# The grammar of meanings in Ibn Sīnā and Frege

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This is not yet the paper; in fact it is barely more than a set of notes. But in submitting a precis of the paper to the journal *Al-Mukhtabat* I undertook to make the acknowledgements and references available here. It will turn into a proper paper covering all the required references as soon as I can manage. The listing follows the sections of the precis.

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#### 1 Ibn Sīnā and Frege as logicians

Gottlob Frege's books *Begriffsschrift* [8] and *Grundgesetze der Arithmetik* [13] mark the beginning and the end of his main involvement with giving formal proofs for arithmetical truths.

Ibn Sīnā's logical writings are in nearly all cases first sections of works covering other disciplines as well. Gutas ([20] Chapter 2) discusses and defends what is now the standard dating of these works. The earliest that I use is *Kitāb al-Najāt* [37], or *Najāt* for short, which was written in around

1013 when Ibn Sīnā was around 33, but it was published a dozen or so years later, probably after some light editing. His major surviving work in logic is the first few volumes of his encyclopedic *Šifā'*, which take the form of commentaries on Aristotle's Organon and were written in the early to mid 1020s. From this work we will use *Madkal* [29] (commentary on Porphyry's *Eisagoge*), *Maqūlāt* [30] (commentary on Aristotle's *Categories*), *°Ibāra* [31] (commentary on Aristotle's *De Interpretatione*), *Qiyās* [32] (commentary on Aristotle's *Prior Analytics*), *Burhān* [33] (commentary on Aristotle's *Posterior Analytics*, *Jadal* [34] (commentary on Aristotle's *Topics*) and *Safsața* [35] (commentary on Aristotle's *Sophistical Refutations*). After publishing *Šifā'* and *Najāt*, Ibn Sīnā no longer bothered to mention the Aristotelian tradition. Two later works in which he develops his own vision of the subject are *Mašriqiyyūn* [38] ('Easterners'), of which sadly only a small section on logic survives, and his laconic but heavy-weight *Išārāt wa-Tanbīhāt* [39].

Metaphysics was always one of Ibn Sīnā's main preoccupations. The section of *Šifā'* covering it is *Ilāhiyyāt* [36], though *Madkal* was written as an introduction to the whole *Šifā'* and contains much of what Ibn Sīnā considered common ground between metaphysics and foundations of logic. This common ground belongs to the part of metaphysics that he described as Universal Science or First Philosophy (see Gutas [20], particularly Chapter 6). But even First Philosophy goes beyond what any logician needs:

It has been the custom to spin out the foundations of logic with things that are nothing to do with logic. These things belong to

(1) the judgmental art, I mean First Philosophy. So I have put off introducing any of that until we come to the proper place for it, so as not to waste time on it. (*Madkal* 10.5–7)

So for example *Madkal* discusses the meanings of common nouns, proper nouns and quantifiers from a very abstract point of view, leaving it to *clbāra* to tie these notions to particular grammatical forms.

The notion of attachment of meanings appears already in *Madkal*, and clearly plays a central role there. For example accidents are described as 'attaching to' things. (Thus typically *Madkal* 15.4 *yalḥaquhā ḥīna'iḏin 'a<sup>c</sup>rāḍun*.) Since Ibn Sīnā never suggests that attaching of meanings is different in logic and in metaphysics, what we say about these attachments in logic has at least to be compatible with what Ibn Sīnā says about them in metaphysics. This is not the proper place to go into the matter in detail, but here is a brief indication of how it should go (in my view). We say that as people get older, the accident of oldness attaches to them. This is a shorthand. To describe someone as an old person, we need to use the concept of an old person.

This concept is formed by attaching the meaning of 'old' to the meaning of 'person', in exactly the sense discussed in this paper. To say that the accident attaches to the particular old person is shorthand for saying that the person, who always satisfied the concept of being a person, comes to satisfy the compound concept of being an old person. No hocus-pocus, but Ibn Sīnā does tend to assume that anything that happens in the world is describable in language, and in some sense the meaning of the description is prior to the happening.

In the first section of *Maqūlāt* Ibn Sīnā lists a number of things that belong to First Philosophy but are of only marginal interest in logic. They include the ten categories. It's true that you can learn the rules of Aristotelian syllogisms without ever invoking the categories. But when Ibn Sīnā lists the various things that can occur as attachments or adjunctions, his lists tend to have a strong whiff of the categories about them. For example at *Mašriqiyyūn* 48.5–8 (in the section on 'Testing the predicate'):

An example is where the genuine predicate carries a condition, that there is a condition attached to it that one fails to notice, and this condition is either a relation or something to do with the nature, or something from the point of view of the difference

(2) between part and whole, or a time or a place or the attachment of a quality or the attaining of a spatial extent or an act or a passion, or a consideration of potency or actuality, or a consideration of a connection to an agent or a consideration of a connection to a patient.

The presence of the categories in the background is rather obvious here. One could criticise Ibn Sīnā for failing to exploit this fact. As we will see, he conspicuously fails to produce any useful catalogue of the kinds of adjunction or attachment, and this is a serious impediment to his attempts to improve on Aristotle's formal logic.

On a place where Frege seems more metaphysical than Ibn Sīnā, see section 5 below on the 'third component' and the notion of incomplete meanings.

Ibn Sīnā believes we can only keep track of meanings by using as labels expressions that mean them:

The particular reason why we have to use expressions is the impossibility of getting meanings arranged in order unless the ex-

(3) pressions for them come to mind together with them. (*Madkal* 22.17f)

Ibn Sīnāinsists on normal usage of language in logic:

One knows that it has become a different proposition, so you [have to] think it through in terms of truth, falsehood, necessity

(4) and lack of necessity all over again. You had it in your two hands and you throw it away. (*Qiyās* 150.12–14)

(I will expand this.)

#### 2 Compositionality

The paper Hodges [25] describes and contrasts Aristotelian compositionality and PTW compositionality. It contains quotations from Al-Fārābī, Abelard and Frege, to which we can add

Each of these phrases [for example 'Some animal is a horse', 'Zayd is walking'] is a compound of two expressions which are parts of it, one of them a description and the other the subject of a description. (10) The meanings of these two expressions are linked together in the soul in the same way as the expressions are linked in the language, with a linkage in the soul that is sim-

(5) alle indee in the language, while a indage in the sour that is similar to the one in the language. Just as the composite sentence is composed of two parts, the thing linked together in the soul is composed of two meanings, the first of them the one signified by the part which is the subject of the description, and the second the one signified by the part of the sentence which is the description. (Al-Fārābī [2] 57.10–15)

(6) The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of parts that correspond to the words. (Frege [17] p. 43)

For Ockham's application of grammar to mental language see Panaccio [46] Ch. 1.

Al-Fārābī's frequent references to language in his logical works may have given rise to the story that he learned linguistics from the distinguished Baghdad grammarian Ibn al-Sarrāj (cf. Zimmermann [48] p. cxviiif). But as his works [2] and [3] illustrate, his interest was almost entirely lexicographical (along the lines of 'The Sogdian for 'is' is *asti*'), and his interest in syntax hardly goes beyond word classes.

In his *Grundlagen der Arithmetik* [9] Frege often refers to the interconnections ('Zusammenhang') of words in a sentence, most famously in his 'context principle' on page x, but also in §§29, 32, 33, 46, 60, 62, 106, 116,

Ibn Sīnā's 'attachment words' deserve a study on their own. But for the present here are some facts about them. They include the following, which Ibn Sīnā sometimes uses interchangeably:

- ( $\alpha$ ) *ziyāda*, pl. *ziyādāt*, 'addition'.
- ( $\beta$ ) šarț, pl. šurūt (also irregular pl. šarā'iț, e.g. <sup>c</sup>Ibāra 74.9), 'condition'.
- (β) yalhaqu, 'is adjoined to';
  lāhiqa, pl. lawāhiq (also related forms luhūq, mulhaq etc.), 'adjunct'.
- (δ) <sup>c</sup>alāqa, pl. <sup>c</sup>alāqāt (also related forms ta<sup>c</sup>alluq, muta<sup>c</sup>alliq etc.), 'attachment'.

This is not a complete list. For example we could have added  $id\bar{a}fa$ , pl.  $id\bar{a}f\bar{a}t$ , '(possessive) relation', 'addition'; *yaqtarinu* 'is linked to'; *nisba* 'connection'; and  $d\bar{a}kil$  'al $\bar{a}$  'put next to'. But Ibn Sīnā uses these four words in other senses too.

Here is a comparison of the number of times words from these groups appear in Ishaq's Arabic translation of Aristotle's *De Interpretatione* (in [4]) and in Ibn Sīnā's *Madkal* and *cIbāra*. The numbers are the number of occurrences per ten pages of the published Arabic.

	ziyāda	šarț	lāḥiqa	<sup>c</sup> alāqa
	etc	etc	etc	etc
De Int	0.5	0	0.7	0
Mad <u>k</u> al	2.1	0.7	2.2	0.2
<sup>c</sup> Ibāra	1	2.7	2	0.4

The commonest of these words, *šarț*, is not in Isḥaq's s Aristotle at all. From the index of Zimmermann [48], it seems not to occur in Al-Fārābī's commentary on *De Interpretatione* either. Al-Fārābī does have the word *šarțI* 'conditional' as the name of a certain kind of proposition, but this is a different use of the root. These are among many indications that this cluster of words is a distinguishing mark of Ibn Sīnā's style in logic. Another indication is that none of the Arabic words or roots mentioned so far in this

section appears in Hugonnard-Roche's survey of the formation of Arabic logical vocabulary [28]

Some of the words were certainly in regular use as technical terms in other disciplines. For example we find  $ziy\bar{a}da$  in the classical linguists, with a purely syntactic meaning. On the other hand <sup>*c*</sup>*atf* is frequent in the linguists for a certain kind of linking, but rare in Ibn Sīnā.

A complete account would also mention the ways in which Ibn Sīnā expresses 'not having anything attached'. They include *muțlaq* 'absolute', *mursal, fī nafsi l-'amr*, X *min ḥayṯu huwa* X. Also he says 'X is a part of the subject' (or predicate) to express that X is attached to the subject (or predicate), as at:

If we adjoin (*'alḥaqnā*) any condition (*šarț*) to the predicate, it is a part of the whole predicate. (*Qiyās* 103.2)

The semantic quotation notation, [ANIMAL] etc., is as in Jackendoff *jack*:1.

The *Madkal* 'every animal' passage.

We say: The idea [ANIMAL] is a meaning in its own right, regardless of whether [an instance of] it exists in the world or it is conceptualised in the mind. Also it is not in itself either universally quantified or existentially quantified. If it was inherently universally quantified, in the sense that the meaning [ANIMAL], just as it is, was universally quantified, then there couldn't be an individual animal. Rather, each animal would be an every animal. And if [ANIMAL] were singular, just because it was [ANI-

(8) MAL], then it wouldn't be possible for there to be more than one individual [animal]. Because of [ANIMAL] being singular, there would have to be just one animal, and it would be impossible for any other individual to be an animal. ... If together with [ [AN-IMAL] ] one conceptualised [EVERY] or [SOME] or something else, then what would be conceptualised would be a meaning added (*zā'idun*) to [ANIMAL], something that happens to [ANI-MAL]. (*Madkal* 65.11–19)

Parallel passages to (8) appear in <sup>c</sup>Ibāra i.7, Ilāhiyyāt 196.6ff and Išārāt iii.4.

On Ibn Sīnā and Skolem semantics, see Hodges [26].

## 3 Dependency grammars

Tesnière [47] is a classic on dependency grammar, and Mel'čuk [42] a more modern account by a noted Russian lexicographer. On dependency grammar as it appears in the Modists, see Covington [6] p. 59. Owens [45] claims that classical Arabic grammar implicitly uses a dependency framework. Kouloughli [41] argues that Owens' account distorts the classical Arabic view by implying that the various things that Owens labels as dependencies have something in common.

Hodges [22] describes *Begriffsschrift* in dependency terms, and cites some of the evidence that stemmata were used for teaching grammatical analysis in 19th century Germany. See also Chiswell and Hodges [5] §3.1, which shows how to give a dependency grammar for propositional logic, and exhibits one of Frege's *Begriffsschrift* formulas as the stemma for a propositional formula.

#### 4 Silent meanings

Frege insisted that for his programme of showing that the truths of arithmetic are analytic, it was essential to make all parts of the reasoning explicit:

... all leaps and bounds are to be avoided in reasoning. That

(9) this condition is so difficult to satisfy lies in the tediousness of proceeding stepwise. Every proof that is only somewhat complicated threatens to assume a monstrous length. ([9] §91)

Ibn Sīnā by contrast was happy to identify places where even scientists leave out part of their meaning, and then to copy them. For example he leaves out arguments of relations:

	Contraries can be simultaneously fals	se, as when we say: 'Equals
(10)	are greater and equals are smaller'.	(Burhān 251.8f, cf. Burhān
	127.2–4, <sup>c</sup> Ibāra 44.2f)	

and objects of verbs, even when he has to supply the missing information immediately afterwards:

(11) The genus doesn't allow but the proprium does — I mean this conversion. (*Madkal* 101.8f)

My impression is that Ibn Sīnā knows how to modulate the degree of explicitness, and he is not at all above making some things deliberately difficult. Gutas [20] Chapter 5.3 'Withholding Knowledge' describes a social context in which it was natural for Ibn Sīnā to 'withhold' some truths from the unedicated masses.

At *Qiyās* 410.11–411.1 Ibn Sīnā gives an analysis of false assumptions made for reductio ad absurdum. He explains there that mathematicians tend not to repeat the false assumption at every stage where it is needed. To see the true structure of the reductio argument we need to restore the assumption throughout. The paper [27] will give an account of all this. Ibn Sīnā's view is strikingly close to Frege's comments on assumptions in [14] and [15]. In *Qiyās* Ibn Sīnā runs through some standard syllogistic moods and writes out how they look when assumptions are added explicitly, for example:

Every C is a B,

(12) and whenever *r* then every *B* is an *A*.So whenever *r*, then every *C* is an *A*. (*Qiyās* 331.14)

A typical example of a sentence which he may be interpreting as being subject to an unspoken 'Suppose' is:

(13) 'No animal is moving voluntarily', i.e. at the time of rest. (*Najāt* 40.9f, translated Ahmed  $\S55$ )

The examples of sentences with silent temporal assumptions are: White things *Qiyās* 22.9; traveller from Rayy to Baghdad *Qiyās* 22.12; breathing in and breathing out *Qiyās* 23.5; horses awake *Qiyās* 44.5. On this last example Ibn Sīnā comments:

If one said 'What makes it true is its holding of all horses', then this doesn't answer the question what makes the sentence true about individual horses. This is because the phrase 'every horse'

(14) embraces the class of horses and quantifies over it, but it doesn't quantify over both the class of horses and the class of times together, because it is a quantifier on the subjects of the universal 'horse', not a quantifier on the two things together. (*Qiyās* 44.9–12)

Ibn Sīnā's discussion of counterfactual conditions is at Qiyas 273.7ff. He says we 'add (yudafa) to the clause in the meaning a condition (*šart*) that

prevents the use of conditions ( $\check{s}ur\bar{u}t$ ) entailing things that are not allowed to follow from the clause' ( $Qiy\bar{a}s$  274.13f/ He has noted the use of reductio in the sciences at  $Qiy\bar{a}s$  273.14).

## 5 Simple propositions and the copular meaning

Ibn Sīnā believes that all simple (i.e. one-clause) propositions split into a subject part and a predicate part. For example

(15) Every simple question splits into a predicate and a subject. (*Burhān* 157.21)

This split more or less coincides with the standard modern decomposition NP + VP for simple sentences.

Ibn Sīnā's fullest discussion of the copula is in his commentary [31] on the *De Interpretatione*, and in particular his discussion of the following passage which raises the question of the unity of the proposition.

The single primary declarative sentence is the affirmation; then after it the denial. And as for all the other sentences, they just

become one through a bond (*ribāț*) that binds them together. (*De Interpretatione* 17a10, the translation is from Ishaq bin Hunayn's Arabic version [4] p. 115.)

Parallel passages in Ibn Sīnā's later works *Easterners* ([38] pp. 60–67) and *Išārāt* ([39] Method iii Remarks 1, 2) show no interest in the unity of the proposition.

Ibn Sīnā believes that the meaning of a simple sentence has three components.

What is objectively needed to make a predicative proposition complete is three items, /38/ namely the meaning of the subject, the meaning of the predicate and the connection between the two meanings. The way the meanings are brought together

(17) Intertwo meanings. The way the meanings are brought together in the mind does not consist just in their being subject and predicate in [the whole], but rather the mind also has to believe in the connection (*nisba*) between the two meanings, which makes [the whole] an affirmation or a denial. (*cIbāra* 37.15–38.3)

So the meaning of the simple declarative sentence is a compound of three parts, a subject meaning, a predicate meaning and a further item which I

am calling the 'copular meaning' in view of Ibn Sīnā's next remarks:

So if it's intended that the [outer] expression should run parallel to the inner heart [of the proposition], the expression needs to contain three signifying elements, an element that signifies the subject meaning, a second element that signifies the predicate meaning, and a third element that signifies the connecting link

(18) between the two meanings. ... So it's clear from this that there is here a meaning which is neither the meaning of the subject term, nor the meaning of the predicate term. This meaning is a connection, and it can properly be signified [by an expression]. An expression that signifies the connection is called a copula, and it plays the role of a particle. (*cIbāra* 38.4–6, 39.4–6)

The name 'third component' probably comes from the Arabic of Aristotle *De Interpretatione* 19b20: 'Here I say that the "is" is a third component'.

(Important material to be added here.)

The textbook that presents Frege's semantics for simple sentences in a form that is actually Ibn Sīnā's is Heim and Kratzer [21].

## 6 Quantifier and modality

According to Ibn Sīnā, both quantifier and modality can be attached to simple sentences.

... violation of one or more of the conditions (*šurūt*) that are ad-

(19) joined (*talḥaqu*) to the terms, in the form of quantifiers, copulas or other things. (*Qiyās* 472.7f)

The terms are the essential parts of the premises, so that when the copula is taken away from the premises, what remains in the

(20) case of predicative sentences is the subject idea and the predicate idea. The quantifier and the modality /54/ are added features (*dawāhil*). (*Qiyās* 53.13–54.1)

Cf. also *Qiyās* 480.10f.

Further on adding modalities, both syntactically and semantically:

The quantifier should be adjacent to the subject, and the copula should be adjacent to the predicate. In the same way the modality should be adjacent to the copula if there is no quantifier. If there is a quantifier there are two places [for the modality], namely [adjacent to] the copula and [adjacent to] the quantifier,

(21) Inamely facilitation to the coputa and facilitation of the quantitier, regardless of whether both places give the same meaning or different ones. You have the choice of attaching the modality in the first place or the second. Thus you can say 'It's possible that everybody is a writer.' and you can say 'Every person can be a writer.'. (*clbāra* 112.15–113.4)

He is clearly talking about Arabic syntax here; in fact he waves aside the question of meaning. But his opening 'should be' (*min haqqihi 'an*, maybe 'is entitled to be') suggests he sees a deeper reason for the syntax.

But in any case <sup>*c*</sup>*Ibāra* tells us almost at once that he does regard the placing of the modality as expressing a difference in meaning.

Thus when we say 'Every human can be a writer.' this is natural (al- $tab\bar{i}^c\bar{i}$ ), and it means that each individual person can be a writer. But if [the modality] was linked to the quantifier and it wasn't intended by this to stretch the language by removing [the modality] from its natural place, but rather it was intended by it to signify that the natural place for [the modality] is adjacent to the quantifier, then it wouldn't be a modality of the copula. Rather it would be a modality of the universal or existential [quantifier], so the meaning would be changed. The meaning would have become a possibility, namely that it's possible that all human beings together are writers. An indication of the dif-

(22) an infinite beings together are writers. Fin indication of the difference of meaning is that nobody at all has any doubts about the first; in fact one knows that no individual human has to be permanently literate or permanently illiterate by nature. But when you say 'It's possible that everybody is a writer.' on the basis that the possibility is the modality of the universal quantifier, then one could doubt that the proposition is true. In fact some people say that it's impossible that everyone is a writer, i.e. that it couldn't possibly be the case that every human is a writer — so that it turned out that nobody at all was not a writer. This shows that there is a testable difference between the two meanings. (*clbāra* 115.2–11)

Modalities can be on the quantifier. Truth or absoluteness on quantifier. The distinction between modality on the copula and modality on (or of) the quantifier reappears several times in  $Qiy\bar{a}s$ , particularly in the sections ii–iv on modal logic. There Ibn Sīnā introduces the idea that the modality of absoluteness can be on the quantifier too ( $Qiy\bar{a}s$  159.13, 189.16). (For this strange modality recall (3) in section 5 above.) He expresses the same notion as'according to the meaning of the quantifier' ( $Qiy\bar{a}s$  151.14f), or as 'truth of the quantifier':

[Aristotle] says after this what he has in mind: that the absolute premises shouldn't refer to their quantifier at all, in such a way

(23) that their absoluteness is that their quantifier is true at some time. So the proposition 'Every *C* is a *B*.', in the meaning that every *C* is a *B* at such-and-such a time, shouldn't be used as an absolute proposition. ( $Qiy\bar{a}s$  193.9–11)

In contrasting modality on the quantifier with modality on the copula, he sometimes calls the latter 'the predicate interpretation' (*al-'i<sup>c</sup>tibāru l-ḥamlī*), as at *Qiyās* 30.16.

Ouhalla [44] pp. 52–72 studies modality, sentence negation and tense as syntactic sentence features that are 'assumed to belong under the I node in the standard analysis'. To these he adds verb agreement.

Nested modality: every human necessarily can be a writer.

In every premise with a modality that you can put outside the predicate, you can [also] make its modality a part of the predi-

(24) cate, and then attach (*talḥaqu*) another modality to it. Thus when you say: 'Every human can be a writer.', you are entitled to go on and say 'Every human necessarily can be a writer.'.

We illustrate modality on the quantifier. At  $Qiy\bar{a}s$  144.10 Ibn Sīnā considers the sentence 'Nothing white is an animal', and he says it could count as true under two interpretations. One is that it means 'Nothing white is an animal because it is white'. The other, which we can see illustrates truth on the quantifier, is that we restrict the sentence to a situation (say M) in which there are no permantly white animals and those that are not permantly white happen all to be black. ( $Qiy\bar{a}s$  144.15–145.2.)

Movahed's paper on modalities de dicto and de re in Ibn Sīnā is [43].

## 7 Compound sentences

## 8 Adjunctions

Ibn Sīnā on interaction between laysa and quantifiers. For example *Mašriqiyyūn* 76.7 where *laysa kullu nsānin* is paraphrased as *laysa ba*<sup>c</sup>*du l-nās*.

Ibn Sīnā on Ethiopians being black. <sup>c</sup>Ibāra 43.13f.

Ibn Sīnā references back to the conditions for contradiction. SEE NEXT.

Sentence not true or even meaningful until conditions specified.

The fact is that a proposition in its intended meaning is not true or false at all, or conceded or rejected, or even conceptualised, to

(25) say nothing of its having an opposite, unless it is determinate in terms of all the attachments (*muta<sup>c</sup>alliq*) to its meaning that we have mentioned. (*cIbāra* 44.3–6)

He evidently regarded this as a key passage. He refers back to it at *<sup>c</sup>Ibāra* (45.9, 102.12) and *Safsata* (22.3, 97.1 and elsewhere), describing the 'attachments' as *šarā'iț*, 'conditions'.

Three is a half of six. (*cIbāra* 44.3)

Alcohol is not prohibited. (*cIbāra* 43.16)

Zayd father + of X. (*Mašriqiyyūn* 48.6–8)

Definition of father can be opened up. (*Išārāt* 67.3–9)

## 9 Scope

Frege on subject not being distinguished argument.

Ibn Sīnā: 'It would be appropriate to speak warily'.

It would be appropriate for us to speak warily: some of the things said in the third book (*cIbāra*) were inadequate. Namely, when ...

(26) we want to take into account the time in the sentence 'Not every B is an A,' since this is one of [the things that have to satisfy] a condition in order to have a contradictory negation, this makes difficulties for us. (*Qiyās* 38.5–7)

Hodges [26] says more on Frege and Ibn Sīnā on scope.

#### **10** The direction of construction

Frege on starting at the top.

According to the conception sketched above, "hydrogen" is the argument and "being lighter than carbon dioxide" the function; but we can also conceive of the same conceptual content in such

(27) a way that "carbon dioxide" becomes the argument and "being heavier than hydrogen" the function. We then need only regard "carbon dioxide" as replaceable by other ideas, such as "hydrochloric acid" or 'ammonia". (*Begriffsschrift* §9)

## **11** Top-level processing

For top-level processing in the Aristotelian tradition in general see Hodges [23]. Boole's contribution to breaking down this restriction is documented in Hodges [24].

Ibn Sīnā autobiography on taking care of conditions:

The next year and a half I devoted myself entirely to reading Philosophy: I read Logic and all the parts of philosophy once again. ... I compiled a set of files for myself, and for each argument that

(28) I examined, I recorded the syllogistic premisses it contained, the way in which they were composed, and the conclusions which they might yield, and I would also take into account the conditions of its premisses ( $'ur\bar{a}^{c}\bar{\imath} \, \check{s}ur\bar{u}\underline{t}a \, muqaddam\bar{a}tih\bar{a}$ ) until I had Ascertained that particular problem.

This is quoted from Gutas [20] p. 27, except that I have removed Gutas' gloss that 'conditions' means modalities. A central point of the present paper is that it means a lot more than modalities.

One notable special inference rule that Ibn Sīnā studies is the descriptional syllogism:

Snow is white for as long as it exists.

Everything white has a colour with wide radiation as long as it (29) is white.

Therefore snow has a colour with wide radiation for as long as it exists. (Paraphrased from *Qiyās* 120.1)

There is some evidence to suggest that Ibn Sīnā knew a version of this syllogism from Theophrastus via Themistius; I will comment on this elsewhere.

There are many other references in Ibn Sīnā to taking care of conditions. A first sample:

On conditions (*šurūț*) of propositions.

One should keep an eye out for  $(yur\bar{a}^c iya)$  ... the status of relations, for example when it is said that '*C* is a father' one should look out for  $(li-yur\bar{a}^c i)$  the question 'whose'? The same goes for

(30) Idea to the form i and j and i and j are question whose i the same goes for time and place and condition (*šar*!). For example when it is said that 'Everything that moves changes', one should look out for (*li-yurā<sup>c</sup>i*) the question 'Is that while it stays moving?'. (*Išārāt* iii.10, Inati p. 89)

At *clbāra* 102.10f he explicitly connects the conditions that one has to 'take care of' with those that have to be checked when forming contradictory negations. At *Maqūlāt* 253.10 he talks of 'taking care of the conditions' when it's a matter of finding not contradictory negations but intermediate cases.

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