

Ibn Sīnā uncovers a subtle mistake in Aristotle's modal logic

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<http://wilfridhodges.co.uk/arabic42.pdf>



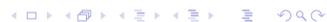
Aristotle's argument

No C is a B

Nec every A is a B



Nec no C is an A



Aristotle claims that the following argument (modal *Camestres*) can't have 'with necessity' added to the conclusion.

No C is a B .

Every A is a B , with necessity.

Therefore no C is an A .

This is at *Prior Analytics* i.10, 30b20–31.

(Aristotle has B, A for A, B . We follow Ibn Sīnā.)



Aristotle's argument

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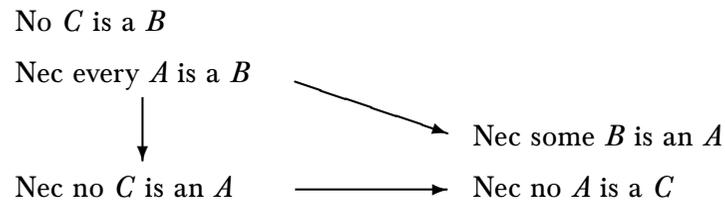
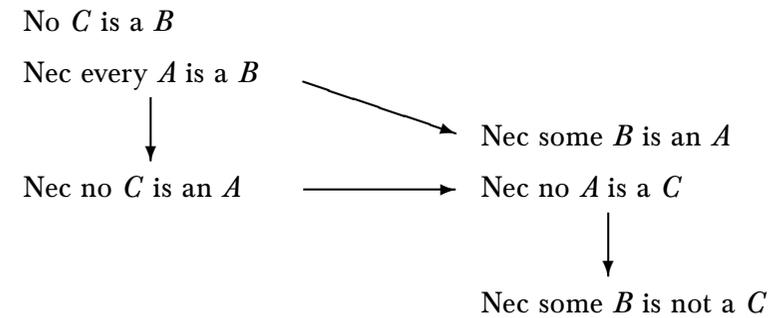
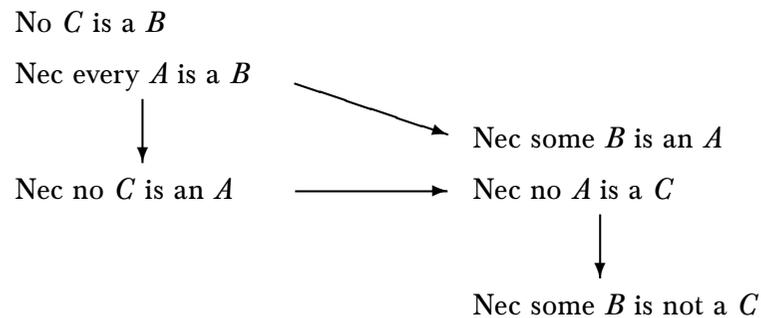


Nec no C is an A



Nec no A is a C



Aristotle's argument**Aristotle's argument****Aristotle's argument**

'But nothing prevents one from choosing a B so that possibly every B is a C .'

So if the conclusion was valid 'with necessity', then we could derive a false conclusion from true premises.

Robin Smith (commenting on *Prior Analytics* i.9, 30a25-28, a parallel argument):

'Aristotle's technique is sophisticated and flawless.'

Ibn Sīnā heads off in a different direction

For Ibn Sīnā, the standard Aristotelian sentence forms

- (a) Every C is a B .
- (e) No C is a B .
- (i) Some C is a B .
- (o) Not every C is a B .

are a gross oversimplification of real language.

In particular Aristotle ignores that there are nearly always implied *time conditions* in both subject C and predicate B . Ibn Sīnā describes several forms that these conditions can take.



- (a-d) Every (sometime-) B is an A all the time it exists.
- (a-ℓ) Every (sometime-) B is an A all the time it's a B .
- (a-m) Every (sometime-) B is an A sometime while it's a B .
- (a-t) Every (sometime-) B is an A sometime while it exists.
- (e-d) Every (sometime-) B is throughout its existence not an A .
- (i-ℓ) Some (sometime-) B is an A all the time it's a B .
- (o-t) Some (sometime-) B is sometime in its existence not an A .

'd', 'ℓ' etc. are based on names suggested by Ibn Sīnā.

In order of decreasing strength:

d = *darur*, ℓ = *lazim*, m = *muwafiq*, t = *mutlaq al-^camm*.



'Two-dimensional' is my shorthand for those examples of Ibn Sīnā's where (1) the time quantification has narrow scope and (2) there are no subtleties connected with natural language existential quantification.

The name 'two-dimensional' comes from Oscar Mitchell who in 1883 independently made a move like Ibn Sīnā's.

2D sentences have besides the a-, e-, i-, o- classification a classification into d, ℓ, m, t depending on the temporal quantification. Examples:



Ibn Sīnā reckons that 'all the time it exists' is a kind of *necessity*,

and 'sometime in its existence' is a kind of *possibility*.

So if Aristotle's modal arguments work at all, they should still work if we put d sentences for 'Necessarily' and t sentences for 'Possibly'.

In his *Qiyas* iii.2 Ibn Sīnā tries this with the argument that Aristotle rejected above.



No C is a B .

Every A is a B , with necessity.

Therefore no C is an A , with necessity.

2D version, using weakest possible (t) for the assertoric premise:

(e-t) Every sometimes- C is sometimes not a B .

(a-d) Every sometimes- A is always a B .

(e-d) Therefore every sometimes- C is always not an A .

VALID.

So Aristotle's refutation must be wrong. Ibn Sīnā checks it:

If every sometimes- C is always not an A ,
then every sometimes- A is always not a C .

VALID.

If every sometimes- A is always a B ,
then some sometimes- B is always an A .

INVALID. BUT ...

If every sometimes- A is always a B ,

then some sometimes- B is sometimes an A .

VALID, and moreover

(i-t) Some sometimes- B is sometimes an A .

(e-d) Every sometimes- A is always not a C .

(o-d) Therefore some sometimes- B is always not a C .

VALID, AND IT'S EXACTLY ARISTOTLE'S CONCLUSION.

!!!

It seems that

- ▶ *Camestres* with necessary conclusion is valid.
- ▶ The steps in Aristotle's refutation of *Camestres* with necessary conclusion are also valid.

Do we have a paradox?

Aristotle claims that his data show we can choose B and C so that a false conclusion is derivable from true premises.

Ibn Sīnā checks what happens if we try to do this, using 2D sentences.

Now add the other premise ‘Every A is always laughing’.
(No matter what A is.)

This creates an inconsistency:
every A must be sometimes human by (2),
hence sometimes not laughing by (1).

Ibn Sīnā’s analysis: we can choose B , C so that
(1) Every sometimes- B is at least once not a C , but
(2) every sometimes- C is at least once a B .

Example:

(1) Every human is at least once not laughing, but
(2) every laugher is at least once human.
Both true.

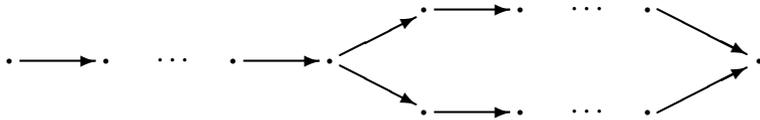
Ibn Sīnā’s conclusion:

“So [Aristotle’s] statement that ‘nothing prevents this’ is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms B and C] on its own.”

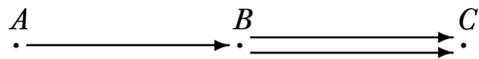
Paul Thom 1996 reaches the same conclusion—
apparently the first Westerner to do so:

“Aristotle’s mistake was to conclude that because ab^a is compatible with the denial of Lab^i , the conjunction of ab^a with Lbc^a must be compatible with the denial of Lab^i .”

With 2D sentences the minimally inconsistent configurations all look like



which allows the above configuration and also



Ibn Sīnā knew this second configuration. In his late *Iṣarāt* i.7 he gives a minimal inconsistent set illustrating it:

- (a,d) Every A is a B throughout its existence.
- (a, ℓ) Every B is a C throughout the time while it's a B .
- (e,d) No B is a C throughout its existence.

Note the use of an ℓ sentence. Ibn Sīnā is right; nothing weaker than an ℓ will work for this configuration.

Ibn Sīnā

