

## Ibn Sīnā's modal logic

Wilfrid Hodges

Hérons Brook, Sticklepath, Okehampton

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<http://wilfridhodges.co.uk/arabic20a.pdf>

We have four main sources for Ibn Sīnā's modal logic:

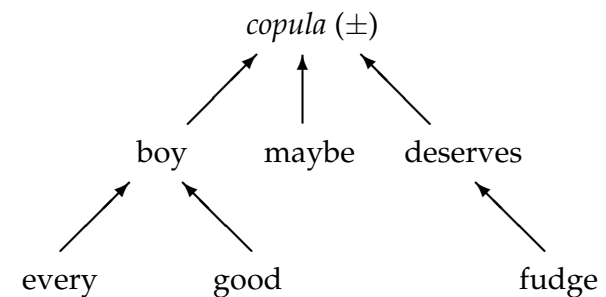
- (1) The *Middle Epitome*, c. 1013, available only in ms.
- (2) The logic of *Najā* ('Rescue'), also written c. 1013 but slightly revised for a later work. Translation *Avicenna's Deliverance Logic*, Asad Q. Ahmed 2011.
- (3) Books ii–iv of *Qiyās* in the *Šifā'*, mid 1020s.  
By far the fullest at about 140 pages.  
About a third in draft translation on my website.
- (4) The logic of *Išārāt wa-tanbīhāt*, late 1020s. Trans. *Ibn Sīnā Remarks and Admonitions: Logic*, S. Inati 1984.

For Ibn Sīnā, logic is a tool for checking the correctness of arguments.

More precisely it checks *inference steps*, starting from their premises (even if what mainly interests us is the conclusion).

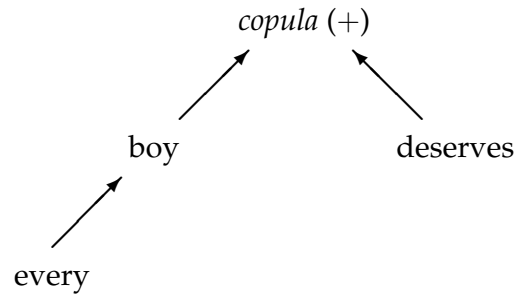
Paradigm case: two premises, both of them simple sentences (i.e. no subclauses).

Sample simple sentence, analysed à la Ibn Sīnā:



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The next step is to cut down to the core (subject word, predicate word, quantifier, affirmative/negative):



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Comparing the two premise cores tells us what figure (if any) the syllogism will be in, by checking where the same word appears in both premises.

Then a 'condition of productivity' (*šarṭu l-intāḥ*) tells us whether there is a conclusion.

If yes, then rules of 'following' tell us the quality and quantity of the conclusion, normally in terms of which premise the conclusion 'follows'.

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If yes so far, then we restore the adjuncts (*lawāḥīq, ziyādāt, šurūt*) that were stripped off the core earlier. We test whether the earlier conclusion still survives, and with what adjuncts.

This is *murā<sup>c</sup>ātu l-šurūt*, 'taking care of the conditions'.

This last stage is where we restore the mode.

There are a rule of productivity for the mode, and a rule identifying the *ibra*, i.e. the premise whose mode the conclusion follows.

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So far, no great disagreement with Aristotle. Probably Ibn Sīnā thought he was taking all this from Aristotle.

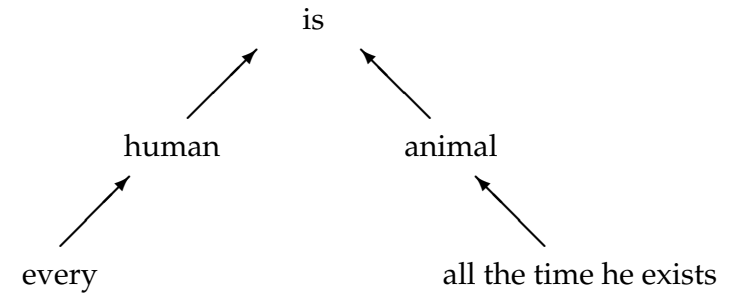
But now comes the major difference, which Ibn Sīnā repeatedly stresses:

The range of adjuncts appearing in normal scientific discourse is *very much more varied* than Aristotle ever admits.

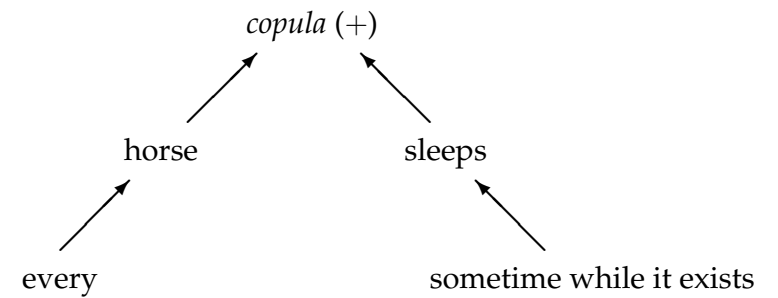
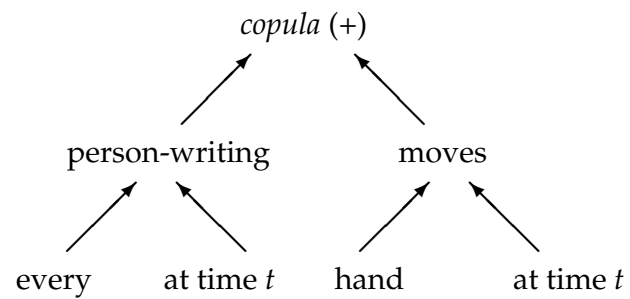
It includes masses more than just modes.

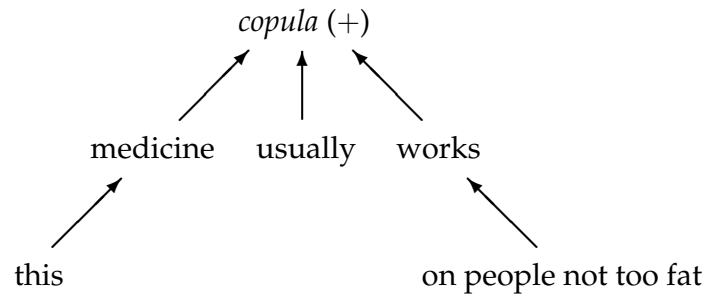
Examples:

Essential sentences:

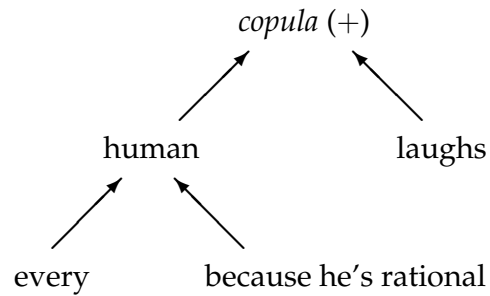
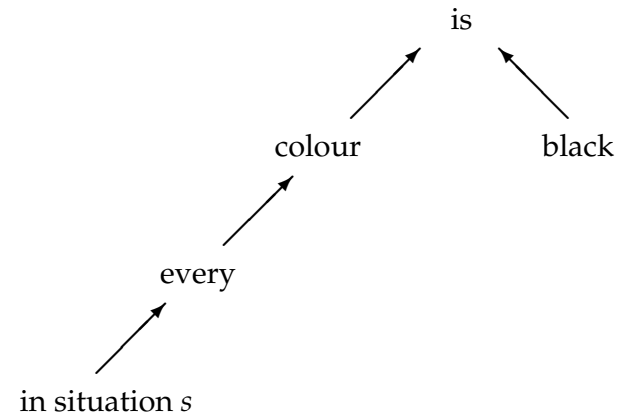


Descriptive sentences:





'Truth on the quantifier':



So when the stripped off adjuncts are restored, taking care of the conditions can be an open-ended task.

This is both because of the variety of sentence-forms, and also because each speaker's intention (*not* formal rules of language, Ibn Sīnā stresses) determines the meaning of the speaker's utterance.

Ibn Sīnā has no comprehensive procedure for it.

Instead:

1. Ibn Sīnā classifies the possible adjunctions into three groups, according as they are

- (a) necessity-like;
- (b) contingency-like;
- (c) absolute, i.e. neither of the above.

Default: all items in one group behave similarly.

Ibn Sīnā notes that already Aristotle groups ‘possibly’ and ‘usually’ with contingency, and ‘permanently’ with necessity.

2. Unlike Aristotle, Ibn Sīnā prepares for logical study of an expression by explaining its meaning.

*But* he never calculates the soundness of an inference by referring to the semantics.

This is deliberate: for him

$$M \models \phi \text{ and } M \models \psi \Rightarrow M \models \chi$$

involves the adjunction “ $M \models$ ” (i.e. truth on quantifier) and hence alters the perceived meaning.

Some of the rules of inference that he adopts are not valid for his own semantics.

Example: the device of putting  $\phi$  in place of ‘possibly  $\phi$ ’ as premise of an argument by *reductio ad absurdum*.

But he never claims that they are valid.

Instead his criteria for adopting inference rules tend to be that they are ‘natural’ or ‘close to our understanding’ or that they give ‘certainty’.

In short, his criteria are cognitive rather than logical.

The guarantee of ultimate correctness is not a calculation but *experience*: by applying the rule many times, we get to learn where it is safe and where not.

‘Be aware that most of what the First Teaching contains about mixtures of modalities consists of tests rather than authoritative rulings. The true facts about them will become clear to you when things that have been said about mixtures of modalities are mentioned or used in later topics, so that you can make a ruling on them in the light of what the facts compel.’ (*Qiyās* 204.10–12)

## Permanent and necessary in Ibn Sīnā

### Permanent = Necessary

Ibn Sīnā, *Najā* 29.8<sup>9</sup>:

The mode ‘necessary’ signifies holding permanently.

(Superscripts refer to the Handout.)

### Permanent ≠ Necessary

Ibn Sīnā, *Iṣārāt* Indication iv.4<sup>11</sup>:

The permanent is not the necessary.  
In fact ‘being a writer’ can be said not to hold of an individual at any time when he exists, let alone at times when he doesn’t exist; but this denial is not a necessity proposition.

(Here and below, a ‘necessity proposition’ is a proposition stating that something is necessary, as opposed to a proposition that is necessarily true.)

### Permanent ≠ Necessary

Ibn Sīnā, *Qiyās* 32.11–13<sup>7.(14)</sup>:

And we say

“Every thing that moves changes, with necessity”,

not that it permanently never did or will cease [to change], ... but while it continues to move.

With some writers (maybe Al-Fārābī, Russell) we have to learn to accept only local consistency. But Ibn Sīnā insists on having a coherent general picture, so his (many) inconsistencies demand explanations.

We will see that Ibn Sīnā himself identifies the problem above, and discusses it in some detail.

But first we should review what was available to Ibn Sīnā in the Aristotelian background.

Theophrastus (student and successor of Aristotle) wrote a *Prior Analytics*. In it he proposed three ways of understanding 'Every  $B$  is an  $A$ ' as a necessary/necessity proposition.

We know about this from

- (a) Alexander's commentary on Aristotle *Prior Analytics*<sup>1,2</sup>,
- (b) a 14th c Hebrew translation by Todros Todrosi of excerpts from an Arabic translation of a commentary by Themistius (4th c)<sup>3</sup>, and
- (c) Ibn Sīnā *Qiyās* 41.5–13<sup>6</sup>.

Passages mentioned by Sharples in *Liverpool Classical Monthly* 3 (1978) 89–91 may also be relevant.

From (a)–(c) we can infer that Theophrastus mentioned three readings of 'Every  $B$  is an  $A$ ' as a 'necessary' statement:

- (1) (Permanent reading) Every  $B$  is at all times an  $A$ .
- (2) (Subject reading) Every  $B$  is an  $A$  while the subject *hupárkhei*.
- (3) (Predicate reading) Every  $B$  is an  $A$  while the predicate *hupárkhei*, i.e. while the  $B$  is an  $A$ .

Ibn Sīnā *Qiyās* 41.5<sup>6</sup> refers to this as the 'three-way division' (*tatlīt fil-qisma*).

We discuss some of Ibn Sīnā's reactions to it.

### A. The predicate reading

Ibn Sīnā treats this reading with contempt, for two reasons.

First (as he comments in several places) it is not really a reading of 'Every  $B$  is an  $A$ ' at all, since we would never suppress the condition 'while it is an  $A$ '.

Second, 'Every  $B$  is an  $A$  while it is an  $A$ ' is tautologous and not known to play a role in any serious argument.

## B. The subject reading

Thanks to several ambiguities, three ways of taking Theophrastus' subject reading appear in the literature:

- (2a) (Essential reading) Every *B* is an *A* so long as it ('the *B*') exists.
- (2b) (Descriptive reading) Every *B* is an *A* so long as it is a *B*.
- (2c) (Nonempty reading) Every *B* is an *A* so long as there is a *B*.

Modern readers of Alexander usually translate as (2c).

## C. Ibn Sīnā versus Theophrastus

Ibn Sīnā in his discussion of the *taḥlīl* opts for the descriptive reading (2b), but elsewhere he frequently discusses the essential reading (2a).

The many places where Ibn Sīnā explains the difference between (2a) and (2b) could be evidence that Ibn Sīnā was the first person to distinguish clearly between them.

Sharples:

'It is perhaps questionable whether the early Peripatetics should be supposed to have been entirely clear about the distinctions between these different types of temporal qualification.'

Ibn Rušd, *Mas'ala* i.3<sup>12</sup>:

'I reckon that ... [Ibn Sīnā] thought that what had been said in order to distinguish different forms of absolute reality had been said in order to distinguish different universally quantified necessity propositions. ... So I reckon this is how this man fell into error. But Allah knows the truth of it.'

Apart from Ibn Rušd's usual view that originality equals error, I think he has it exactly right.

Ibn Sīnā converted what had been semantics proposed for 'Every *B* is an *A*' into a range of new *sentence forms*.

Several recent writers refer to 'Ibn Sīnā's *wasfī* sentences'. In fact Ibn Sīnā had no name for these sentences, apart from a brief attempt in *Mašriqiyyūn* to call them 'adherent' (*lāzim*).

The name 'descriptive' (*wasfī*) was used by later writers, e.g. Tūsī.

Ibn Sīnā calls attention to an argument form involving descriptive sentences.

We can refer to it as the *descriptive syllogism*.

It's a form of modal *Barbara*.



The descriptive syllogism<sup>7</sup>:

- ▶ Every *C* is a *B* so long as it exists.
- ▶ Every *B* is an *A* so long as it is a *B*.
- ▶ Therefore every *C* is an *A* so long as it exists.

Given Theophrastus' known interest in modal *Barbara*, and the fact that Ibn Sīnā claims no originality for this argument, my guess is that some confused form of this argument goes back to Theophrastus, though the form above is Ibn Sīnā's.

Against this, Theophrastus is known to have proposed the *peiores* rule:

In a valid syllogism the properties of the conclusion match those of the weaker premise.

But the descriptive argument violates *peiores*. Its conclusion is essential and its major premise is descriptive, although the descriptive reading is clearly weaker than the essential.

This point vanishes in face of the evidence that Theophrastus probably didn't distinguish between the essential and the descriptive readings.

#### D. Theophrastus or Aristotle?

Ibn Sīnā attributes the *tatlīt* not to Theophrastus but to 'the First Teaching', i.e. Aristotle.

Easily explained. First, Alexander in his commentary<sup>2</sup> claims (perhaps correctly) that Theophrastus took at least his predicate reading from Aristotle's *De Interpretatione*.

Second, the Arabs tended to believe (perhaps correctly) that Theophrastus and Eudemus would have discussed their views with Aristotle, so that their writings are indirect evidence for Aristotle's own later views<sup>4</sup>.

#### E. Necessary or absolute?

Ibn Sīnā describes the *tatlīt* not as a classification of modal necessity propositions, but as a classification of *absolute*, i.e. non-modal, propositions.

Elsewhere he gives exactly the same propositions as examples of necessity propositions, apparently thinking that the ambiguity goes back to Aristotle.

This is not an aberration in Ibn Sīnā. Themistius<sup>3</sup> had also presented the three readings first as readings of necessary/necessity propositions, and then as readings of non-modal propositions.

Here we come close to our original problem.

In fact Ibn Sīnā brings matters to a head by observing that<sup>8.(24)</sup>

- ▶ in *Qiyās* (his commentary on the *Prior Analytics*) permanent sentences count as necessity propositions but essential and descriptive sentences count as absolute,
- ▶ whereas in *Burhān* (his commentary on the *Posterior Analytics*), essential and descriptive sentences count as necessity propositions.

To introduce his explanation of this discrepancy, we note some distinctions that he makes.

## F. Explicit versus intended

Ibn Sīnā constantly distinguishes between what we say and what we mean.

Normally we mean more than we say.

In practice language users are well aware of this.

He consistently takes the view that logical inference is *between intended meanings in the mind*, not between syntactic sentence forms.

Nevertheless he regularly gives sentences with their intended meaning partly suppressed.

A frequent example is

Everything that moves changes.

where a temporal ‘condition’ is added in the meaning.

This is an awful nuisance but clearly deliberate.

He believes logicians should follow normal usage.

## G. Modal in act

For Ibn Sīnā a proposition is modal ‘in act’ (*bil-fi‘l*)<sup>8</sup> if its *meaning* contains a modal meaning ‘necessarily’ or ‘possibly’.

Note that permanent, essential and descriptive propositions are in general not modal in act.

But here is a proposition that is modal in act<sup>8.(23)</sup>:

Some trees grow leaves in the spring.

On Ibn Sīnā’s reckoning the intended meaning is that these trees *have to* grow leaves in the spring. (I’m not convinced, but it’s arguable.)

This modality *is not conveyed by the temporal clause*.

Proof (my example based on Ibn Sīnā): Compare with

Some people sit down when they eat.

So in the tree case the intended meaning has to include a mention of modality,  
and hence the proposition is modal in act.

Now consider Ibn Sīnā *Burhān* 123.14–16<sup>8.(26)</sup>:

‘Someone might say: In the book *Qiyās*, when you considered those propositions which are necessary in the sense that [the predicate holds] so long as the subject [individual] continues to fit the [subject] description, you took them to be absolute propositions. In that book there were universally quantified propositions that are [called] absolute although their universality is not qualified by any gap in the time at which they apply.’

A page later (*Burhān* 124.16):

‘So this hard-to-describe (‘*awīs*) problem has been solved.’

Note:

(i) The objection is addressed to ‘you’, i.e. Ibn Sīnā. In fact although Ibn Sīnā can trace the original ambiguity to Aristotle, he defends his own treatment of it without any reference to Aristotle.

(ii) The problem is ‘hard to describe’ (as opposed to ‘hard to solve’). This seems to rule out some obvious explanations. It rules out that the propositions described as absolute are not the same as those described as necessity propositions. It rules out that the problem rests on an ambiguity in the word ‘necessary’ or the word ‘absolute’.

This leaves Ibn Sīnā with very little wriggle room.

Ibn Sīnā’s ‘solution’ is difficult to decipher, and not one that can be straightforwardly read off. We have to conjecture what he might have been aiming at, and then check whether his text fits our conjecture.

Under these circumstances any explanation of his solution has to be provisional.

But here is one.

### H. Modal in possibility (*'imkān*)

A proposition is modal 'in possibility' if it could (though in general need not) have a modal condition added without changing the meaning<sup>8</sup>.

From a modern point of view, this is as if the proposition was factual, and the added modal condition represented an evaluation that doesn't change the facts.

Note for example Ibn Sīnā's closing comment at *Burhān* 124.13–15<sup>8</sup>:

'Such premises do occur in *Burhān* with the modality of necessity suppressed [in the expression] but present in the mind. They really are absolute when the modality of necessity is suppressed [in the expression] and not present in the mind either, and the proposition is regarded as purely factual (*nuzīra ilā l-wujūd faqat*).'

If this is correct, then the distinction between absolute and necessary in Ibn Sīnā is, at least sometimes, not a distinction between different kinds of proposition. It is closer to a distinction between ground-level and more subtle aspects of *the same propositions*.

When we add 'necessarily' to a non-modal statement, we need not in general be changing the factual content or truth conditions.

Rather, we may be labelling the statement as suitable for giving scientific knowledge and certainty. This accounts for the change of terminology in *Burhān*, which is about epistemology and scientific method.

Such views don't appear in *Najā* (written a dozen years before *Qiyās*, published later).

Instead *Najā* treats necessity and permanence as parallel concepts.

The main difference between them is that propositions about *individual objects* can be permanent without being necessary.

This can't happen with truly universal propositions<sup>10</sup>.

This view is repeated by Ibn Rušd<sup>12</sup> and by Tūsī (commentary on *Išārāt*).

The *Najā* view, though interesting, doesn't come near explaining the complex relations between permanence and necessity in the *Šifā'*.

For example the propositions that express necessity but not permanence.

Maybe it represents an earlier layer of Ibn Sīnā's thinking.

If so, what effect would the development of the *Šifā'* view have had on his modal logic?

Instead he presents the categorical syllogisms, and as he describes each figure he examines the effect of adding 'conditions' of various kinds, modal or other. (*murā'ātu l-šurūṭ*, already a leading theme in *Šifā'*.)

Like much of *Išārāt*, the treatment is sketchy. It would be extremely interesting to know how Ibn Sīnā himself would have expanded it if he had had the time and the inclination.

It's plausible that the logical relations between propositions depend mainly on their truth conditions, and less on their evaluative content.

If Ibn Sīnā took that view, we would expect it to lead him in later work to downplay the role of modalities in logical rules.

But this is exactly what we find in the late *Išārāt*, where Ibn Sīnā abandons Aristotle's scheme of treating modal rules separately.