the narrower proposition q is false too, but not conversely. When the narrower proposition q is true, then the broader proposition p is true, but not conversely. 85.10

[2.1.25] Let us set out a table of the unquantified propositions too: 85.10

The human is just	The human is not just
The human is not unjust	The human is unjust
The human is not oppressive	The human is oppressive

[2.1.26] The sentence

(46) The human is just.

is true when all humans are just, and when some of them are just and the remainder are not; it is false when there are no humans, and when there is no just human regardless of how they happen to be, allowing that they may be a mixture. 85.15

[2.1.27] The sentence 85.16

(47) The human is not just.

is true if /86/ there are no humans; or no humans at all are just, regardless of what else they are; or some humans are not just, regardless of what else they are, and regardless of whether the others are just or unjust or anything, including that some of them are nonexistent or oppressive or intermediate, or only potentially just, or not receptive to being just. It is false only when all humans are just. These two 'bent' propositions can coincide in being true in the same situation, which is a coincidence that one can't demand in general.

[2.1.28] And the sentence: 86.4

(48) The human is oppressive.

is true when all humans are oppressive, or some of them are oppressive and some of them are not just, in any way at all, and so (47) can be true whenever (48) is, regardless of how (48) is true. But the converse fails, because (47) can be true when all humans are neither just nor oppressive, regardless of how they happen to be, allowing that they may be a mixture, and in this case (48) is not true. 86.5

[2.1.29] The sentence

(49) The human is not oppressive.

is true when there are no humans, or no humans at all are oppressive, or only some of them are oppressive, and in general if some of them are nonexistent or just or intermediate or not receptive to being just, and the rest can be any way at all. It is false only when all humans are oppressive. So it is broader than the sentence (46). The sentence 86.9
86.10

(50) The human is unjust.

is true when no humans at all are just, regardless of what else they are, and including the possibility that they are a mixture; or some humans are not just regardless of what else they are — so long as some humans are oppressive or intermediate or only potentially just or not receptive to being just; it could be that only one human is just. It is false if there are no humans, or all humans are just. This is broader than the sentence (48) but narrower than the sentence (49). The sentence 86.15

(51) The human is not unjust.

is true when there are no humans, or all of them are just, or some of them are just regardless of what the remainder are, and in general even when some of them are nonexistent or just. It is false if none of them at all are just /87/ regardless of what else they are. It is true in more cases than the sentence (46) but narrower than the sentence (49) because the sentence (49) is true when there are no humans or all of them are just or some of them are just, and it is also true when all of them are

intermediate or only potentially just or not receptive to being just, and in this case the sentence (51) is false. So all the bent sentences agree in being true when some humans are just and some are not. 87.5

[2.1.30] And for the fellow-travellers: the affirmative simple proposition and the affirmative privative proposition coincide in being true when some humans are just and some unjust. The affirmative simple proposition and the affirmative metathetic proposition coincide when some humans are just, and [besides the just ones] there exist other humans, who can fit the other cases in any way. The simple negative proposition and the privative negative proposition coincide in being true when no humans at all are either just or oppressive, or some humans are just and some are oppressive. The simple negative proposition and the metathetic negative proposition coincide in being true when there are no humans, or some humans are just and some are not just. The affirmative privative proposition and the negative simple proposition coincide in being true when some humans are oppressive, with no restriction on those who are not oppressive. The negative privative proposition and the affirmative metathetic proposition agree in being true when [humans exist but] there are no just humans and no oppressive humans, or there are some just humans and some oppressive ones. 87.7
87.10

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