

# Notes on a remark of Street

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On page 152 of his paper ‘An outline of Avicenna’s syllogistic’ ([2], 2002) Tony Street discusses Ibn Sīnā’s proof of the syllogistic mood *Barbara* with possibility minor premise and necessity major premise and conclusion. We now know — as Street did not in 2002 — what Ibn Sīnā meant by his modal syllogistic moods, and we also have a good modern proof theory for this system. This note will look back at Street’s treatment of that mood in the light of the more recent results. I will give only as much of the background theory as we need.

Street comments ‘The proof in [*Qiyās* [1]] makes sense, though it may not be valid’. He gives a translation of that proof, but to save explaining his notation I expand his formulas, using the three modalities ‘With necessity’, ‘With actuality’ and ‘With possibility’. Thus:

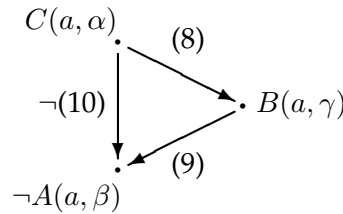
- (1) With necessity, every  $B$  is an  $A$ . (Premise)
- (2) With possibility, every  $C$  is a  $B$ . (Premise)
- (3) Not: With necessity, every  $C$  is an  $A$ . (Assumption)
- (4) With possibility, some  $C$  is not an  $A$ . (3)
- (5) With actuality, some  $C$  is not an  $A$ . (4)
- (6) With necessity, some  $C$  is not a  $B$ . (1, 5, *Baroco*)
- (7)  $\perp$  (clash between 2 and 6).

For comparison, here is what we now know Ibn Sīnā meant by the relevant syllogism. I symbolise it in a two-sorted first-order language, with a ‘roman’ sort of *objects* and a ‘greek’ sort of *times*. The expression  $E(a, \alpha)$

means ‘ $a$  exists at time  $\alpha$ ’, a notion that appears often in Ibn Sīnā’s logic.

- (8)  $\forall x\forall\tau(C(x, \tau) \rightarrow \exists\sigma(E(x, \sigma) \wedge B(x, \sigma)))$ .  
 (9)  $\forall x\forall\tau(B(x, \tau) \rightarrow \forall\sigma(E(x, \sigma) \rightarrow A(x, \sigma)))$ .  
 $\vdash$  (10)  $\forall x\forall\tau(C(x, \tau) \rightarrow \forall\sigma(E(x, \sigma) \rightarrow A(x, \sigma)))$ .

A modern proof is as follows:



But of course I will need to explain at once what this picture means.

As any mathematical logician will tell you, in proofs it’s good to lay out all the usable information early on. In particular if you have an assumption that something exists with a certain property, it’s good to put a name on that thing so that you have an object in hand to discuss.

So faced with the premises and conclusion of *Barbara*, it’s sensible to take the premises (8), (9) and the negation  $\neg(10)$  of the conclusion:

$$\neg(10) \quad \exists x\exists\tau(C(x, \tau) \wedge \exists\sigma(E(x, \sigma) \wedge \neg A(x, \sigma)))$$

and aim for a contradiction. Taking  $\neg(10)$  is a particularly helpful move, because the resulting proposition begins with an existential quantifier, and we can instantiate it. Actually there are three existential quantifiers that we can instantiate, and we need distinct letters for all three constants. This will give us three atomic sentences:

$$C(a, \alpha), \quad E(a, \beta), \quad \neg A(a, \beta).$$

The atomic sentence  $E(a, \beta)$  plays a bookkeeping role, and we leave it aside for the moment. We place  $C(a, \alpha)$  and  $\neg A(a, \beta)$  at top left and bottom left of the picture, with ‘ $\neg(10)$ ’ on the left between them to say where they came from.

Now we can use these sentences to get some information out of (8) and (9). We could go in either order, but taking (8) first allows us to move forwards directly rather than backwards contrapositively. So we take (8) and apply  $C(a, \alpha)$ :

$$\exists\sigma(E(a, \sigma) \wedge B(a, \sigma))$$

and hence by instantiating again with a new greek symbol of sort *time*

$$E(a, \gamma), B(a, \gamma).$$

Again we set  $E(a, \gamma)$  aside for the moment, and we write  $B(a, \gamma)$  against the righthand node, with '(8)' between it and the top node to express that we got  $B(a, \gamma)$  by applying  $C(a, \alpha)$  to proposition (8). All the nodes are now labelled with information that we have extracted. It remains to use the other premise (9) to get a contradiction from  $B(a, \gamma)$  and  $\neg A(a, \beta)$ . Instantiating  $x$  and  $\tau$  in (9) and using  $B(a, \gamma)$  gives

$$\forall \sigma (E(a, \sigma) \rightarrow A(a, \sigma)).$$

Clearly for a contradiction we need to take  $\sigma$  to be  $\beta$ , and use the fact that we put on one side earlier, that  $E(a, \beta)$ . This gives

$$A(a, \beta)$$

which contradicts the bottom sentence, and we are done.

The metatheory of these proofs tells us that if a sentence of the form  $E(-.-)$  is needed, then it will always be available. So the prover can take these sentences for granted.

Now for some comparisons. The part of this argument that Ibn Sīnā achieves in the proof quoted by Street is simply to bring out  $\neg(10)$ . The appeal to *Baroco* corresponds to the rest of the argument. So strictly the proof quoted by Street is not a proof of *Barbara* at all, but a preliminary adjustment to set out the materials. That adjustment is well motivated by the heuristic points I made earlier. But it would have been better if Ibn Sīnā had gone on to prove the relevant case of *Baroco* from first principles, which is essentially what we did above.

Actually it would have been even better if Ibn Sīnā had proved *Bocardo* from first principles instead. This is because the proof starts with the arrows marked  $\neg(10)$  and (8), and these would be the premises of *Bocardo* if Ibn Sīnā had reduced to *Bocardo* instead of *Baroco*. The reason we chose to take  $\neg(10)$  and (8) first rather than  $\neg(10)$  and (9) (the premises of *Baroco*) can be seen if we quickly try the *Baroco* option. Given  $\neg A(a, \beta)$  and (9), we would need to think (9) backwards, say as

$$\forall x \forall \tau (\exists \sigma (E(x, \sigma) \wedge \neg A(x, \sigma)) \rightarrow \neg B(x, \tau)).$$

Instantiating  $x$  and rearranging the quantifiers,

$$(\exists \sigma (E(a, \sigma) \wedge \neg A(a, \sigma)) \rightarrow \forall \tau \neg B(a, \tau)).$$

Now  $\neg A(a, \beta)$  and  $E(a, \beta)$  allow us to deduce

$$\forall \tau \neg B(a, \tau).$$

But then applying (8) as above gives us  $\exists \sigma B(a, \sigma)$ , which gives our contradiction. Clearly this is a rougher route.

Unfortunately Ibn Sīnā had run out of steam by the time he came to *Bocardo*, so we will never know how he proved it. But his statement of which syllogistic moods are valid and which aren't is so consistently correct that he clearly had a method. Since he certainly lacked the kind of modern technique described above, and his use of Aristotelian reduction methods is manifestly not up to the mark, I would guess he worked out a few hundred examples by first principles using naked intuition. More's the pity that he never thought to explain what intuitions gave him *Bocardo*.

Street goes on to say that 'Avicenna apparently realises he is treading the ragged edge of circularity', because in the Aristotelian scheme of things *Baroco* presupposes *Barbara*. We should stand back a bit to appreciate what is going on here. Ibn Sīnā has at his disposal a highly original system of logic, consisting of modal syllogistic moods. When Aristotle described his assertoric syllogistic moods, he added another ingredient: a kind of proof calculus that would derive these moods, taking *Barbara* and *Celarent* as axioms. In *Qiyās* books iii and iv Ibn Sīnā is trying to do for his system what Aristotle had done for his, and find a proof calculus generating the modal syllogistic moods. He starts by trying to copy Aristotle, but he quickly finds that it doesn't work; for example Aristotle relied on conversions that simply aren't valid for the new sentences that Ibn Sīnā is working with. So he has to cast around for new approaches. What Street has correctly detected was a hint of a new approach, namely *Don't take Barbara and Celarent as axioms*. In fact for reasons given above, if you want to justify your moods by first principles, the best place to start is likely to be the third figure and not the first.

Ibn Sīnā never took that plunge. But we can't blame him for taking the first steps in that direction. It seems that the move to third figure axioms wouldn't have helped him without another change which he shows no signs of having contemplated, namely ecthesis of times as well as of objects.

Ibn Sīnā always kept a rigorous separation between logic and psychology. But this is one place where some crossover might be helpful. A proof calculus for generating moods will only serve its purpose if it actually gen-

erates the moods in an easier and safer way than unaided intuition. But it's a matter of human psychology what we do find easier and safer. I think the main relevant point that we can take from Ibn Sīnā's psychology is the distinction between the *wahm* and the *ʿaql*. The *wahm* yields first principles quickly and intuitively (for example that 'The whole is greater than the part'), but has no methods of reflection and hence is error-prone. The *ʿaql* can make deliberate choices of how to proceed and how to backtrack over its own previous decisions, and hence has powers of self-correction.

When Aristotle labelled the first figure moods as 'perfect', one thing he seems to have had in mind was that we can 'think' these moods intuitively, they come naturally. In terms of Ibn Sīnā's psychology, they are available to the *wahm*. That's safe, because they are so clearly valid. But for Ibn Sīnā's new first figure moods the position is completely different. In a few cases we can justify them by an easy jingle: for example Ibn Sīnā offers 'possibly possible is possible' to justify *Barbara* with possibility premises and conclusion. But that's obviously not an intellectually sound justification of the mood. In the case that Street discusses above, the full proof that we gave is way beyond the powers of the *wahm*; only a person with a serious *ʿaql* could ever undertake it. So the justification for taking first figure syllogisms as axioms vanishes. Instead one should ask what is best for the reflective intellect, and here the third figure moods win hands down.

One other small point is worth mentioning. At proposition (5) in his report of the proof, Street switches from a possibility proposition to an actuality proposition. This looks as if it might be an instance of some procedure for handling possibilities. But — subject to one reservation below — nothing in our proof of the mood corresponds to such a switch. In fact I think it is a misreading. Ibn Sīnā says we should take a certain possibility proposition as *mawjūd*. I think he is just saying that we should assume it's true, not that we should convert it to some other kind of proposition.

The reservation is this, and I don't know how significant it is. Ibn Sīnā's semantics for existential quantification belongs to the style that today is known as *choice function semantics*. Thus he takes a statement 'A man came to see me' as meaning 'X came to see me', where the speaker can have different degrees of definiteness in pinning down in her mind who the man X is. Likewise the statement 'Every horse sometimes breathes in' means what we might express as 'Every horse  $x$  breathes in at time  $F(x)$ ', where again the speaker may be able to spell out the function  $F$  precisely, but in this case probably can't at all. This business of degrees of definiteness keeps appearing in Ibn Sīnā's discussions of existential quantifiers, and it causes

no end of trouble.

The proposition that Ibn Sīnā says we should take as *mawjūd* is  $\neg(10)$  above. It has three existential quantifiers. So his remark might just possibly indicate that we should assume individuals for some or all of these quantifiers, a kind of ecthesis. That would be exactly the logical move that we did make in our modern proof.

## References

- [1] Ibn Sīnā, *Al-Qiyās*, ed. I. Madkour et al., Našr Wizāra al-Ṭaqāfa wal-ʿIršād al-Qūmī (1964).
- [2] Tony Street, 'An outline of Avicenna's syllogistic', *Archiv für Geschichte der Philosophie* 84 (2) (2002) 129–160.