

# Ibn Sīnā states and applies properties of temporal logic

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[4 June 2016: This paper as it stands is virtually the same as the handout at the talk in Cambridge in May 2014 where the results were first presented. The results in it were definitive and don't need any change, but the implications need spelling out better. So far I have only updated the title. I am still hoping to be able to submit the paper to the Proceedings of the SIH-SPAI conference in Paris 2014, where it formed the substance of a plenary lecture.

On the day when the sad death of Muhammad Ali is announced, maybe a little blowing of own trumpets can be allowed in his honour. I believe the main result in this paper is the most incisive discovery about Ibn Sīnā's formal logic since his death in 1037. It confirms beyond any reasonable doubt both the identity of his two-dimensional logic and his high professional standing as a formal logician (though there is plenty of other evidence for both these points).]

## Abstract

We examine a couple of pages from *Qiyās* iii.2 in Ibn Sīnā's *Šifā'*, on necessary conclusions of second figure modal syllogisms. At first sight the passage seems rambling. But if we place it in the context of (1) the argument of Aristotle that Ibn Sīnā is reviewing and (2) Ibn Sīnā's own statement in *Qiyās* i of the temporal predicative sentences that he proposes to discuss, Ibn Sīnā's whole argument falls tightly into place. Ibn Sīnā uses the temporal sentences to examine the correctness of a meta-argument given by Aristotle, and shows that Aristotle relies on a false assumption (it seems generally missed in the West, though Thom made the same criticism of Aristotle in 1996). Aristotle's assumption would have been true for assertoric sentences, but Ibn Sīnā demonstrates that it is false for temporal ones, with the implication that it is in need of justification for modal ones. A revised

version of Ibn Sīnā's main conclusion appears in a prominent place in *Iṣārāt* i.7. Ibn Sīnā's argument is both exact and radical. Nothing else of this logical calibre has yet been recognised in Ibn Sīnā's discussions of modal syllogisms, but this example raises the hope that other jewels may be found when Ibn Sīnā's arguments are placed in their proper context.

## 1 Critique of Aristotle

In *Qiyās* 140.8–141.2 [1] Ibn Sīnā reports an argument used by Aristotle in *Prior Analytics* i.10, 30b18–31, to show that in *Cesare* and *Camestres* in second figure, if the affirmative premise is a necessity statement and the negative premise is not, then the conclusion can't validly be taken to be a necessity statement. Ibn Sīnā makes a brief reply in 141.3–9, and a more substantial one in 142.15–144.5. There are further arguments against Aristotle's position in the surrounding text, but they are less direct.

In fact Aristotle used the same argument at *Prior Analytics* i.9, 30a23–28 and 30b1–5 to show that in first figure, if the minor premise is a necessity statement and the major premise is not, then the conclusion can't validly be taken to be a necessity statement. But at least in *Qiyās*, Ibn Sīnā ignores the application to first figure syllogisms. For the second figure moods he restricts his attention to *Camestres*, though the case of *Cesare* is closely parallel.

This pair of passages in *Qiyās* is very hard to get into, and one might quickly gain the impression that Ibn Sīnā is 'bashing around at random' (as he accuses other logicians of doing at *Qiyās* 495.15f). But it rewards patience, and in fact Ibn Sīnā turns out to be making a sharply focused point that may not have been made in the West until the 1990s.

Here is the passage that Ibn Sīnā is commenting on; I follow Striker's translation of Aristotle's Greek.

- (1) For let  $A$  belong to every  $B$  of necessity but merely belong to no  $C$ . ... if the conclusion is necessary, the result is that  $C$  does not belong to some  $A$  of necessity. For if  $B$  belongs to no  $C$  of necessity, neither will  $C$  belong to any  $B$  of necessity. Yet it is necessary for  $B$  to belong to some  $A$ , given that  $A$  also belonged to  $B$  of necessity. So it is necessary for  $C$  not to belong to some  $A$ . But nothing prevents one from choosing an  $A$  such that  $C$  may belong to all of it. ([5] p. 15, *Prior Analytics* i.10, 30b20–31)

Ibn Sīnā changes the lettering to his usual convention:  $C$  minor term,  $B$

middle term,  $A$  major term (or in Arabic  $j, b, a$ ). Here is a brief exposition of Aristotle's argument, with the lettering as in Ibn Sīnā's version.

We have a valid syllogism in *Camestres*,

No  $C$  is a  $B$ .

(2) Every  $A$  is a  $B$ , with necessity.

Therefore no  $C$  is an  $A$ .

Aristotle claims to show as follows that the mood got by adding 'with necessity' to the conclusion is not valid. He argues: Suppose it is valid. Then we would have

(3) No  $C$  is an  $A$ , with necessity.

By e-conversion of necessary sentences we infer

(4) No  $A$  is a  $C$ , with necessity.

But also by conversion of the second premise

(5) Some  $B$  is an  $A$ , with necessity.

These last two sentences yield

(6) Some  $B$  is not a  $C$ , with necessity.

But this can't be right, because nothing prevents us choosing the matter of the first premise in such a way that every  $B$  is a  $C$ , with possibility. In other words we can choose  $B$  and  $C$  so that in fact no  $C$ s are  $B$ s, but every  $B$  could be a  $C$ . If we choose the matter of the syllogism in this way, then we have succeeded in deducing a falsehood from true premises.

In *Cesare* the argument would be the same, except that the conclusion is 'No  $A$  is a  $C$ ', so the conversion from (3) to (4) becomes unnecessary. In *Celarent* the premise 'No  $C$  is a  $B$ ' becomes 'No  $B$  is a  $C$ ', which converts to 'No  $C$  is a  $B$ ' and hence allows the same argument. Ibn Sīnā doesn't discuss the application of Aristotle's argument to the other first figure moods, though we will see in Section 5 below that he manages to exploit the case of *Barbara* without mentioning it explicitly.

## 2 Ibn Sīnā's two-dimensional sentences

Ibn Sīnā has told us in *Qiyās* i and in *Easterners* (written a few years later) what kinds of sentence he will study in his predicative logic. These sentences resemble the familiar assertoric a-sentences, e-sentences, i-sentences

and o-sentences of Aristotle’s logic, but they all have an extra ingredient, namely a quantification over times. His full picture is quite complicated, but it contains a solid core which we can describe as follows. Suppose the subject term is  $B$ . Then the predicate ‘is (is not) an  $A$ ’ is replaced by one of the following four forms:

- (7)
- d:** ‘is, for all the time while it exists, (not) an  $A$ ’.
  - $\ell$ :** ‘is, for all the time while it is a  $B$ , (not) an  $A$ ’.
  - m:** ‘is, at some time while it is a  $B$ , (not) an  $A$ ’.
  - t:** ‘is, at some time while it exists, (not) an  $A$ ’.

Since the terms now carry a temporal variable, the subject term needs a temporal qualification too; we read the subject term ‘ $B$ ’ as ‘ $B$  at some time during its existence’. Combining the Avicennan types  $d$ ,  $\ell$ ,  $m$  and  $t$  with the Aristotelian types  $a$ ,  $e$ ,  $i$ ,  $o$ , we get sixteen sentence forms  $a$ - $d$ ,  $a$ - $\ell$ ,  $a$ - $m$ ,  $a$ - $t$ ,  $e$ - $d$  etc. I will call sentences of these forms *two-dimensional sentences*, stealing the name from some similar but much later work by Peirce’s student Oscar Mitchell.

We are taking the letters  $d$ ,  $\ell$ ,  $m$  and  $t$  mainly from the initial letters of the sentence descriptions in *Easterners*. For example Ibn Sīnā says that by *ḍarūrī* (‘necessary’) he means what we have described as the  $d$  form;  $\ell$  is for *lāzim*,  $m$  for *muwāfiq* and  $t$  for *mutlāq* (in the *‘āmm* sense). When he comes to review the inference forms in *Qiyās* ii–iv, Ibn Sīnā correlates the  $d$  sentences with Aristotle’s ‘necessary’ (again *ḍarūrī* in Arabic), so that their duals, the  $t$  sentences, have to count as ‘possible’. The  $\ell$  and  $m$  forms get rough treatment by being dumped together under the head ‘absolute’. (Later Arabic logicians rightly criticised Ibn Sīnā for his laxity with the  $\ell$  and  $m$  forms.) Ibn Sīnā scatters throughout this part of *Qiyās* reminders that his primary reading of ‘necessary’ and ‘possible’ is as temporal terms; *Qiyās* 142.15–17 in the passage we are considering is a typical example.

Faced with Aristotle’s argument, Ibn Sīnā will certainly have checked directly whether Aristotle’s claims hold good for two-dimensional sentences. Presumably he will have expected his more serious students to check these facts for themselves too. So we should do likewise.

Aristotle’s initial claim, that the conclusion of the syllogism (2) can’t be validly taken as necessary, is demonstrably false. Thus suppose (and here

and below I suppress the understood qualification ‘while it exists’):

- (8) (e-t) Every sometimes- $C$  is at least once not a  $B$ .  
(a-d) Every sometimes- $A$  is always a  $B$ .

Suppose the individual  $c$  is at some time a  $C$ . Then by the first premise,  $c$  is at least once not a  $B$ , and so by the second premise it is never at any time an  $A$ . In other words

- (9) (e-d) No sometimes- $C$  is ever an  $A$ .

So for these sentences, Aristotle is wrong; in *Camestres* we can always infer a necessity conclusion from a possibility minor premise and a necessity major premise. (Ibn Sīnā asserts this for the parallel case of *Cesare* at *Qiyās* 217.12–14 without bothering to give a proof.)

In this argument we chose to represent the non-necessary minor premise of (2) as a t sentence. This is the weakest kind of two-dimensional sentence that we could have taken here, so the same conclusion applies *a fortiori* if we choose a stronger representation, for example as  $\ell$  or  $m$ .

Since the claim that Aristotle is refuting is in fact true, there must be a mistake somewhere in his refutation. We check the refutation too. Aristotle supposes we have a necessity conclusion (9). This sentence is equivalent to

- (10) No sometimes- $A$  is ever a  $C$ . (= (36) below)

So Aristotle’s e-conversion works.

Next we convert the second premise. Here Aristotle’s conversion fails: if every sometimes- $A$  is always a  $B$ , it certainly doesn’t follow that at least one sometimes- $B$  is always an  $A$ , even given that the second premise is understood to imply that there is at least one  $A$ . The best we can get by conversion is

- (11) Some sometimes- $B$  is at least once an  $A$ . (= (37))

Never mind: Ibn Sīnā will note at *Qiyās* 204.1f, and we can easily confirm it directly, that (11) and (10) together entail

- (12) Some sometimes- $B$  is never a  $C$ . (= (38))

And this was exactly Aristotle’s conclusion. Moreover Aristotle is clearly right that we can find  $B$  and  $C$  so that every sometimes- $C$  is at least once not a  $B$  (= (39)), but also every sometimes- $B$  is at least once a  $C$  (= (41)).

(For example everything that breathes in is at least once not breathing out, but everything that breathes out will at least once breathe in.) So we can choose a matter just as Aristotle claims in his refutation.

This creates a strange situation. For two-dimensional sentences, Aristotle seems to have both a false metatheorem and a correct proof of that metatheorem. In fact there is no doubt at all that the metatheorem is false for two-dimensional sentences, so there must be something wrong in Aristotle's proof of it.

It will not help to point out that Aristotle was not talking about two-dimensional sentences. Ibn Sīnā has followed the logic of Aristotle's argument exactly as Aristotle presents it, and he has reached a false conclusion. So there is a mistake somewhere in Aristotle's procedure, and Ibn Sīnā's discussion is mainly devoted to pinning down what that mistake is.

A methodological footnote: we noted that one of Aristotle's conversions doesn't work for two-dimensional sentences, and that the best one can get is a t converse rather than the claimed d converse. We also noted that Ibn Sīnā at this point in his analysis uses a sentence (37) which doesn't assert necessity. If we didn't realise that Ibn Sīnā was using two-dimensional sentences to track Aristotle's argument, we would have had to suppose that the lack of necessity in (37) was a piece of minor carelessness on Ibn Sīnā's part. Details like this are a significant help for confirming that we are on the same wavelength as Ibn Sīnā.

### 3 Ibn Sīnā tracks down Aristotle's mistake

Working with two-dimensional sentences, Ibn Sīnā can take it as a fact that the syllogism (2) does yield a necessity conclusion. So he can feed this fact into Aristotle's argument as an established truth, not a hypothesis to be disproved by *reductio ad absurdum*. Doing this, he can ask exactly what Aristotle's proof has established. Here is the result of the analysis, in a modern notation.

Aristotle starts with two propositions,  $p(A, B)$  with terms  $A, B$  and  $q(B, C)$  with terms  $B, C$ . His argument establishes that we can indirectly deduce from these two propositions a third proposition  $q'(B, C)$  with terms  $B, C$  which is not a consequence of  $q(B, C)$  alone. Since  $q'(B, C)$  is not a consequence of  $q(B, C)$ , 'nothing prevents one' (to use Aristotle's phrase) from choosing terms  $B, C$  so that  $q(B, C)$  and  $\text{not-}q'(B, C)$  are both true. In

other words, the formal condition

(13)  $q(B, C), \text{not-}q'(B, C)$

is consistent. Nevertheless the formal condition

(14)  $p(A, B), q(B, C), \text{not-}q'(B, C)$

is inconsistent. Aristotle apparently assumed that the consistency of (13) implies the consistency of (14) too. But Ibn Sīnā can give an example which shows that Aristotle was wrong to assume this.

Here is Ibn Sīnā's example:

- |      |                 |   |
|------|-----------------|---|
|      | $q(B, C)$       | Every (sometimes-)human is sometimes not laughing.<br>(= (46), (53))  |
| (15) | Not- $q'(B, C)$ | Every sometimes-laugher is (sometimes) human.<br>I.e. Not: Some sometimes-laugher is never human.<br>(= (48)) |
|      | $p(A, B)$       | Every $A$ laughs all the time.<br>(= (47))  |

The first two propositions are certainly formally consistent, since they are both true. But they are jointly incompatible with the third. It doesn't matter what term we put for  $A$ , so Ibn Sīnā leaves it unspecified. Ibn Sīnā offers a second example to back this up (*Qiyās* 144.3):

- |      |  |  |
|------|--|--|
|      |  | Every (sometimes-)human is sometimes not moving.   |
| (16) |  | Everything that sometimes moves is (sometimes-)human.<br>Every heavenly sphere moves all the time. |

The logical relationships are the same in both examples, but in the second example the second sentence is false and the third is supposedly true, the other way round from the first example.

Ibn Sīnā's first example comes from the argument in *Camestres*:

- |      |       |  |
|------|-------|--|
|      | (e-t) | Every (sometimes-)human sometimes doesn't laugh. |
| (17) | (a-d) | Every $A$ laughs all the time.                   |
|      | (e-d) | Therefore no $A$ is ever human.                  |

The argument is clearly valid. The reader can construct an argument in *Camestres* for the second example too.

Ibn Sīnā sums up his conclusion as follows:

- So [Aristotle's] statement that 'nothing prevents this' is not true.  
(18) The fact is just that nothing prevents it if one takes [the pair of sentences with terms  $B$  and  $C$ ] on its own. (*Qiyās* 144.5)

Ibn Sīnā's analysis is certainly correct. But it also seems to be almost unique in the literature. The earliest version that I know of it in the Western logical literature is by Paul Thom in 1996:

- Aristotle's mistake was to conclude that because  $ab^a$  [ $q(B, C)$  in our analysis] is compatible with the denial of  $Lab^i$  [of  $q'(B, C)$  in our analysis], the conjunction of  $ab^a$  with  $Lbc^a$  [ $p(A, B)$  in our analysis] must be compatible with the denial of  $Lab^i$ . ([6] p. 125)

As far as I know, Ibn Sīnā's observation is not in any of the standard modern commentaries.

Most traditional logicians and their modern commentators seem to agree with Aristotle that *Camestres* with non-necessary minor premise and necessary major premise can't be given a necessary conclusion. Probably the fact that they agree with the conclusion of Aristotle's refutation makes them careless about checking the details of that refutation. Thus Ross:

- In order to prove that a certain conclusion does not follow, he supposes that it does, and shows that if it did, it would lead to knowledge which certainly cannot be got from the original premisses. ([4] p. 319)

No. Aristotle shows (using Ibn Sīnā's lettering) that the conclusion would lead to knowledge about  $B$  and  $C$  which certainly can't be got from the original premise about  $B$  and  $C$ . But Aristotle does nothing to check that the knowledge can't be got by taking both of the original premises together. Several other commentators follow Ross in this misrepresentation. So there is an easy explanation of why at least the commentators failed to spot the mistake. It probably doesn't explain why Aristotle himself made the mistake, since he at least will have taken his argument seriously.

In this passage in *Qiyās* iii.2, Ibn Sīnā confines himself to criticising Aristotle. But he returns to the matter in *Išārāt*. Here there is no mention of Aristotle; instead Ibn Sīnā presents his conclusion as a new fact of logic. Before

we turn to that, it will be helpful to review some facts about syllogistic in general. These facts give a more plausible and interesting explanation of why Aristotle himself made his mistake.

## 4 The shapes of syllogisms

We consider formal assertoric sentences, i.e. assertoric sentences with term letters  $A, B, C$  etc., not meaningful terms. Likewise we consider formal assertoric syllogisms, which are syllogisms built up from such sentences. A formal assertoric syllogism is valid if and only if it yields a valid inference whenever the term letters are consistently replaced by meaningful terms. We follow the traditional convention that each assertoric sentence contains two distinct terms.

A valid simple assertoric syllogism can be converted into an inconsistent set of assertoric sentences by replacing the conclusion by its contradictory negation. Ibn Sīnā was of course well aware of this. Quite often he begins his analysis of a syllogism by saying *wa-'illā* ('for otherwise'), and then switching the conclusion to its negation. So we can classify the valid syllogisms by classifying inconsistent sets of assertoric sentences. In fact all the simple syllogisms recognised as valid in traditional logic give rise to inconsistent sets of sentences with a further property: they are *minimally inconsistent*, in the sense that no proper subset is inconsistent.

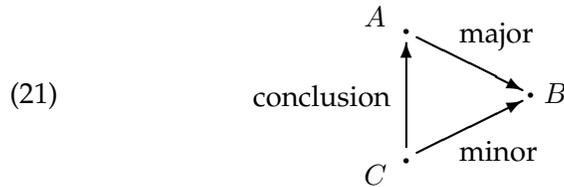
So valid simple assertoric syllogisms give rise to minimally inconsistent sets of three assertoric sentences. The number three plays no role in the facts of the case, so I will generalise to arbitrary finite minimally inconsistent sets of assertoric sentences.

Let  $T$  be a set of assertoric sentences. A useful tool for analysing  $T$  is the 'graph'  $\Gamma(T)$  of  $T$ . This graph is a diagram of dots and arrows, written as follows. We put a dot for each term occurring in sentences of  $T$ . For each sentence  $\phi$  in  $T$  we draw an arrow from the dot representing the subject of  $\phi$  to the dot representing the predicate of  $\phi$ . If there are two sentences with subject term  $A$  and predicate term  $B$ , we draw two arrows from  $A$  to  $B$ .

Note that the contradictory negation of a sentence  $\phi$  has the same subject term and the same predicate term as  $\phi$ . So we get the same graph if we take the set of all sentences in a syllogism  $\Sigma$  as we get from the set of sentences got by replacing the conclusion of  $\Sigma$  by its contradictory negation. Thus it makes good sense to talk of the 'graph' of a syllogism.

This notion of graphs extends to other kinds of sentence which have a subject term and a predicate term, for example modal sentences. Thus the

syllogism (2) above has the following graph:



Note that if we travel forwards along the arrows, there is a trail from  $C$  to  $A$  and then to  $B$ , and a second trail from  $C$  direct to  $B$ . The two trails are distinct but have the same initial point  $C$  and the same terminal point  $B$ .

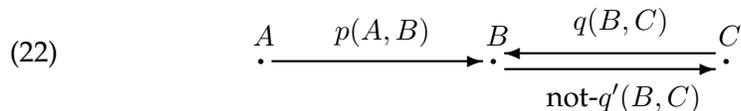
We call a graph with these features a *two-trail graph*; the two distinct trails have the same initial point and the same terminal point, and together they include all the arrows in the graph. One can show:

*FACT ONE. Every minimally inconsistent set of assertoric sentences has a two-trail graph where the two trails have no arrows in common, so that the graph can be drawn as a circle.*

For Ibn Sīnā’s assertoric syllogisms this is strictly provable, using his convention that when the subject term is empty, an affirmative sentence is false and a negative sentence is true. In Aristotle’s case one has to make some plausible assumptions about what he intended. For completeness I add that we have to allow the case where all the arrows lie in one trail, from the initial point back to itself, so that the second trail has length 0; this is the case of fourth figure syllogisms.

*FACT TWO. The case overlooked by Aristotle in the previous section can never occur with assertoric sentences.*

To be precise, Aristotle’s case consists of two sentences  $p(A, B)$  and  $q(B, C)$  which are together inconsistent with a third sentence  $\text{not-}q'(B, C)$ , though  $q(B, C)$  is consistent with  $\text{not-}q'(B, C)$ . Taking the graph of this set of sentences, note that in  $q(B, C)$  the subject and predicate are switched around:  $C$  is the subject and  $B$  is the predicate. So the graph is as follows:



Assuming that the sentences are assertoric, Fact One shows that the set of

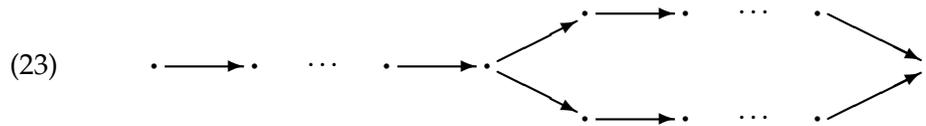
three sentences is not minimally inconsistent, so we can remove one sentence without damaging the inconsistency. If we remove  $p(A, B)$  then we are left with two sentences that Aristotle chooses to be consistent. If we remove either  $q(B, C)$  or  $\text{not-}q'(B, C)$  we are left with a graph that by Fact One still doesn't correspond to any minimally inconsistent set. Also individual assertoric sentences are always consistent. This proves Fact Two.

Fact Two seems by far the most likely reason why Aristotle and his more conscientious commentators failed to notice the gap in Aristotle's reasoning. These logicians wouldn't have been able to give strict proofs of Facts One and Two, but they would surely have known at least Fact One from experience. (Cf. Thom [6] p. 24; in Thom's words, the circular form is 'one of the utterly basic properties of Aristotle's assertoric syllogistic'.)

Ibn Sīnā's example (15) shows that Fact One is no longer true for two-dimensional sentences. Instead one can show:

*FACT THREE. Every minimally inconsistent set of two-dimensional sentences has a two-trail graph where the two trails can have a common initial segment (but no common final segment).*

In other words, the graph of a minimally inconsistent set of two-dimensional sentences always looks like this:



where either or both of the tail on the left and the lower half of the circle may have length 0.

So for the graphs of minimally inconsistent sets of two-dimensional sentences, the one new feature is the possible tail on the left, which can have any length. It seems pretty certain that Ibn Sīnā would not have been able to prove Fact Three, which makes it all the more impressive that he did notice and illustrate the new case. In Ibn Sīnā's example (15) above, the tail has length one and the lower half of the circle is empty.

## 5 *Iṣārāt* reconsiders the point

When he came to write *Iṣārāt*, Ibn Sīnā decided to emphasise this new feature of his two-dimensional logic. So he gave it prominence by including

it in his statement of the *qawānīn* for validity of first-figure syllogisms. The example he gives there is a little different; its graph is



Here is Inati's translation:

- You must know that if the minor premise is necessary and the major purely concrete, belonging to the genus of the concrete, in the sense that as long as the subject is qualified by that with which it is qualified, no syllogism with true premises is formed.
- (25) For the major premise is false, since if we say, "Every C is by necessity B", and then say, "Every B is qualified as A as long as it is qualified as B, and not always", we judge that all that which is qualified as B is qualified thus, only at a certain time, and not always. This is opposite the minor premise. ([2] 145.5–9, [3] p. 399f)

The lefthand sentence here is the minor premise

- (26) Every *C* is a *B* throughout its existence.

The major premise is a conjunction of two sentences which provide the two sentences on the right:

- (27) Every *B* is an *A* all the time that it is a *B*.  
Every *B* fails to be an *A* at least once during its existence.

Note that the first sentence here is an *ℓ* sentence. Ibn Sīnā's previous example used only d and t sentences.

The general theory of two-dimensional sentences throws some light on this situation. One can show (there is no space to show it here) that any two-dimensional counterexample to Aristotle's assumption must have one of the two forms (22) or (24). The lefthand sentence must be a d sentence, though it could be i-d rather than a-d (as Ibn Sīnā himself notices at 143.12). In case (22) the two righthand sentences can both be t, as in Ibn Sīnā's example. In case (24) one of the two righthand sentences can be t, but the other needs to be at least as strong as *ℓ*. (In order of decreasing strength the Avicennan forms are d, *ℓ*, m, t.) So Ibn Sīnā by his examples shows that he understands the range of possibilities.

Probably Ibn Sīnā's example in *Iṣārāt* came by examining how Aristotle's refutation works for *Barbara* with a d minor premise and an *ℓ* major

premise; we can deduce a d conclusion from these premises, but not from anything weaker. Aristotle doesn't spell out the details, but presumably he would have reasoned as follows. Suppose given

- (28) Every  $C$  is a  $B$ , with necessity.  
Every  $B$  is an  $A$ .

Suppose we can deduce

- (29) Every  $C$  is an  $A$ , with necessity.

Then by conversion of the first premise

- (30) Some  $B$  is a  $C$ , with necessity.

From (30) and (29) we deduce

- (31) Some  $B$  is an  $A$ , with necessity.

But this is a statement about  $B$  and  $A$  not deducible from the second premise, etc.

For Ibn Sīnā with two-dimensional sentences, the conclusion can be taken as d provided that we take the second premise at least as strong as  $\ell$  (as Ibn Sīnā illustrates at *Qiyās* 129.1):

- (32) Every sometimes- $C$  is always a  $B$ .  
Every  $B$  is an  $A$  so long as it is a  $B$ .

Conversion of the first premise gives only

- (33) Some sometimes- $B$  is sometimes a  $C$ .

But as at *Qiyās* 203.10, this together with the conclusion

- (34) Every sometimes- $C$  is always an  $A$ .

yields the conclusion

- (35) Some sometimes- $B$  is always an  $A$ .

exactly as Aristotle supposes. We get the example in *Iṣārāt* by taking the two premises in (32) together with the negation of (35).

You may have noticed that Ibn Sīnā gives the two righthand arrows of (24) not as two separate sentences but as the two conjuncts of a single

premise. He might be making the following point. Aristotle presented a number of syllogisms in which a premise expresses that something is contingent. These premises are conjunctions, and you might be tempted to think (for example using Fact One) that anything that followed from the premises would already follow from just one of the conjuncts together with the other premise. This would be a sound observation if the sentences in question were all assertoric. But Ibn Sīnā's example shows that the observation is not sound for two-dimensional sentences, and so there is no reason to expect it to be sound for modal sentences in general.

## 6 Some observations

1. The role that Ibn Sīnā's two-dimensional sentences play in relation to Aristotle's modal syllogistic deserves some comment. The definition of the *d*, *ℓ*, *m* and *t* sentences makes it completely determinate what logical relations hold between these sentences. So Ibn Sīnā can use them as a testbed for Aristotle's modal claims. In this respect they play a similar role to Kripke structures in relation to modal statements. This is a completely separate point from any claim that Ibn Sīnā himself thinks of his two-dimensional sentences as describing anything like a Kripke frame or an indexed family of possible worlds. No such claim is needed for making sense of the passage of *Qiyās* under discussion.

2. For making sense of Ibn Sīnā's discussion, it was in effect necessary not only to call on Ibn Sīnā's two-dimensional sentences, but also to know some of their metatheory. The indications are that Ibn Sīnā knew these sentences well enough to be able to use various metatheoretical properties of them.

3. Where such methods do work, they are likely to depend on some assumption about the relation between the two-dimensional sentences and Aristotle's modal sentences. It would have been hugely helpful if Ibn Sīnā himself had been more up front about this. As it is, there seems to be no strong case for arguing that Ibn Sīnā thought Aristotle wanted his modal sentences to be read as *meaning* temporal sentences. *Qiyās* 142.15–17 could be a methodological remark rather than a historical one. But Ibn Sīnā's method of discussion may well presuppose that the two-dimensional sentences, understood as temporal, are one allowed interpretation of Aristotle's modal sentences.

4. I think we don't yet know what other parts of Ibn Sīnā's modal theorising can be explained in a similar way. It seems high priority to check

this, given that the points which Ibn Sīnā makes in the present discussion are of a high calibre in exactness, methodology and independent insight.

## 7 Translation

Then there is the approach using absurdity, which was referred to in the proof of their claim about an absolute proposition following from a premise-pair consisting of a negative universally quantified minor premise and a universally quantified affirmative [necessary] major premise. [They claim that it] entails a negative universally quantified absolute conclusion, saying: 140.8  
140.10

- (36) If with necessity no  $C$  is an  $A$ , then with necessity no  $A$  is a  $C$ .

Also the sentence

- (37) Some  $B$  is an  $A$ .

is true, so that

- (38) With necessity not every  $B$  is a  $C$ ,

when it had been [assumed] that

- (39) No  $C$  is a  $B$ , without necessity.

But there is nothing to prevent its converse

- (40) No  $B$  is a  $C$ , without necessity.

from being a negative statement that is not true with necessity of anything at all. And in that case /141/ there is nothing to prevent its being the case that

- (41) Every  $B$  could be a  $C$ .

But our assumption [that the conclusion can be taken as necessary] has forced it to be the case that with necessity not every  $B$  is a  $C$ .

The first thing to be said to them is that when [they claim that], given that 141.3

(42) No *C* is a *B*, without necessity.

there is nothing to prevent the converse statement 'Every *B* is a *C*' from being true without necessity, so that on their own this sentence could be true together with (39) — [when they claim this], it doesn't have to be the case that there is nothing to prevent that when a certain matter is taken into account. Even if we grant that in some matter there is nothing to prevent that, still we can ask, given a premise-pair like this one, why there shouldn't be something that forces the matters that fit with this premise-pair to be restricted to matters containing something that does prevent that? Perhaps, given that it is true that 141.5

(43) Every *A* is a *B* with necessity.

this does prevent its being true conversely that

(44) Every *B* is a *C*.

...

[Aristotle] just meant that the lack of necessity of the truth [of 'Every *B* is a *C*'] should be in terms of the predication, not in terms of the quantifier. So his statement 142.15

(45) No *C* is a *B*.

is meant in the sense that each *C* has *B* denied of it at some time, and not denied of it at some time; so it's not required that [*B*] is denied [of *C*] permanently, but rather it's possible that *B* is a property that *C* has but not permanently. So let us consider how /143/ a syllogistic premise pair can be composed of a proposition of this kind together with a necessity premise in such a way that this absurdity follows from it.

We say the following. Suppose we say

(46) Every human is not actually laughing, i.e. at times when he is not laughing.

and then we say:

(47) Every *A* with necessity actually laughs.

giving the required syllogism, where we have to say that

(48) Everything that actually laughs is human.

So it follows that

(49) Every *A* is human.

and then it follows that

(50) Some human is *A*.

But [we assumed that]

143.5

(51) Every *A* is a laugher with necessity.

So

(52) Some human is a laugher with necessity.

But we had that

(53) No human fails to have laughing sometimes false of him.

This is an absurdity.

So therefore the truth of the sentence

(54) Every *B* is a *C*.

prevents the truth of the sentence

(55) With necessity, every *A* is a *B*.

And the truth of the sentence

(56) Every *A* is a *B*, with necessity.

prevents the truth of the sentence

(57) Every *B* is a *C* (not with necessity).

If the two were true together, the aforementioned impossibility would occur. So then when it is true that

143.10

(58) Every *B* is a *C*.

then it has to be false that

(59) Every *A* is a *B* with necessity.

and the truth of the former prevents the truth of the latter.

In matters like this it is impossible for there to be something that actually laughs with necessity, so that *A* can be [taken to be] this laugher. The truth is that ‘actually laughing’ is a description that can only be taken as applying to humans, and for humans it is not a description that holds with necessity. If something other than a human was said to laugh with necessity, it would be true by conversion that some laugher was not human, and how could that be? And if something besides a human was said to laugh with necessity, so that the description was taken to include humans, /144/ then there is no way that it could be true conversely that humans sometimes don’t laugh (absolutely) and sometimes do laugh. 143.15

For an example where the sentence

(60) Every *A* is a *B*.

is true and [makes it] impossible to affirm the converse of the other sentence, take *C* to be ‘human’, *B* to be ‘actually moving’ and *A* to be ‘the heavenly sphere’. But nothing would justify you in saying

(61) Everything that actually moves is a human.

So [Aristotle’s] statement that ‘nothing prevents this’ is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms *B* and *C*] on its own. 144.5

## 8 Notes on the text

140.9 ‘their claim’: Aristotle’s claim.

140.10 Following several mss, add *ḍarūriyyatin* after *mūjibatin*.

141.3 Read *al-’umūri*, as in the parallel passage at 144.5.

- 142.15 ‘in terms of the quantifier’: Ibn Sīnā uses this phrase to refer to sentences in which the temporal quantification has wider scope than the standard quantification, so that the temporal modality is in effect *de dicto*.
- 143.1 ‘absurdity’: At 140.8f Ibn Sīnā described Aristotle’s argument as ‘the approach using absurdity’. That description makes sense: Aristotle wants to show that a certain syllogistic mood with a necessity conclusion is invalid, and he reasons by assuming it is valid and claiming to deduce the logical impossibility that the premises entail more than they entail. But the ‘absurdity’ that Ibn Sīnā himself will deduce at 143.7 below is not this one. Rather, he is going to show up a false assumption in Aristotle’s argument by constructing a set of sentences that is inconsistent in a way Aristotle had overlooked. So the reference to ‘absurdity’ here is a little playful. Text: the mss reading of *yālzamuhu* is probably preferable to *yālzamu*.
- 143.3 ‘the required syllogism’: the reference is to Aristotle’s example as stated by Ibn Sīnā earlier. Here he put ‘actually laughing’ for *B* and ‘human’ for *C*. Aristotle had said that nothing prevents us from choosing *B* and *C* so that every *B* is sometimes a *C*; Ibn Sīnā’s choice of *B* and *C* has that property, because everything that laughs is (sometimes) human, as Ibn Sīnā points out in the next line.
- 143.7 Ibn Sīnā is careless over the key point. (54) is not on its own incompatible with (55), but it is incompatible with (55) given the initial assumption (46) that every *C* is sometimes not a *B*. The fact that Ibn Sīnā doesn’t feel it necessary to repeat an initial assumption is typical for him; for example it plays a central role in his explanation of *reductio ad absurdum* in *Qiyās* viii.3.

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