The architecture of Ibn Sīnā's logic (1)

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http://wilfridhodges.co.uk/arabic38.pdf

Ibn Sīnā's logic splits down into distinct 'logics' defined by the kinds of sentence involved:

- assertoric — *mašhūr*
- predicative 2D (narrow time-scope) — *ḥamliy*
- predicative (wide time-scope) — *ᶜalā l-sūr*
- meet-like — *muttaṣil*
- difference-like — *munfaṣil*
- ⋮ — ⋮

Each 'logic' has broadly the same components, which it's convenient to classify as:

1.	propositions	*qaḍāyā*
2.	listing of valid inference forms	*taᶜdīd*
3.	criteria of validity	*qawānīn*
4.	explanation	*bayān*
5.	analysis of arguments	*taḥlīl*

In this talk we concentrate on 3 and 4.

Sketch of the components

1. *Propositions*: This part describes the standard forms of the sentences studied in the logic. *Analysis* will study dos and don'ts for paraphrasing other sentences into the standard forms. (We won't consider *Analysis* further.)

2. *Listing* and 3. *Criteria*: The simple inference forms (= valid moods) studied in the logic are those involving sentence forms of the kinds listed in *Propositions*.
The logic must describe what these moods are.
Listing and *Criteria* do this in different ways.

2. *Listing* lists the valid moods. Aristotle gave a listing for the assertoric moods, and Ibn Sīnā follows or adapts this listing.

An essential early step in understanding an Avicennan logic is to determine what the propositions are in enough precision to allow us to calculate what are the valid moods, and then check that our calculation agrees with Ibn Sīnā's listing.

This is OK for the assertorics, where Ibn Sīnā follows Aristotle almost totally.
Likewise for the meet-like (*muttaṣil*) propositions, whose logic is isomorphic to the assertoric.
We have a characterisation of the two-dimensional logic which totally agrees with Ibn Sīnā's reports of validity and invalidity.

3. *Criteria* by contrast states *necessary and sufficient conditions* for a mood to be valid.
It states a uniform algorithm for determining validity.

The algorithm is required only to give the right answers, not to provide evidence of validity.
(Like Łukasiewicz's algorithm for the logic of 'if and only if': a proposition is valid if and only if each propositional variable occurs an even number of times.)

4. *Explanation* provides, for each valid mood, reasons for believing that the mood is valid.
This part of Ibn Sīnā's logics is highly problematic, particularly the two-dimensional part in *Qiyās* ii–iv, since we have very little idea what criteria he is working from.
Much seems unconvincing.

Ibn Sīnā's template is the calculus introduced by Aristotle for the assertorics.
Aristotle identifies some moods as perfect, i.e. self-evident.
He deduces the other moods from these, by conversion etc.

For two-dimensional logic, *no* moods are self-evidently valid, since the two levels always require thought.

Two-dimensional (2D) sentences

In *Qiyās* i.3 and *Easterners*, Ibn Sīnā introduces sentences of a-, e-, i- or o-form which also have a (sometimes implicit) quantification over times or situations.

'Two-dimensional' is my shorthand for those sentences where (1) the time quantification has narrow scope and (2) there are no subtleties connected with natural language existential quantification. Ibn Sīnā didn't develop the logic of these subtleties, but some later Arabic logicians tried.

The name 'two-dimensional' comes from Oscar Mitchell 1883, who developed a similar set of sentences,
but too late in the day to be interesting.

2D sentences have besides the a-, e-, i-, o- classification a classification into d, ℓ, m, t depending on the temporal quantification. Examples:

(a-d) Every (sometime-)B is an A all the time it exists.
(a-ℓ) Every (sometime-)B is an A all the time it's a B.
(a-m) Every (sometime-)B is an A sometime while it's a B.
(a-t) Every (sometime-)B is an A sometime while it exists.
(e-d) Every (sometime-)B is throughout its existence not an A.
(i-ℓ) Some (sometime-)B is an A all the time it's a B.
(o-t) Some (sometime-)B is sometime in its existence not an A.

'd', 'ℓ' etc. are based on *Easterners*. E.g. d = *ḍarūrī*, ℓ = *lāzim*.

Criteria of validity

It seems that Ibn Sīnā was the first logician to consider these an essential part of logic.
In *Qiyās* i.2 he describes logic as a tool of the other sciences, because it provides *rules* (*qawānīn*) for determining whether an inference, presented in standard form, is valid.

In the West Leibniz (late 17th century) is credited with emphasising a decision procedure for validity.
Leibniz credited Llull, mainly for the algorithmic emphasis.

Forward comparisons (Leibniz, 1930s Tarski)

For all three, the purpose of the criteria is to resolve doubt about whether a given argument is valid.

For Leibniz the doubt is a dispute between two people.
For Tarski the aim is 'to replace subjective scrutiny of definitions and proofs by criteria of an objective nature'.

Ibn Sīnā says little about why resolving the doubt is useful.
But in practice (*ᶜIbāra* ii.5, *Letter to Vizier*) he uses the criteria as a weapon in disputes.

Leibniz's criterion is algorithmic
and hence uses numbers ('*calculemus*').
For both Ibn Sīnā and Tarski the criteria are syntactic and work directly with the sentence forms.

Ibn Sīnā recommends internalising the criteria so that one produces valid inferences from the start.
Tarski follows the custom (Leśniewski etc) of writing names of rules when they are used,
with a similar effect to Ibn Sīnā's recommendation.
Leibniz has nothing similar (as far as I know).

Backward comparisons

The algorithm that Ibn Sīnā offers in *Qiyās* i.2 has two parts:
(1) check whether the premises yield a syllogistic conclusion,
(2) check what is the best conclusion that they yield.

The forms of (1), (2) that he uses for assertoric logic from *Najāt* onwards are taken from Philoponus.

For Philoponus (i.e. Ammonius?), (1) is a set of facts collected up from Aristotle which is helpful for counting the number of syllogisms (i.e. not for a validity algorithm).
Confirmed by Sherwood who quotes (1) without (2).

In Philoponus the rules (1) are not uniform: there are some uniform for all figures, and some for specific figures.

Ibn Sīnā follows Philoponus exactly,
apart from adding a redundant all-figure rule.
Perhaps he adds this in an unsuccessful attempt to find uniform rules.

The later Western rules of distribution and quality are uniform for all figures.
(Ibn Sīnā never had the notion of distribution.)

Several logicians before Ibn Sīnā (Elias, Yahyā, Mattā, apparently Al-Fārābī) claim that logic is a tool through providing criteria of *truth*.
Some (Elias, Yahyā, probably Mattā) added *right and wrong action*.
Stupid propaganda. Ibn Sīnā reprimands Al-Fārābī for this.

Alexander, Ammonius, Philoponus and Al-Fārābī describe logic as a tool or source of tools for the other sciences, but in the loose sense that an education in logic improves reasoning powers.

These comparisons place Ibn Sīnā at the start of the trend to see logic as built around algorithms.
Cf. his proof search algorithm for compound syllogisms (*Qiyās* ix.6), which makes him a significant figure in the history of algorithms generally.

He has been badly served by descriptions like ‘Logic is primarily concerned with intelligibles, not expressions’ (Black 1991).

Explanation

We concentrate on one item: Ibn Sīnā's discussion in *Qiyās* iii.2 of modal *Camestres* with non-necessary minor premise, necessary major premise and necessary conclusion:

No C is a B.
Every A is a B, with necessity.
Therefore no C is an A, with necessity.

Aristotle said that adding 'with necessity' to the conclusion makes this mood invalid.

Aristotle's argument:
Assume the premises and suppose no C is an A, with necessity.
Then no A is a C, with necessity.
By second premise, some B is an A with necessity.
So some B is not a C, with necessity.
But 'nothing prevents' our choosing B and C in the first premise to make this false.

Ibn Sīnā checks out Aristotle's claims, using 'necessary' two-dimensional sentences as he defined them in *Qiyās* i. So 'necessary' is d and 'possible' is t.

First, *Camestres* itself:

(e-t) Every sometimes-C is sometimes not a B.
(a-d) Every sometimes-A is always a B.
(e-d) Therefore every sometimes-C is always not an A.
VALID.

So Aristotle's refutation must be wrong. We check it:

If every sometimes-C is always not an A,
then every sometimes-A is always not a C.
VALID.

If every sometimes-A is always a B,
then some sometimes-B is always an A.
INVALID. BUT . . .

If every sometimes-A is always a B,
then some sometimes-B is sometimes an A.
VALID, and moreover

(i-t) Some sometimes-B is sometimes an A.
(e-d) Every sometimes-A is always not a C.
(o-d) Therefore some sometimes-B is always not a C.
VALID, AND IT'S EXACTLY ARISTOTLE'S CONCLUSION.
!!!

It seems that *Camestres* with necessary conclusion is valid,
and that the steps in Aristotle's refutation of this mood
are also valid.

Do we have a paradox?

Since Aristotle's assumption (that the mood is valid –
for contradiction) is in fact correct for 2D sentences,
Ibn Sīnā can check out the facts for these sentences
and see where Aristotle went wrong.
The mistake has to be at the very end, where Aristotle claims
that his data show invalidity.

Ibn Sīnā's analysis: we can choose B, C so that
(1) Every sometimes-B is at least once not a C, but
(2) every sometimes-C is at least once a B.

Example:
(1) Every human is at least once not laughing, but
(2) every laugher is at least once human.
Both true.

Now add 'Every A is always laughing'.
(No matter what A is.)

This creates an inconsistency:
every A must be sometimes human by (2),
hence sometimes not laughing by (1).

Ibn Sīnā's conclusion:

"So [Aristotle's] statement that 'nothing prevents this' is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms B and C] on its own."

"Aristotle's mistake was to conclude that because ab^a is compatible with the denial of Lab^i, the conjunction of ab^a with Lbc^a must be compatible with the denial of Lab^i."
Paul Thom 1996.

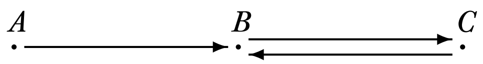
Why did Ross, Smith, Striker etc. all miss Aristotle's mistake?

Probable answer: They accepted Aristotle's conclusion about *Camestres*, so they didn't bother to check his argument.
(Also Striker didn't check Thom's book.)

Ibn Sīnā had the advantage of knowing that Aristotle's conclusion was false for 2D sentences.

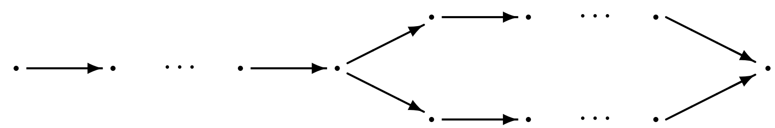
Why did Aristotle make the mistake?

Probable answer: the minimally inconsistent configuration

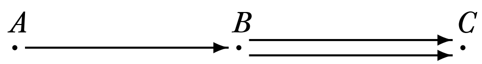


can't occur with assertoric sentences. Every minimally inconsistent set of assertorics has a circular configuration.

With 2D sentences the minimally inconsistent configurations all look like



which allows the above configuration and also



Ibn Sīnā gives an example illustrating this second configuration in *Išārāt* i.7, in the section stating the criteria of validity for first-figure syllogisms.

So we come round again from Explanation to Criteria, closing the circle and the talk.

Wilfrid Hodges, 'Ibn Sina on analysis: 1. Proof search', in *Fields of Logic and Computation: Essays Dedicated to Yuri Gurevich on the Occasion of his 70th Birthday*, Lecture Notes in Computer Science 6300, Springer, Heidelberg 2010, pp. 354–404.

Wilfrid Hodges, *Mathematical Background to the Logic of Ibn Sīnā* (alpha version exists, will go on my website).

Paul Thom, *The Logic of Essentialism*, Kluwer, Dordrecht 1996.