Ibn Sīnā uncovers a subtle mistake in Aristotle's modal logic

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St Andrews, July 2014

`http://wilfridhodges.co.uk/arabic42.pdf`

Aristotle claims that the following argument (modal *Camestres*) can't have 'with necessity' added to the conclusion.

No C is a B.
Every A is a B, with necessity.
Therefore no C is an A.

This is at *Prior Analytics* i.10, 30b20–31.
(Aristotle has B, A for A, B. We follow Ibn Sīnā.)

Aristotle's argument

$$\begin{tikzcd}
\text{No } C \text{ is a } B \\
\text{Nec every } A \text{ is a } B \arrow[d] \\
\text{Nec no } C \text{ is an } A
\end{tikzcd}$$

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\text{Nec every } A \text{ is a } B \arrow[d] & \\
\text{Nec no } C \text{ is an } A \arrow[r] & \text{Nec no } A \text{ is a } C
\end{tikzcd}$$

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\begin{tikzcd}
\text{No } C \text{ is a } B & \\
\text{Nec every } A \text{ is a } B \arrow[dd] \arrow[dr] & \\
 & \text{Nec some } B \text{ is an } A \\
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\end{tikzcd}
$$

'But nothing prevents one from choosing a B so that possibly every B is a C.'

So if the conclusion was valid 'with necessity',
then we could derive a false conclusion from true premises.

Robin Smith (commenting on *Prior Analytics* i.9, 30a25–28, a parallel argument):

> 'Aristotle's technique is sophisticated and flawless.'

Ibn Sīnā heads off in a different direction

For Ibn Sīnā, the standard Aristotelian sentence forms

(a) Every C is a B.
(e) No C is a B.
(i) Some C is a B.
(o) Not every C is a B.

are a gross oversimplification of real language.

In particular Aristotle ignores that there are nearly always implied *time conditions* in both subject C and predicate B. Ibn Sīnā describes several forms that these conditions can take.

'Two-dimensional' is my shorthand for those examples of Ibn Sīnā's where (1) the time quantification has narrow scope and (2) there are no subtleties connected with natural language existential quantification.

The name 'two-dimensional' comes from Oscar Mitchell who in 1883 independently made a move like Ibn Sīnā's.

2D sentences have besides the a-, e-, i-, o- classification a classification into d, ℓ, m, t depending on the temporal quantification. Examples:

(a-d) Every (sometime-)B is an A all the time it exists.
(a-ℓ) Every (sometime-)B is an A all the time it's a B.
(a-m) Every (sometime-)B is an A sometime while it's a B.
(a-t) Every (sometime-)B is an A sometime while it exists.
(e-d) Every (sometime-)B is throughout its existence not an A.
(i-ℓ) Some (sometime-)B is an A all the time it's a B.
(o-t) Some (sometime-)B is sometime in its existence not an A.

'd', 'ℓ' etc. are based on names suggested by Ibn Sīnā.
In order of decreasing strength:
d = *ḍarūrī*, ℓ = *lāzim*, m = *muwāfiq*, t = *muṭlaq al-ʿāmm*.

Ibn Sīnā reckons that 'all the time it exists' is a kind of *necessity*,
and 'sometime in its existence' is a kind of *possibility*.

So if Aristotle's modal arguments work at all,
they should still work if we put d sentences for 'Necessarily' and t sentences for 'Possibly'.

In his *Qiyās* iii.2 Ibn Sīnā tries this with the argument that Aristotle rejected above.

No C is a B.
Every A is a B, with necessity.
Therefore no C is an A, with necessity.

2D version, using weakest possible (t) for the assertoric premise:

(e-t) Every sometimes-C is sometimes not a B.
(a-d) Every sometimes-A is always a B.
(e-d) Therefore every sometimes-C is always not an A.
VALID.

So Aristotle's refutation must be wrong. Ibn Sīnā checks it:

If every sometimes-C is always not an A,
then every sometimes-A is always not a C.
VALID.

If every sometimes-A is always a B,
then some sometimes-B is always an A.
INVALID. BUT . . .

If every sometimes-A is always a B,
then some sometimes-B is sometimes an A.
VALID, and moreover

(i-t) Some sometimes-B is sometimes an A.
(e-d) Every sometimes-A is always not a C.
(o-d) Therefore some sometimes-B is always not a C.
VALID, AND IT'S EXACTLY ARISTOTLE'S CONCLUSION.
!!!

It seems that

- *Camestres* with necessary conclusion is valid.
- The steps in Aristotle's refutation of *Camestres* with necessary conclusion are also valid.

Do we have a paradox?

Aristotle claims that his data show we can choose B and C so that a false conclusion is derivable from true premises.

Ibn Sīnā checks what happens if we try to do this, using 2D sentences.

Ibn Sīnā's analysis: we can choose B, C so that
(1) Every sometimes-B is at least once not a C, but
(2) every sometimes-C is at least once a B.

Example:
(1) Every human is at least once not laughing, but
(2) every laugher is at least once human.
Both true.

Now add the other premise 'Every A is always laughing'. (No matter what A is.)

This creates an inconsistency:
every A must be sometimes human by (2),
hence sometimes not laughing by (1).

Ibn Sīnā's conclusion:

"So [Aristotle's] statement that 'nothing prevents this' is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms B and C] on its own."

Paul Thom 1996 reaches the same conclusion—
apparently the first Westerner to do so:

"Aristotle's mistake was to conclude that because ab^a is compatible with the denial of Lab^i, the conjunction of ab^a with Lbc^a must be compatible with the denial of Lab^i."

Why did Ross, Smith, Striker etc. all miss Aristotle's mistake?

Probable answer: They accepted Aristotle's conclusion about *Camestres*, so they didn't bother to check his argument.
(Also Striker didn't check Thom's book.)

Ibn Sīnā had the advantage of knowing that Aristotle's conclusion was false for 2D sentences.

Remark

Ibn Sīnā's d and t sentences are formally almost identical with the *semantical interpretations* that Spencer Johnston gives for Buridan's divided Necessity and Possibility statements.

- Ibn Sīnā's 'times' correspond to Johnston's 'worlds'.
- Ibn Sīnā's 'while it exists' corresponds to Johnston's o function, which is in the semantics but I think not explicit in Buridan himself.
- Johnston defines an accessibility relation on the worlds. Ibn Sīnā has nothing corresponding, but I think it's redundant in Johnston's semantics anyway.

Roughly speaking,
Ibn Sīnā's 2D sentences can play the role of Kripke structures, but in the object language, not in a logical metalanguage.

In general Ibn Sīnā was very resistant to metatheory.
He believed that all inferences in logic rest on direct intuitions of implications between propositions expressed in natural language.

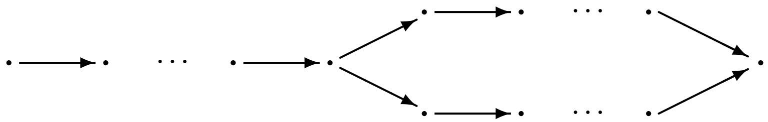
We return to Aristotle. Why did he make his mistake?

Probable answer: the minimally inconsistent configuration

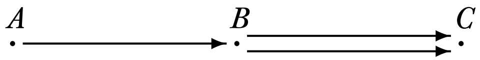
$$\overset{A}{\bullet} \longrightarrow \overset{B}{\bullet} \rightleftarrows \overset{C}{\bullet}$$

(where an arrow from A to B represents a sentence with subject term A and predicate term B)
can't occur with assertoric sentences.
Every minimally inconsistent set of assertorics has a circular configuration.

With 2D sentences the minimally inconsistent configurations all look like



which allows the above configuration and also



Ibn Sīnā

Ibn Sīnā knew this second configuration.
In his late *Išārāt* i.7 he gives a minimal inconsistent set illustrating it:

(a,d) Every A is a B throughout its existence.
(a,ℓ) Every B is a C throughout the time while it's a B.
(e,d) No B is a C throughout its existence.

Note the use of an ℓ sentence. Ibn Sīnā is right; nothing weaker than an ℓ will work for this configuration.