Ibn Sīnā states and applies properties of temporal logic

Wilfrid Hodges
SIHSPAI, Paris October 2014
http://wilfridhodges.co.uk/arabic45.pdf

We consider Ibn Sīnā, Qiyās i–iv from his Šifa’.

In these books Ibn Sīnā introduces two forms of logic.

The second (mainly in Qiyās iii, iv) is Aristotle’s logic of ‘mixed syllogisms’ as reported in the Arabic Aristotle.

This logic uses three alethic modalities: ‘necessary’ (darūt), ‘possible/contingent’ (mumkin) and ‘absolute’ (muṭlaq).

The first logic is one built around some sentences that Ibn Sīnā introduces in Qiyās i.3 and in the parallel passage of Maṣriqiyūn, written a little later than Qiyās but before ṯārāt as we have it.

I will call these sentences two-dimensional, following Oscar Mitchell who studied similar sentences in the 1880s.

The second dimension is time. Example:

Everybody who writes moves his hand all the time he is writing.

Formalised examples:

(a–d) Every (sometime-)B is an A all the time it exists.
(a–ℓ) Every (sometime-)B is an A all the time it’s a B.
(a–m) Every (sometime-)B is an A sometime while it’s a B.
(a–t) Every (sometime-)B is an A sometime while it exists.
(e–d) Every (sometime-)B is throughout its existence not an A.
(i–ℓ) Some (sometime-)B is an A all the time it’s a B.
(o–t) Some (sometime-)B is sometime in its existence not an A.

‘d’, ‘ℓ’ etc. are based on names suggested by Ibn Sīnā.

In order of decreasing strength:
d = darūt, ℓ = lazim, m = muwaqqīq, t = muṭlaq al-c amm.
**Major Problem:** To disentangle the two-dimensional logic from the alethic modal logic in *Qiyās* iii, iv and show how Ibn Sīnā relates the two logics.

Most (all?) discussions in print solve this problem by ignoring or downgrading the two-dimensional logic.

One possible reason is that *Mašriqīyyūn* is generally not taken seriously.

Ibn Sīnā himself compares *Mašriqīyyūn* with Šīfa’ in prefaces to both:

Šīfa’ is more detailed, but biased towards the Peripatetics. *Mašriqīyyūn* removes that bias.

(More details in Gutas’ book.)

That account seems exactly right. Will somebody please get us a properly edited text of *Mašriqīyyūn*?

---

I think I can solve the Major Problem in broad framework. A scientific account will take time and effort (in progress!), and I won’t attempt it here.

It depends crucially on understanding what Ibn Sīnā means by the *qawāntn* (rules) of logic, a topic he emphasises in *Qiyās* i.2.

If I am right, there will always be work to do on fitting particular passages into the framework.

---

We’ll concentrate on one passage, *Qiyās* iii.2, pp. 140–144. In this passage Ibn Sīnā uses the two-dimensional logic to solve a previously unrecognised problem in Aristotle’s text, and to show a novel fact about the possible shapes of modal inferences—a fact that he will develop in *Iṣarat*.

His use of two-dimensional logic in this passage is sophisticated and accurate to the fine detail. It could still be accepted as a research contribution in a modern logic journal.
Aristotle claims that the following argument (modal *Camestres*) can’t have ‘with necessity’ added to the conclusion.

No *C* is a *B*.
Every *A* is a *B*, with necessity.
Therefore no *C* is an *A*.

This is at *Prior Analytics* i.10, 30b20–31. (Aristotle has *B*, *A* for *A*, *B*. We follow Ibn Sinā.)

**Aristotle’s argument**

```
No C is a B
Nec every A is a B  
   |           |           |
   |           | Nec some B is an A
Nec no C is an A  
   |           |           |
   |           | Nec no A is a C
Nec some B is not a C
```

‘But nothing prevents one from choosing a *B* so that possibly every *B* is a *C*.’

So if the conclusion was valid ‘with necessity’, then we could derive a false conclusion from true premises.

Robin Smith (commenting on *Prior Analytics* i.9, 30a25–28, a parallel argument):

‘Aristotle’s technique is sophisticated and flawless.’

Ibn Sinā reckons that ‘all the time it exists’ is a kind of *necessity*, and ‘sometime in its existence’ is a kind of *possibility*.

So if Aristotle’s modal arguments work at all, they should still work if we put *d* sentences for ‘Necessarily’ and *t* sentences for ‘Possibly’.

In his *Qiyas* iii.2 Ibn Sinā tries this with the argument that Aristotle rejected above.
No $C$ is a $B$.
Every $A$ is a $B$, with necessity.
Therefore no $C$ is an $A$, with necessity.

Two-dimensional version, using weakest possible ($t$) for the absolute premise:

(e-t) Every sometimes-$C$ is sometimes not a $B$.
(a-d) Every sometimes-$A$ is always a $B$.
(e-d) Therefore every sometimes-$C$ is always not an $A$.
VALID.

So Aristotle's refutation must be wrong. Ibn Sinā checks it:

If every sometimes-$C$ is always not an $A$, then every sometimes-$A$ is always not a $C$.
VALID.

If every sometimes-$A$ is always a $B$, then some sometimes-$B$ is always an $A$.
INVALID. BUT . . .

If every sometimes-$A$ is always a $B$, then some sometimes-$B$ is sometimes an $A$.
VALID, and moreover

(i-t) Some sometimes-$B$ is sometimes an $A$.
(e-d) Every sometimes-$A$ is always not a $C$.
(o-d) Therefore some sometimes-$B$ is always not a $C$.
VALID, AND IT’S EXACTLY ARISTOTLE’S CONCLUSION. !!!

It seems that

- *Camestres* with necessary conclusion is valid.
- The steps in Aristotle's refutation of *Camestres* with necessary conclusion are also valid.

Do we have a paradox?
Aristotle claims that his data show we can choose $B$ and $C$ so that a false conclusion is derivable from true premises.

Ibn Sīnā checks what happens if we try to do this, using two-dimensional sentences.

Ibn Sīnā’s analysis: we can choose $B$, $C$ so that

1. Every sometimes-$C$ is at least once not a $B$, but
2. every sometimes-$B$ is at least once a $C$.

Example:
1. Every human is at least once not laughing, but
2. every laugher is at least once human.
Both true.

Now add the other premise ‘Every $A$ is always laughing’.
(No matter what $A$ is.)

This creates an inconsistency: every $A$ must be sometimes human by (2), hence sometimes not laughing by (1).

Ibn Sīnā’s conclusion:

“So [Aristotle’s] statement that ‘nothing prevents this’ is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms $B$ and $C$] on its own.”

Paul Thom 1996 reaches the same conclusion—apparently the first Westerner to do so:

“Aristotle’s mistake was to conclude that because $ab^a$ is compatible with the denial of $Lab^b$, the conjunction of $ab^a$ with $Lbc^a$ must be compatible with the denial of $Lab^b$.”
Why did Aristotle make his mistake?

Probable answer: the minimal inconsistent configuration

\[ A \rightarrow B \rightarrow C \]

(where an arrow from \( A \) to \( B \) represents a sentence with subject term \( A \) and predicate term \( B \))

can't occur with assertoric sentences.

Every minimal inconsistent set of assertorics has a circular configuration.

With 2D sentences the minimal inconsistent configurations all look like

\[ \cdots \rightarrow A \rightarrow B \rightarrow C \rightarrow \cdots \]

which allows the above configuration and also

\[ A \rightarrow B \rightarrow C \]

Ibn Sinā knew this second configuration.

In his later \( Isārat \) i.7 he gives a minimal inconsistent set illustrating it:

\( a-d \) Every \( A \) is a \( B \) throughout its existence.

\( a-\ell \) Every \( B \) is a \( C \) throughout the time while it's a \( B \).

\( e-\ell \) No \( B \) is a \( C \) throughout its existence.

Note the use of an \( \ell \) sentence. Ibn Sinā is right; nothing weaker than an \( \ell \) will work for this configuration.

Broad observation: Ibn Sinā is here using his extensional (\( 'bīl fīl \)) two-dimensional logic as a testbed for Aristotle's intentional alethic modal logic.

Following Ibn Sinā involves understanding the two-dimensional logic itself.

That includes inference rules for multiple quantification, a topic barely touched in the West before the 19th century.

Further details at
http://wilfridhodges.co.uk/arabic44.pdf
a book in progress.