Ibn Sīnā's alethic modal logic

Wilfrid Hodges
Herons Brook,
Sticklepath,
Okehampton
Devon EX20 2PY
England
wilfrid.hodges@btinternet.com

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Everything below is under construction. As of 6 April 2015, the main things that are not yet included at all are:

(1) The detailed treatment of the necessary/possible fragment and the necessary/absolute fragment (though some conclusions about them are stated).

(2) An analysis of Ibn Sīnā's treatment of arguments involving wide time scope, as it affects alethic modal logic.

(3) An analysis of the reactions of Ibn Sīnā's successors, up to and including Rāzī, to his treatment of alethic modal logic.

Chapter 1

Introduction

This monograph is a report on Ibn Sīnā's logic of necessary, possible, contingent and absolute, which we will refer to as his *alethic modal logic*. We describe what he did and why he did it. Two new features of our account are, first, a description of the logical properties of the two-dimensional (temporal) logic which he sets out in *Qiyās* i.3 and *Mašriqiyyūn*, and second, a review of his account of logic as a science. The two-dimensional logic was a major innovation in its own right, and it had the potential to revolutionise logic if Ibn Sīnā's successors had recognised it for what it was. The account of logic as a science and the logic itself have generally been treated in isolation from each other, but in fact neither makes full sense without being closely tied to the other.

We separate Ibn Sīnā's treatment of alethic modal logic into three parts: first the listing of moods, second the proof of these moods where the proof is internal to the science of logic, and third the justification where it relies on something other than logical proof, such as drawing principles from First Philosophy. The first and second parts are a highly accurate report of the facts of two-dimensional logic. The third part is strictly not formal logic at all. It is best accounted for as an attempt to derive axioms for an alethic modal logic which is abstract in the sense that it applies to modalities of any category (including both temporal and ontological); and it is illuminated by Ibn Sīnā's own remarks about how the science of logic can proceed in such cases.

In the literature on Ibn Sīnā's alethic modal logic, much has been said about Ibn Sīnā's attitude to Aristotle's modal logic, and about the relationship of Ibn Sīnā's logic to that of his successors from the late twelfth century onwards. Both these enquiries should be based on an account of what Ibn

Sīnā's alethic modal logic consists of in its own right. His references to Aristotle fall broadly into two groups. Firstly there are attempts to mine Aristotle and the Peripatetic literature for intuitions and heuristics to support finding axioms. Secondly there are a number of passages which are criticisms of Aristotle or other Peripatetic writers, but these criticisms are not essential to the alethic logic itself. One could delete them without altering the logical content, as Ibn Sīnā himself does in *Išārāt*.

There is one overriding difference between Ibn Sīnā's work and that of his successors from Fak̲r-al-Dīn al-Rāzī (a century and a half after Ibn Sīnā) onwards. Rāzī took the view that in practice it is impossible to do properly motivated work in modal logic if we don't know precisely which modal category we are dealing with at any one time. He developed a new paradigm for modal logic which allows most of Ibn Sīnā's work to be included, but only with reference to two modalities, which are always clearly distinguished: temporal and ontological. Myself I am strongly in sympathy with Rāzī here. Maybe Ibn Sīnā's abstract modal logic was always a will o' the wisp, though as often with Ibn Sīnā it raises original and deep questions. The flood of original research that followed Rāzī's proposals is in sharp contrast with the lack of progress in the period between Ibn Sīnā and Rāzī.

The broad outlines of this monograph were obtained in January 2014 and circulated to a number of people; I thank Zia Movahed and Saloua Chatti in particular for their responses. But it has taken all the time since then to fill in details, and that process continues. Part of the problem is that there is not yet a published body of sifted data about Ibn Sīnā's logic that one can refer back to. As it is, the monograph makes several demands on the reader's acquiescence. But it became tiresome not to be able to give people an account of the matter.

Chapter 2

Preliminaries

2.1 The problem of Ibn Sīnā's modal logic

In one sense we know exactly what Ibn Sīnā's modal logic is. Namely, we know exactly what modal syllogistic moods he accepted. He lists them in several works; the lists are unambiguous and consistent with each other. You can read them at the ends of Street [49] and [50].

For safety I listed them myself without consulting Street, and then compared his lists with mine. They agreed completely. That was using *Najāt*, *Qiyās* and *Išārāt*. Later I took a list from *Muḵtaṣar*, which Street didn't have, and again it came out to be the same list. So the list is a hard datum. (Thom has raised some queries about it, but I don't think they alter the findings; we will consider them in due course.)

But as soon as we ask what the list means, we run into difficulties. Some obvious questions:

(a) What do Ibn Sīnā's modal sentences mean?

(b) What reasons does Ibn Sīnā have for accepting just these moods and for rejecting others?

(c) Are these reasons sound?

(d) Does he have a proof theory that generates just these moods, and if so, how does it go and how are its proof rules justified?

(e) What are the intended applications of the moods?

Not that there is any lack of material for answering these questions. We have between two and three thousand pages of Ibn Sīnā's text, mostly in Arabic, to give us answers. But let me give some personal impressions of this text.

First, there is the lack of a clear overall picture. Ibn Sīnā loves details. He spends pages and pages chasing up the finer nuances of this mood or that mood. But he tends to do this for each mood separately, and it's hard to discern any larger themes. His style of writing doesn't help; for example in *Išārāt* he explains what should be a rule to help list the moods that he accepts. We read there:

(2.1) The conclusion agrees with the major premise in its quality and aspect in each case of the syllogisms of this figure, except when ...; or else if ... with an exception that we will mention. ... the conclusion follows the worse of the two premises ... in quality and quantity, and with the exception mentioned above. (Details given.) But in this case the conclusion doesn't follow the major premise, so this is another exception. (*Išārāt* i.7.4, 144.14–145.10)

How many exceptions is that in all, and what is the rule that they are exceptions to? Why does Ibn Sīnā mention more exceptions here than he seems to at *Qiyās* 108.11? One could be forgiven for thinking sometimes that he makes up the details as he goes along.

Whether the reason is style or something deeper, it just is very difficult to make out a global strategy in his logic. Even the list of moods has no obvious overall shape or pattern.

Second, the intended use of the system is unclear. One might guess that as grand master of the metaphysics of necessity, Ibn Sīnā would want to use his modal logic as a tool in those metaphysical arguments. But I know of no case outside his logical writings where Ibn Sīnā uses a modal syllogism to justify an argument. Within logic the examples seem aimed at justifying the formal moods rather than the conclusions of any particular instances.

We can get a measure of this problem by looking at the semi-final chapter of Gutas' book *Avicenna and the Aristotelian Tradition*, where Gutas gives two illustrations of how Ibn Sīnā applied 'the strictest possible demonstrative method, notably [in] his commentaries' ([15] p. 353). Gutas provides two examples. In one of them (his p. 355) the mood used is assertoric *Barbara*, and in the other (his p. 357) it is modus ponens, with a subsidiary syllogism in modus tollens. We already had assertoric *Barbara* in Aristotle's logic; modus ponens and modus tollens are much older than Ibn Sīnā, and Ibn Sīnā himself treats them with some disdain. No modal syllogism is visible here at all. If these examples from Gutas are typical of how Ibn Sīnā applied his logic, why did he bother with modal logic at all?

Third there is the Archimedes problem: where can we put our fulcrum? Everything in Ibn Sīnā depends on everything else. To understand his modal sentences we need to understand his semantics, which sends us off to ^{c}*Ibāra*, his commentary on the *De Interpretatione*. To put any of his arguments in their proper context we need to understand his scientific methodology as he describes it in *Burhān*, his commentary on the *Posterior Analytics*. To understand anything at all, we need to make sense of his logical vocabulary.

In what we might call mainstream western Ibn Sīnā studies (Goichon, Marmura, Gutas etc.) it has long been recognised that in order to understand what Ibn Sīnā means by a word or a phrase, we need to examine how he uses the word or phrase in a range of contexts; each context needs to be assessed and fed into an overall picture. This work takes many years, even with the benefit of modern search engines. In the field of Ibn Sīnā's logic the work has hardly started—one still often sees a phrase explained in terms of a single text in *Išārāt*, taken out of context; or on the say-so of a commentary written two hundred years after Ibn Sīnā; or with an appeal to modern Arabic usage.

2.2 Some strategies

Some of the problems mentioned above will not be solved in my lifetime. But for the present it seems we can alleviate the worst difficulties.

For example I decided that in these notes I would work as far as possible with the original Arabic. So we sidestep the need to produce justified and commented translations; the down side is that fewer people are going to read these notes. But so far as this language barrier allows, I have aimed to make the evidence public and checkable, which is why the index of citations is as long as it is.

Also I reckoned that it was important to read widely through Ibn Sīnā's logical texts. This paid off, because some of Ibn Sīnā's messages become loud and clear through repetition, though you will have to read the texts yourself if you want to verify this. For example it becomes clear that Ibn Sīnā is completely committed to Aristotle's assertoric logic, down to fine details of the proof theory, and that he accepts it as a basis for all his innovations in formal logic.

It also becomes clear that Ibn Sīnā makes a sharp separation between ontology and formal logic. For example we never find him stating a rule of logic that refers to a distinction of Aristotelian categories—the rules of logic are just the same for qualities as they are for substances. The reader also becomes aware that this separation is not just an accident of Ibn Sīnā's practice; Ibn Sīnā himself is keen to draw our attention to it. (For most modern logicians it would seem obvious that the rules of logic don't vary for different classes of being; but although Ibn Sīnā shared this modern view, his predecessor al-Fārābī probably didn't.)

So we can add these two items—the primacy of Aristotle's assertoric logic and the separation between ontology and formal logic—as solid data alongside the list of accepted modal moods. Maybe the Archimedes problem is not so severe after all. At the ends of some chapters I have put statements of what has been established so far. These statements may give a misleading impression that the progress is more monotonic than it really is. In practice one has to keep going back to earlier statements to check that they still hold water in the light of things established later.

It was also helpful to take seriously Ibn Sīnā's own statements about what he reckons he is doing in his writings, logical and other. For example his comments on *Qiyās*, *Mašriqiyyūn* and *Išārāt* send a strong message that we should take *Qiyās* as his fullest account and *Mašriqiyyūn* (as much of it as we have) as his most straightforward; and that we should regard *Išārāt* with caution. In Ibn Sīnā's prologue to the *Šifā'* (which contains *Qiyās*) we read:

(2.2)
> I also wrote another book ..., in which I presented philosophy as it is naturally [perceived] and as required by an unbiased view which neither takes into account [in this book] the views of colleagues in the discipline, nor takes precautions here against creating schisms among them as is done elsewhere; this is my book on Eastern philosophy. But as for the present book, it is more elaborate and more accommodating to my Peripatetic colleagues. Whoever wants the truth [stated] without indirection, he should seek the former book; whoever wants the truth [stated] in a way which is somewhat conciliatory to colleagues, elaborates a lot, and alludes to things which, had they been perceived, there would have been no need for the other book, then he should read the present book. (*Madḵal* 10.11–17, trans. Gutas [15] p. 44f)

Of course it would be naive to take all Ibn Sīnā's statements about himself at face value. But here he is describing his intentions, not boasting of his achievements, so there is less likelihood of distortion. In any case a reading of *Qiyās* and *Mašriqiyyūn* will confirm what he says about their relationship. (I have to state this as bald fact. At the time of writing I don't have any evidence that since the pioneering work of Amèlie-Marie Goichon, anybody other than Riccardo Strobino and me has actually advanced any further into *Mašriqiyyūn* than the prologue.) Ibn Sīnā's description of the bias in *Šifā'* will be helpful to us below.

Taking *Qiyās* and *Mašriqiyyūn* as primary leads us to the main thing that the surviving texts of these works have in common, which is the two-dimensional temporal logic. In all Ibn Sīnā's major logical works this logic keeps popping up alongside the alethic modal logic of necessity and possibility, and gets entwined with it in various ways. It's clear that the two-dimensional logic plays a central role in Ibn Sīnā's thinking about alethic modal logic.

I hived off some preliminary work that had to be done but hardly depends on the modal logic. One part of this is the mathematical theory of two-dimensional logic, and another is the general form of Ibn Sīnā's proof theory so far as we can reconstruct it. The first of these items is or will be covered in detail in [20], and the second (joint with Amirouche Moktefi) will appear in [21], both at present on my website. Ibn Sīnā himself didn't have the mathematical theory of two-dimensional logic, but he will certainly have had its results in terms of lists of valid inferences. He will have checked these directly, case by case, as he expected his students to do. You can do likewise. Here are two examples to try.

The first is *Disamis(d,t,m)*:

(2.3) Everything that is sometimes a B is a C throughout its existence;
Something that is sometimes a B is sometimes an A.
Therefore something that is sometimes a C is also sometimes both a C and an A.

The second is $(o\text{-}t)$-ecthesis:

(2.4) Suppose something that is sometimes a C is at some time not a B. Say that a thing is a D at a time τ if that thing is not a B at time τ but is a C at some time. Then the following hold:
Something that is sometimes a C is sometimes a D.
Nothing that is sometimes a B is never a D at the same times as it is a B.

On the other hand we make no use at all of the views of later Arabic logicians as evidence for the views of Ibn Sīnā. The reason for this is very simple: they are *not* evidence for the views of Ibn Sīnā. The one possible exception to this is Bahmanyār, who was Ibn Sīnā's student; we will discuss his input in Chapter 15 below.

I would add: the relationship between the logic of Ibn Sīnā and that of his Arabic-speaking successors is an important question both for the history of logic and for understanding medieval Arabic culture. To study this relationship we need to have an account of Ibn Sīnā's views which is not contaminated with views of those successors.

2.3 Remarks on modalities

We will need some notions that I had thought were common currency. But reading around and talking to some people has convinced me that they are not, so it would be better to be explicit about them. (My thanks to Yde Venema for a useful discussion, but as always, don't blame him if anything is incoherent.)

Georg Henrik von Wright on page 2 of his classic work [57] on modal logic presents a table:

(2.5)

alethic	*epistemic*	*deontic*	*existential*
necessary	verified	obligatory	universal
possible		permitted	existing
contingent	undecided	indifferent	
impossible	falsified	forbidden	empty

The individual items under each of the heads are, in his terminology, *modes* or *modalities*; this seems to be standard usage. He says that the columns represent four *modal categories*. This is his own usage, and it has not fared so well. Perhaps no modal logicians use the term 'category' this way today.

The term is found among linguists. For example

(2.6) Modal logic has to do with the notions of possibility and necessity, and its categories epistemic and deontic concern themselves with these notions in two different domains. (Bybee and Fleischman [9] p. 4)

But notice the difference from von Wright: the categories of epistemic and deontic are now *kinds* of necessity or possibility. The alethic modes don't form a category; rather they are words that (as some other linguists tell us) can be *used* to express items in modal categories. On this view the alethic modes do have a meaning of a sort, because for example 'possibly' has to be incompatible with 'necessarily not', and both 'necessary' and 'contingent' have to imply 'possible'. So at least they have enough meaning to carry some logical relationships between them.

In what follows I will go with von Wright to the extent of using 'category' for a family of modal notions that provide a necessity notion, a matching possibility notion etc. Von Wright's epistemic and deontic are two standard examples. Ibn Sīnā would surely add temporal and ontological:

(2.7)

alethic	*temporal*	*ontological*
necessary	permanent	essential, by nature
possible	occurring	acceptable
contingent	temporary	separably accidental

or something similar. Von Wright's alethic modes don't form a category in this sense. They play more the role of abstract place-holders, but they do have meanings of a kind.

Often in this paper we will find Ibn Sīnā treating 'necessary' and 'permanent' as in some way equivalent notions. So we should set on record at once that he was perfectly capable of distinguishing between them. We give two quotations to show this; both of them will be useful to us later. The first is from *Išārāt*:

(2.8)

> An example of that which endures and is non-necessary is something like the affirmation or negation, applicable to an individual [of a quality] accompanying him in a non-necessary manner as long as he exists: as you may correctly say that some human beings have white complexions as long as their essence [[is satisfied]], even though that is not necessary.
> He who believes that non-necessary predication is [[not]] found in universal propositions has committed an error. For it is possible that universal propositions have that which is applicable, affirmatively or negatively, to every individual subsumed under them ... at a determined time as that of the rising and the setting of the [[planets]] and that of the eclipse of the sun and the moon; or at an undetermined time as that which belongs to every born human being such as respiration ... (*Išārāt* [34] 89f; trans. Inati [22] pp. 93f)

The double brackets are my emendations of Inati. Her missing 'not' may mean she is translating from a different Arabic text, but the sense surely requires 'not' here.

And the second quotation is from *Qiyās*:

(2.9)

> ... being permanent is not the same as being necessary. [A thing is] necessarily what it is by its nature, and this requires that if it is false of an individual then it is permanently false of that individual; while [a thing is] permanent either by its nature or because it just happens to be. But it is not for the logician as logician to know the truth about this. (*Qiyās* 48.14–17)

So for Ibn Sīnā there is a difference between being permanent and being necessary, but this is not a difference for the 'logician as logician'. What can he mean?

Below we will find an answer in Ibn Sīnā's own account of what logicians do as logicians. He is very articulate about this. But a prerequisite for understanding what he says is that we know some of the art of logic itself; so we begin with a chapter on assertoric logic, what Ibn Sīnā learned from Aristotle and how he adjusted it for his own use.

Chapter 3

Assertoric logic

3.1 What Ibn Sīnā inherited

Ibn Sīnā knew Aristotle from the Arabic translations of his works. Most of the classical Arabic translations of Aristotle were due to a team of Syriac-speaking translators associated with Ḥunayn b. Isḥāq and his son Isḥāq b. Ḥunayn in the 9th century. One translation of the *Prior Analytics* has come down to us from this period; we have it in two manuscripts, one in Paris and the other in Istanbul. Lameer ([40] pp. 3f) assembles evidence that points to this translation being the work of one Theodorus, a member of the Syriac team somewhere around the mid 9th century. Very likely the translation was made first into Syriac and then into Arabic. I will cite the translation as 'the Arabic Aristotle', using the edition of Jabre [38] but giving references to the Greek original. The default assumption must be that Ibn Sīnā worked from a version of Theodorus' translation; though I know of no research to confirm this, and it might be difficult given Ibn Sīnā's preference for saying everything in his own words.

For example Ibn Sīnā refers to 'the Philosopher's [i.e. Aristotle's] 'habit' (*ᶜāda*) of saying *bil wujūd*' (*Masā'il* 94.6). To the best of my knowledge the phrase *bil wujūd* never appears in our text of Theodorus' translation. But the word *mawjūd* is very frequent there; would Ibn Sīnā have counted this as close enough? My guess is yes, but you may disagree.

An added complication is that the text in the Paris manuscript may have been corrected in the light of Ibn Sīnā's own commentary. For example at *Qiyās* 197.5f Ibn Sīnā says that the text in front of him reads *bil ḍarūra lā* when it should read *laysa bil ḍarūra*. The Paris manuscript reads *laysa bil ḍarūra*. Aristotle's Greek at 34b28 has *mēdenì ex anágkēs*.

According to the Arabic Aristotle (24a14) there are three kinds of sentence, namely universal (*kullī*), particular (*juz'ī*) and unquantified (*muhmal*). The name *juz'ī* covers both existentially quantified sentences and singular sentences about a named individual. Ibn Sīnā will distinguish these and refer to the first kind as existential (again *juz'ī*) and the second as singular (*šaḵsī* or *maḵṣūṣ*). Ibn Sīnā maintains that within formal logic the singular sentences behave as if they were universal and the unquantified sentences behave as if they were existential (*Qiyās* 109.11–13), so we can save paper by concentrating below on the universal and the existential. Ibn Sīnā will refer to the properties of being universal or existential as the 'quantity' (*kamm, kammiyya*) of a sentence; in the Arabic Aristotle this expression is found only in the chapter headings, which may have been added later.

The Arabic Aristotle (24a20) also distinguishes between sentences in which something is said of something and sentences in which something is 'not said' (i.e. is denied) of something. Ibn Sīnā will read this as a distinction between affirmative (*mūjib*) and negative (*sālib*) sentences. Being affirmative or negative is the 'quality' (*kayfa, kayfiyya*) of the sentence; *kayfiyya* is found already in the text of the Arabic Aristotle.

So there are four kinds of sentence:

(3.1)
(a) : 'Every B is an A'.
(e) : 'No B is an A'.
(i) : 'Some B is an A'.
(o) : 'Not every B is an A'.

At least this is how Ibn Sīnā read the Arabic Aristotle. Aristotle himself rarely spelt out the sentences, and when he did he usually used a technical vocabulary that put the A before the B. This accounts for the backwards ordering of the letters in Ibn Sīnā and other Arabic logicians. The labels (a) for 'universal affirmative', (e) for 'universal negative', (i) for 'existential affirmative' and (o) for 'existential negative' are a later Latin invention, but they give us a useful shorthand.

Every sentence has a 'contradictory negation' (*naqīḍ*, 34b29) that denies what the sentence affirms, or vice versa. The contradictory negation of 'Every B is an A' is 'Not every B is an A', and conversely; the contradictory negation of 'No B is an A' is 'Some B is an A', and conversely.

We will call the sentence forms above the *assertoric* sentence forms, and their logic will be *assertoric logic*. The Arabic Aristotle has no distinguishing name for them; Ibn Sīnā sometimes refers to them as the 'standard' (*mašhūr*) forms. In the Arabic Aristotle it is not clear whether the schemas above are themselves objects of interest, or whether they are regarded as shorthand for longer sentences that have vernacular text in place of the letters A, B. We will need to make this distinction; I will refer to the schemas as *formal sentences* as opposed to the *concrete sentences* that are got by putting text in place of the letters. This text, or its meaning, is called *matter* (*mādda*). The Arabic Aristotle describes B as the 'subject' (*mawḍū*^{c}) of the sentences (e.g. at 24b29) and A as their 'predicate' (*maḥmūl*, e.g. at 24a27); these names may refer either to the letter or to the matter assigned to it. The subject and the predicate are referred to as 'terms' (*ḥudūd*, singular *ḥadd*, 24b17).

Although the Arabic Aristotle seems to be consistent in applying the expression *mawḍū*^{c} to a *term* of sentences, Peripatetic logicians developed a habit of using it to refer to the *individuals that fall under the subject term*. For example the sentence 'Every horse sleeps' has the subject term 'horse', but one says also that horses are subjects of it. To avoid this confusion I will speak of the horses as the *subject individuals*, as opposed to the *subject term*. Ibn Sīnā has his own ways of resolving this ambiguity.

The Arabic Aristotle defines a 'syllogism' (*qiyās*) as a piece of discourse in which when two or more sentences are proposed, something else follows from their being true, of necessity and intrinsically (24b29f). The proposed sentences are called 'premise' (*muqaddama*, 24a23). The something else that follows is called 'conclusion' (*natīja*, 30a5) or occasionally 'goal' (*maṭlūb*, 42a40). In practice he limits himself to syllogisms with just two premises, at least in the part of the *Prior Analytics* that concerns us here.

In sections i.4–6 (25b27–29a17) the Arabic Aristotle runs through a list of all the syllogisms; the syllogisms are expressed using formal assertoric sentences or paraphrases of them, and they are classified by 'figure' (*šakl* 26a14). There are three figures. For a conclusion with subject C and predicate A, the first figure has a premise with subject C and predicate B, and a premise with subject B and predicate A; the second figure has premises with B the predicate in both; the third figure has premises with B the subject in both. It will be helpful to speak of a formal syllogism, expressed with formal assertoric sentences, as a *mood*, and a pair of formal sentences as a *premise-pair*; Ibn Sīnā will use *ḍarb* for 'mood' and *qarīna* for 'premise-pair'. When a premise-pair fails to produce a conclusion in a given figure, the Arabic Aristotle says that it is 'not a syllogism'; it will be helpful if we adopt a term used by Ibn Sīnā and say that the premise-pair is *productive* (*muntij*) if it does yield a conclusion in the given figure.

In this context the Arabic Aristotle describes the term C as the 'minor extreme', the term B as the 'middle' and the term A as the 'major extreme' (25b35, 26a19). ('Extreme' is *ra's*, literally 'head'; 'minor' is *ṣaḡīr*; 'middle' is *awsaṭ* and 'major' is *kabīr*. Variants later in the Arabic text are *ṭaraf* for extreme, *aṣḡar* for minor and *akbar* for major; these are the expressions that Ibn Sīnā will normally use.) The premise containing C is the 'minor premise' (*ṣuḡrā*) and the premise containing A is the 'major premise' (*kubrā*). Since the Arabic Aristotle rarely sets out concrete examples of syllogisms, there is room for reading either the minor premise or the major premise as the 'first' premise; in practice the Arabic logicians took the minor premise as first, the opposite way to the Latins.

The Arabic Aristotle tells us ([38] 24b24–28) that

(3.2) A perfect (*kāmil*) syllogism is a syllogism which needs, for proving what must be the case given its premises, the use of something other than those premises. And a syllogism that is not perfect is one which needs—for proving what must be the case given its premises—the use of one thing, or a combination of things, which must be the case given the premises that compose the syllogism, but which has not been used in the premises.

Moreover all the first figure syllogisms are perfect ([38] 26b28), but none are perfect in the second ([38] 27a1) or third ([38] 28a5) figure. The syllogisms in second and third figure are 'made perfect' (*yukmalu*) by having certain things 'attached' (*ulḥiqa*) to them.

There are three kinds of attachment, as follows. One is 'conversion' (*ᶜaks*, 30a5), which consists of replacing a premise or conclusion ϕ by a sentence ψ whose subject is the predicate of ϕ and its predicate is the subject of ϕ, where ψ follows from ϕ. Giving them their usual names, there are three forms of conversion:

- e-conversion, taking 'No B is an A' to 'No A is a B';
- i-conversion, taking 'Some B is an A' to 'Some A is a B';
- a-convesion, taking 'Every B is an A' to 'Some A is a B'.

When conversions are 'attached' to a syllogism, this means the following. First a premise of the syllogism is converted to a new premise in such a way that the new premise-pair 'reduces' (*rajaᶜa*) to a premise-pair in first figure. The conclusion of this second premise-pair either is the conclusion needed from the original syllogism, or it entails that needed conclusion through a further conversion.

A second kind of attachment is 'ecthesis', where a new term is 'posited' (*wuḍiᶜa*) or 'stipulated' (*yufraḍu*). In Aristotle this seems to cover more than one kind of argument. We will say more on it later.

A third kind of attachment is contraposition (literally 'denying the statement', *rafᶜ al-kalām*, or 'absurdity', *k̲alf* in the spelling that Ibn Sīnā favoured). When we use contraposition, we show that one of the premises of the syllogism, together with the contradictory negation of the conclusion, entails the contradictory negation of the other premise. This method can be used when the rearranged syllogism has already been shown correct, for example if it is in first figure.

So for every assertoric syllogistic mood, the Arabic Aristotle either states that it is self-evidently correct, or he proves the correctness by some kind of reduction to a mood whose correctness is self-evident. For premise-pairs that he regards as not a syllogism (i.e. not productive), he uses a method which he calls 'terms' (*ḥudūd*) to prove that no conclusion follows from them in their figure. The method is subtler than first appears, and there is evidence that Ibn Sīnā struggled to understand it. But briefly, suppose the figure requires a conclusion with subject C and predicate A. Then the method consists in setting out two examples of concrete premise-pairs of the given form, both consisting of true sentences, where in the first case the sentence 'Every C is a B' is true, and in the second case the sentence 'No C is a B' is true. The examples are specified by giving concrete terms for them—hence the name 'terms'. As the Arabic Aristotle says at [38] 26b19f, 'It is clear that when there are terms fitting this description, then there is not a syllogism'.

At [38] 27b38 the Arabic Aristotle offers the same set of terms to eliminate several different formal premise-pairs at the same time. Aristotle may have intended nothing more than saving a little effort, but we will see that this move had a significant effect on Ibn Sīnā.

3.2 Ibn Sīnā's logical writings

Among the works of Ibn Sīnā's maturity that have come down to us, six are particularly relevant to formal logic. I summarise briefly what they are, with references to the Inventory of Avicenna's Works in Gutas [15].

Muk̲taṣar Gutas [15] p. 433 names this the *Middle Summary on Logic*. We have no precise dating, but a date around the early 1010s is plausible. The work has not been printed, and I thank Alexander Kalbarczyk for giving me access to the Nuruosmaniye manuscript.

Najāt This is an encyclopedia, called *The Salvation* in Gutas [15] p. 115. It was published soon after *Qiyās* below, but we know that its logic section is taken from an earlier work, the *Shorter Summary on Logic* from around 1014, with a few probable editorial changes. There is a translation of the logic section [3] by Asad Q. Ahmed.

Qiyās This is *Syllogism*, a volume of the encyclopedia which is *Šifā'* in Arabic and *The Cure* in Gutas [15] p. 420. *Qiyās* is by far the most detailed of Ibn Sīnā's accounts of formal logic. Ibn Sīnā himself describes the whole of *Šifā'* as 'somewhat conciliatory to colleagues' (*Madḵal* 10.16, cf. Gutas [15] p. 45—the 'colleagues' are the Peripatetic logicians who follow Aristotle). *Qiyās* is dated to around 1024 (Gutas [15] p. 107).

Mašriqiyyūn Gutas [15] p. 119 calls this work *The Easterners*, but with some misgivings about the title. The work was a survey of various areas of philosophy; from the logic section fewer than a hundred pages survive, roughly corresponding to the first of the nine books of *Qiyās*. But Ibn Sīnā advertises the work as more direct and less biased in favour of the Peripatetics than *Qiyās*, and this is borne out by the contents. Its main contributions are a full and integrated discussion of definitions, and the best-organised presentation of what below we will call Ibn Sīnā's two-dimensional logic. Gutas [15] p. 132 dates it to 1027–8.

Dānešnāmeh This work, the *Philosophy for ᶜAlā'-ad-Dawla*, is a relatively elementary summary of philosophy written in Persian at some time between 1023 and Ibn Sīnā's death in 1037 (Gutas [15] p. 118). The first section is on logic, treated from a practical point of view. There is a French translation of the whole work, [1].

Išārāt This late work is called *Pointers and Reminders* and dated to 1030–4 (Gutas [15] p. 155). It covers a range of philosophical topics, beginning with logic. In logic the differences from *Qiyās* and *Mašriqiyyūn* are very visible, but I believe they are mainly in presentation rather than content. One of them is extreme brevity, which made the work prime material for later commentators. There is a translation of the logic section [22] by Shams Inati.

These are by no means the only surviving writings in which Ibn Sīnā discusses logic. See the index of citations at the end of this paper for some other examples.

3.3 What Ibn Sīnā added to Aristotle

One of Ibn Sīnā's most important additions to Aristotle is most fully treated not in the works above, but in his ^{c}*Ibāra*, the volume of the *Šifā'* that comes immediately before *Qiyās*, corresponding to Aristotle's *De Interpretatione*. (See ^{c}*Ibāra* [27] 79.11–80.12 and the discussion in [18].) Here Ibn Sīnā explains the meanings of the assertoric sentence forms, in enough detail to justify the following translations into first-order sentences. In the diagram (3.3) the righthand column gives the first-order translations of the (a), (e), (i) and (o) sentences, and the lefthand column gives convenient abbreviations of these formulas.

$$(3.3)\quad \begin{array}{lcl} (a)(B,A) & : & (\forall x(Bx \to Ax) \wedge \exists x Bx) \\ (e)(B,A) & : & \forall x(Bx \to \neg Ax) \\ (i)(B,A) & : & \exists x(Bx \wedge Ax) \\ (o)(B,A) & : & (\exists x(Bx \wedge \neg Ax) \vee \forall x \neg Bx) \end{array}$$

Given these meanings, one can check that the fourteen moods listed by Aristotle are exactly those where the premise-pairs are productive and the conclusion is the strongest conclusion (with the appropriate terms for the given figure) that can be deduced from the premises. We will refer to the clauses $\exists x Bx$ in the first sentence and $\forall x \neg Bx$ in the fourth as the *augments*, respectively the *existential augment* in the first and the *universal augment* in the fourth.

Maybe this is the best place to note that neither Aristotle nor Ibn Sīnā operates with the modern notion of *validity* in dealing with syllogisms. For us an inference is *valid* if and only if its conclusion is a logical consequence of its premises. For both Aristotle and Ibn Sīnā the operative notions are first that the premises are productive in a figure (i.e. there is a valid inference from them to a conclusion in that figure), second that a sentence follows validly in the given figure, and third that a sentence is the strongest that can be drawn in that figure. When Ibn Sīnā writes out a syllogistic mood as one that he accepts, he is normally taking it to be *conclusion-optimal*, i.e. it is valid and its conclusion is the strongest that can be validly drawn in the relevant figure. (There is no requirement that the premises are the weakest that will allow that conclusion.) I will use the notion of validity because it is more versatile than these older notions; but one should be aware that this often involves some paraphrasing of the originals.

Ibn Sīnā reports the contents of *Prior Analytics* i.4–6 in several places, most straightforwardly in *Muk̲taṣar* 49b9–53a6, *Najāt* 57.1–64.3, *Qiyās* ii.4, 108.12–119.8 and *Dānešnāmeh* 67.5–80.2. Besides these four accounts, we also have a report in *Išārāt* i.7, 142.10–153.9 ([22] 135–143) which is sketchier and mixed with modal material. In *Qiyās* vi.4, 296.1–304.4 Ibn Sīnā repeats the entire scheme in detail, but with a version of propositional logic in place of Aristotle's assertoric sentences.

In all these accounts Ibn Sīnā reports the same fourteen moods as Aristotle, in the same order (apart from some slight variation in *Išārāt*). Moreover the justifications that he offers are almost exactly the same as Aristotle's. (This is fully documented in Appendix A of [20].) In first figure he tells us, following Aristotle, that all the moods are perfect. In second and third figures he repeats Aristotle's justifications by conversion, ecthesis and contraposition, with only a very few variations, mostly insignificant.

In fact the only significant variation from Aristotle is that Ibn Sīnā introduces a proof of second-figure *Baroco* by ecthesis. This proof appears in all six of his reports. As I read him, he intends a proof along the following lines, where the deductions are direct from top to bottom and Dx is defined as $(Cx \wedge \neg Bx)$:

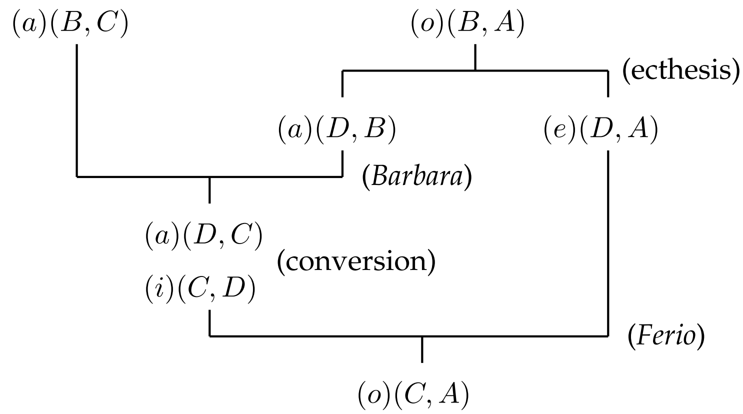
(3.4)
$$\dfrac{(o)(C,B)}{(i)(C,D) \qquad (e)(B,D)}\ \text{(ecthesis)} \qquad (a)(A,B)$$
$$\dfrac{(e)(B,D) \qquad (a)(A,B)}{(e)(A,D)}\ (\textit{Celarent})$$
$$\dfrac{(e)(A,D)}{(e)(D,A)}\ \text{(conversion)}$$
$$\dfrac{(i)(C,D) \qquad (e)(D,A)}{(o)(C,A)}\ (\textit{Ferio})$$

Strictly the ecthesis is a non sequitur, because by (3.3), $(i)(C,D)$ implies that at least one thing is a C and $(o)(C,B)$ doesn't imply this. But the procedure can still be justified, as for example in [20].

Though Ibn Sīnā never discusses the point, the introduction of this proof for *Baroco* has the effect that he can give justifications of all the second- and third-figure moods without ever invoking contraposition. Not that he objects to contraposition; he mentions it in all the cases where Aristotle did. But contraposition uses some propositional logic—as is particularly clear in his analysis of it in *Qiyās* viii.3—and this would certainly not be the only place where Ibn Sīnā aims to set up the foundations of a logic without invoking other logics.

In fact there already is an ecthetic justification for third-figure *Bocardo* in the Arabic Aristotle (28b20f), with a remark that it makes contraposition unnecessary. I believe Ibn Sīnā reads this argument as follows:

(3.5)



He takes Dx to mean $(Bx \wedge \neg Ax)$, following the guidance of the Arabic Aristotle that it is 'the some of B taken from what is not in A'. His argument for *Baroco* is a straightforward rearrangement of this argument.

Ibn Sīnā also refers to a proof of *Darapti* by 'ecthesis' but without any reduction to another mood. He presumably took this from 28a24–26 in the Arabic Aristotle. From Ibn Sīnā's general usage I guess that he intends a two-step semi-formal argument as follows:

(3.6)
$(a)(B,C)$, $(a)(B,A)$
Therefore some determinate D, namely B, is a C which is an A.
Therefore $(i)(C,A)$.

If this is right, then it must be what Ibn Sīnā is referring to at *Qiyās* 77.10–78.3 where he says that the contradiction between 'D is a C', 'D is a B' and 'No C is a B' doesn't require a syllogism in third figure.

This ties into a problem with Aristotle's comments on the conversions that are needed for deriving the second- and third-figure syllogisms from the first-figure ones. These derivations require that we can infer from 'No B is an A'' to 'No A is a B' (e-conversion) and from 'Some B is an A' to 'Some A is a B' (i-conversion). On the face of it, the Arabic Aristotle justifies e-conversion by i-conversion (at 25a15–19) and i-conversion by e-conversion (at 25a20–22). Alexander of Aphrodisias in his commentary on the *Prior Analytics* ([4] 32.4–33.2, [5] pp. 87f) sought to escape from this circle by finding in Aristotle's derivation of e-conversion from i-conversion a hint of an independent proof of i-conversion by ecthesis. Ibn Sīnā's comments at *Qiyās* 77.10–78.3 are almost certainly an endorsement of Alexander's suggestion. For the health of Aristotle's assertoric syllogistic it hardly matters, because one could reasonably claim that both e-conversion and i-conversion for assertoric sentences are self-evidently valid. But Ibn Sīnā will rely on Alexander's suggestion when it comes to justifying the modal moods, so this is an issue we will come back to.

Ibn Sīnā makes some further changes in the general layout. He includes two items that were not in Aristotle, namely *conditions of productivity* (*šarā'iṭ al-'intāj*) and *rules of following*. The conditions of productivity are necessary and sufficient conditions for a pair of sentences in a figure to be productive. As Ibn Sīnā presents them, there are a set of conditions that apply to all three figures, together with a further set of conditions that apply just to one figure. For example a condition applying to all three figures is that at most one of the premises is negative; for the second figure we have the stronger condition that exactly one of the premises is negative. Ibn Sīnā states the conditions precisely as they appear in Philoponus [47] 70.1–21, except that Ibn Sīnā usually includes a further condition applying to all three figures; this further condition is correct but redundant. From remarks in Philoponus it appears that the conditions were first assembled from Aristotle's proofs of non-productivity, by noting where Aristotle handled a group of formal premise-pairs together (as we remarked at the end of Subsection 3.1 above).

The rules of following tell us, given a productive premise-pair in a particular figure, what is the strongest conclusion in that figure that can be drawn from the premises. The Peripatetic logicians had a tendency to assume that each logical property of the conclusion is inherited from one of the premises, and so the conclusion can be described by saying which of the premises it 'follows' (in Arabic *yatba*^{c}*u*) for each of its logical properties. Ibn Sīnā also has a further piece of terminology, which as far as I know he introduced himself. In the case of modalities he says that the premise whose modality is inherited by the conclusion is the premise with the ^{c}*ibra*. Because of the obvious analogy with genetics I translate ^{c}*ibra* as *dominance*, and I refer to the Peripatetic assumption that the conclusion inherits its modality (and other features) from one or other premise as the *genetic hypothesis*. The link to genetics is not just a modern fancy; Ibn Sīnā uses ^{c}*ibra* in this genetic sense in his biological essay *Ḥayawān* 159.7.

Ibn Sīnā states the rule of following for assertoric logic in several places, and nearly always in a form that is wrong for *Darapti* and *Felapton*, which don't inherit their quantity from either parent. It's hardly likely that he was unaware of this exception, and in fact he gets it right in *ᶜUyūn al-ḥikma* [35] 50.2f, where he explains that there is an *ᶜibra* for quality (but by implication not also for quantity). Probably the error is the result of a common tendency to be careless about minor counterexamples.

3.4 Conclusions so far

Conclusion 3.1 Ibn Sīnā accepts Aristotle's assertoric logic, both the lists of moods in each figure and the verifications that Aristotle gives for the listed moods.

Conclusion 3.2 The assertoric moods that Ibn Sīnā lists in each figure are those where (1) the conclusion follows validly from the premises, and (2) no stronger conclusion in the same figure follows from those premises (i.e. the moods are conclusion-optimal). He includes moods where the same conclusion could be proved from weaker premises in the same figure.

Conclusion 3.2 is at present only an observation on the list of assertoric moods. Ibn Sīnā could have listed just these moods because he found them listed in the Arabic Aristotle. But in Section 8.1 we will confirm this conclusion by seeing that it holds for the two-dimensional moods. Since these moods were Ibn Sīnā's own discovery, he couldn't be following anybody else's list when he lists them.

Conclusion 3.3 Ibn Sīnā adds to Aristotle's assertoric procedures an ecthetic proof, as a result of which he can deduce all the second- and third-figure assertoric moods from first-figure moods by conversion and ecthesis, without needing to use contraposition (though he does accept contraposition as a valid method).

Chapter 4

The science of logic

(This is a footnote that I hope to be able to remove sooner rather than later. In this section I am moving outside my comfort zone; my expertise is in mathematical logic and classical languages, not in epistemology or philosophy of science. So I would welcome any advice and corrections, but subject to two reasonable requirements. First, attempts to formulate a description of the science of logic are unlikely to be helpful if they are not informed by knowledge of the facts of logic, the relevant logic here being Ibn Sīnā's logic. Second, attempts to establish Ibn Sīnā's views on any topic are unlikely to be successful if they are based on an unrepresentative sample of his available writings on the topic. Of the published modern discussions of the issues raised in this section, I know of none that address the first requirement at all, and none that are fully satisfactory on the second. So we have here a real opportunity to increase our understanding.)

4.1 The structure of a science

Ibn Sīnā regards logic (*manṭiq*) as a 'theoretical art' (*ṣinā*^{c}*a naẓariyya*, *Najāt* 8.8), and also as a 'science' (^{c}*ilm*, *Qiyās* 10.11f). Every science or theoretical art has 'principles' (*mabādi'*, singular *mabda'*) and 'theorems' (*masā'il*, singular *mas'ala*, literally 'question'). Both principles and theorems are propositions which the science guarantees to be true. The difference between them is that the theorems are demonstrated in the science using premises that are already principles or theorems of the science; the principles are either proved using premises from a 'higher' science, or they are not proved at all because they are self-evidently true. (*Burhān* 155.1–7). I ignore here what Ibn Sīnā describes as the 'rare' case of principles proved in a lower science, though I am not sure what he is referring to.

Ibn Sīnā calls logic both a science (^{c}*ilm*) and an art (*ṣinā^{c}a*). There is a difference between these two descriptions. To learn a science, we learn a class of true propositions and we learn how to demonstrate their truth. To learn an art, we learn a skill that consists in acting according to certain 'rules' (*qawānīn*, singular *qānūn*, *Jadal* [30] 21.11). The chief principles and theorems of any theoretical science are universally quantified (e.g. *Qiyās* 4.4, *Burhān* 220.8 and *passim*), since these sciences deal with causes and not with particular instances. But Ibn Sīnā also describes the rules of an art as 'universal' (*kullī*, *Jadal* 21.11), and at least in the case of logic it seems that he makes no consistent distinction between principles and theorems on the one hand, and rules on the other. At least for the case of logic, it will be helpful to lump together the principles, the theorems and the rules as the *truths* of logic.

As the mention of 'higher' sciences indicates, Ibn Sīnā puts the sciences into a hierarchy of higher and lower; a principle of a science, if it is not self-evident, is deduced using truths of a higher science. He also has a relation of inclusion between sciences at the same level, as for example anatomy is included in medicine and planar geometry is included in geometry.

Ibn Sīnā is clear that there is one science that is above all other sciences, namely the part of metaphysics that he describes as First Philosophy (*al-falsafa al-ūlā*). This science investigates the properties of basic meanings such as [EXISTS] and [ONE], as opposed to the more specific topics of the other sciences (*Burhān* 166.1f). Ibn Sīnā refers to it as 'providing the principles of the other sciences' (*Ilāhiyyāt* 5.7f), and it is presumably the part of metaphysics that Ibn Sīnā describes at *Aqsām al-^{c}ulūm* 112.15–17 as 'investigat[ing] the bases and principles of such sciences as physics, mathematics and the science of logic, and refut[ing] false opinions about these'. In various places Ibn Sīnā talks about the borderline between First Philosophy and logic, often to say that some things which are commonly regarded as logic should be referred back into First Philosophy (e.g. *Maqūlāt* 5.1–9, *Qiyās* 13.6f, *Burhān* 188.8f). Nor does Ibn Sīnā ever suggest that there is any science intermediate between First Philosophy and logic. So we infer that logic lies directly below First Philosophy in the hierarchy.

There is a complication. Presumably some of the universal sentences expressible in the language of a science will be false, and so their contradictory negations, which are existential sentences, will be true. For example in logic some formal premise-pairs will be unproductive, which is to say that *there are* counterexamples to various putative conclusions. Now for universal statements Ibn Sīnā makes a distinction between those which express accidental truths (for example that all the planets are in the ascendent today, cf. (2.8) above) and essential truths. Every essential truth ϕ has a cause, and it's the task of the relevant science to locate that cause and feed it into a demonstration (*burhān*) of ϕ—to show not just *that* ϕ is true but also *why* ϕ is true. But Ibn Sīnā's picture of science has no corresponding distinction for existential sentences. The fact that such-and-such a premise-pair is unproductive is no more or less scientific than the fact that all the planets will be in the ascendent tomorrow. This has to be reckoned a blind spot in Peripatetic scientific theory.

It is certainly not a coincidence that the one place where Ibn Sīnā can be convicted of significant formal errors of logic is in his treatment of non-productive premise-pairs in propositional logic. It never occurred to him that Aristotle's method of terms needs a scientiific justification. If he had tried to work out a justification, he would have realised at once that the method needs adjustment when one applies it to the propositional logic of *munfaṣil* sentences. But he died before he realised this.

One might try hiding the quantifiers inside the definition of 'productive'. But then for example the statement 'No premise-pairs of such-and-such a form are productive' is a negative statement, and Ibn Sīnā's account of negative truths in science is hardly better than his account of existential ones.

There is a further point before we leave Ibn Sīnā's general theory of the sciences. Ibn Sīnā certainly doesn't believe that we learn new facts only by deriving them from already known principles. Often our first awareness of new facts comes from hands-on experience. (Here we touch on what is often referred to as Ibn Sīnā's 'empiricism'—see Gutas [16], McGinnis [43].) Mostly we learn from hands-on experience; key words in his accounts of this are *tajriba* ('experience' or 'experiment'), *imtiḥān* ('testing') and *istiḵrāj* ('working out'). He applies all of these words both to medical and to logical discovery. For example he tells us:

(4.1) As for us, without seeking any help we worked out (*istaḵrajnā*) all the syllogisms that yield propositional compound goals, and this without needing to reduce them to predicative syllogisms; and we enumerated all the propositional compound propositions. We invite those of our contemporaries who claim to practise the art of logic to do likewise, and to compare all of their findings with all of ours. (*Masā'il* [36] 103.12–14)

In Ibn Sīnā's view, we have an intellectual facility for converting our experience of many and varied instances into concepts for describing what happens in these instances, in such a way that if the concepts are added to the foundations of a science, they allow us to deduce theorems that account for the instances. For him, this is how science advances. (But he has no conception of using experience to correct mistakes in the foundations of a science; you can't correct what is known to be true. His sciences are pre-Galilean.)

4.2 Fitting logic into the picture

So Ibn Sīnā places the science of logic immediately below First Philosophy. But there is a complication. Although logic takes principles from First Philosophy, First Philosophy has to rely on logic for the validation (*taḥṣīl*) of its arguments. Ibn Sīnā is never in any doubt that First Philosophy is a rational discipline: it has 'demonstrations' (*Burhān* 179.12f) and 'syllogisms' (*Burhān* 188.8) and 'proofs' (*Burhān* 87.13). See also the wealth of references in Bertolacci [6] Chapter Six on the demonstrative content of metaphysics; some of these references must certainly refer to First Philosophy. But logic is the art which establishes the principles by which we test whether a demonstration does derive its conclusion from its premises.

This is not yet a paradox, but it does need some sorting. If the justification of the arguments of logic rests on the arguments of First Philosophy, and the justification of the arguments of First Philosophy rests on those of logic, then we have a vicious circle, and neither branch of science can claim that its arguments are properly justified. Ibn Sīnā's response is that since we clearly do have justified arguments in both these sciences, there have to be some arguments that need no justification from other arguments; in fact there must be truths that are self-evidently true and not in need of any justification. 'So it is clear that not all knowledge comes through demonstration, and that some of what is known is known through itself and directly' (*Burhān* 118.18). Also we can know that a demonstration is valid without having to count a statement of its validity as one of its premises (*Qiyās* 11.11–12.2).

So Ibn Sīnā says enough to guard against immediate threats of circularity. But there are still a number of loose threads to tie up in this area. One is that we run into principles of logic that are not self-evident, so they need some kind of justification, but no non-circular derivation is available. Ibn Sīnā recognises this point, and his answer (*Qiyās* 16.2–5) is that in such cases the justification has to consist of a 'preparation' (*'iᶜdād*). This 'preparation' is an idea that need not be directly relevant to the rule being derived, but when planted in our minds at the same time as the rule being considered, it can serve as a catalyst to induce in us a certainty that the rule is true.

Myself I think Ibn Sīnā gives himself far too much rope here. The thing is too subjective: one person's 'preparation' is going to be another person's codswallop. And in fact there are several arguments in Ibn Sīnā's treatment of modal logic that do seem to fall into this slot. I think they are codswallop, and so did many of Ibn Sīnā's successors in Arabic logic; in some cases there are indications that Ibn Sīnā didn't believe the arguments himself. There is a problem for historians of logic here. If these arguments were only ever intended as preparations, then there is not much point in trying to find any valid logical content in them. On the other hand are we entitled to dismiss parts of Ibn Sīnā's text in this way?

Before we go any further, we need to identify the things that Ibn Sīnā would count as being truths of logic. On his account, these truths will fall into three classes: (1) those that are self-evident (*bayyin bi-nafsih*) and need no proof, (2) those that are proved wholly within the science of logic (we will say that these are *proved internally*), and (3) those whose demonstrations rely on one or more premises from First Philosophy. There may also be (4) truths of First Philosophy that logic takes over and uses.

We ought to be able to look at Ibn Sīnā's logical writings and make some plausible guesses about what exactly he takes the truths of logic to be, and which of the classes (1)–(3) he puts them in. In fact I recommend this as a very healthy exercise.

Not that we need to rely just on plausible guesses. Ibn Sīnā himself takes us some of the way. For example in *Qiyās* i.2 he discusses how logic helps the other sciences by providing rules that 'measure' whether inferences are sound or not. In his first example he says

(4.2) [Logic] helps by being a measure which tells us that this premise-pair is productive. (*Qiyās* 11.17)

The specific premise-pair that he mentions is a particular case; presumably logic provides a general rule which says that such-and-such premise-pairs are productive, and Ibn Sīnā's example fits the conditions. So the rule is a condition of productivity. Ibn Sīnā's next example in *Qiyās*— i.2 illustrates how logic can confirm that a certain conclusion follows; here logic is providing a rule of following.

In both these examples the rules are being used affirmatively, to show that a given premise-pair is productive and that a given sentence follows from the pair. Generally Ibn Sīnā justifies the affirmative side of his conditions of productivity and rules of following by running through all the relevant moods and checking each mood. At *Qiyās* 108.10, after stating part of these rules for assertoric logic, he says 'You will learn these things later as we consider the separate cases'. So the validation of the affirmative content of these rules rests on establishing, for each of the moods, that it is in fact a mood.

What is the form of the statement that assertoric *Barbara* is valid? Using the definition of *Barbara* we can write it out:

(4.3) For all C, B and A, if it is posited that every C is a B and that every B is an A, then it follows that every C is an A.

So we have a universal truth, which quantifies over C, B and A. What exactly is being quantified over? The fact that there are three variables here is no worry for Ibn Sīnā; he regularly follows the advice of Alexander of Aphrodisias, that a triple of universal quantifiers can be read as a single universal quantifier over triples. But still the truth needs a subject term; it needs a value for X in the paraphrased form

(4.4) Given any triple of Xs, if the (a) sentence with subject the first element of the triple and predicate the second, and the (a) sentence with subject the second element and predicate the third, are both posited, then there follows the (a) sentence with subject the first element and predicate the third.

(Cf. *Qiyās* 184.2f for this use of 'first element', 'second element', 'third element'.)

We know Ibn Sīnā's answer to this question, because he tells us in several places (*Madḵal* 15.4–7, *Ilāhiyyāt* 10.17–11.2, *Mašriqiyyūn* 10.15 among them). The subject term X is 'meaning' (*ma*^{c}*nā*) or 'whatness' (*māhiyya*, the 'quiddity' or definitional core of a meaning); sometimes he adds 'well-defined' (*ma'qūl*, literally 'intellected'). In other words, the subject individuals of logic are well-defined meanings. (This is one place where we need to be clear about the difference between the subject term and the subject individuals.)

We can check that Ibn Sīnā's description works for the affirmative side of the conditions of productivity and the rules of following:

(4.5) Given any triple of well-defined meanings, if the sentence with subject the first element and predicate the second, and the sentence with subject the second and predicate the third, satisfy the following conditions [namely those for first figure], then the premise-pair consisting of the first sentence and the second is productive.

(4.6) Given any triple of well-defined meanings [etc. as above], the sentence which has subject the first element and predicate the third, and has such-and-such a quantity and such-and-such a quality, is a consequence of the aforementioned sentences.

Strictly these sentences should be tightened up to restrict the meanings to ones of the appropriate type for assertoric logic; for example they should be of noun or verb type, not proposition or particle type. This kind of restriction on the subject term is a very good illustration of what Ibn Sīnā says at *Najāt* 135.12–136.3 about how the subject terms of the truths of a science adapt the subject term of the science as a whole.

In a later section we will dig deeper into Ibn Sīnā's description of the subject term of logic. But already we have enough to start fitting assertoric logic into Ibn Sīnā's scheme of a science. For example the theorems expressing that the second- and third-figure assertoric moods are valid are prime candidates to be theorems with internal proofs. As we saw, Aristotle proves them by using the validity of first-figure moods and of conversions and ectheses; and all of these can be written down as theorems of logic that were justified before the second- and third-figure moods.

The next two sections consider two other kinds of principle: those expressing the validity of first-figure moods and those that account for ecthesis.

4.3 The logician as logician

The main thing that logicians do as logicians is to formulate and apply the rules of logic. So we should in theory be able to reach a better understanding of what Ibn Sīnā means by the phrase 'the logician as logician' if we set alongside each other the places where he uses this or similar phrases, and the places where he explains what the rules of logic look like. This enterprise really deserves a paper of its own, or perhaps several, since it ties in closely with Ibn Sīnā's general notion of a science. The present section is a holding operation.

Most of what Ibn Sīnā tells us about the form of the truths of logic is wrapped up in his description of the subject term of logic. When he tells us that the subject individuals of logic are well-defined meanings, he adds two other points.

The first point is that the subject individuals have to be taken in the second of what he calls 'the two *wujūd*s' (*Madkal* 15.3, 19f, 16.1, 34.7–9, 13, *Maqūlāt* 4.15f). This is an ontological notion and we must explore it in a moment. But first, please be clear that there are not two different classes of meanings, those in first *wujūd* and those in second *wujūd*. The meanings in these two *wujūd*s are *the same meanings* but with a different ontological status. This is very clear for example in the discussion at *Madkal* 34.5–16 (as at *Madkal* 34.8–10 'The propria and accidents which belong to the *māhiyya* can be attached to it in each of the two *wujūd*s'). Marmura ([41] p. 46) translates *wujūd* in this context as '[kind] of existence'.

Ibn Sīnā gives his main explanation of second *wujūd* at *Madkal* 15.1–7. He explains there that a whatness can be considered in three different ways. The first is on its own; the second and third are the first and second *wujūd*s. In the first *wujūd* a whatness is considered as being true of (or 'attaching to') things in the world. In the second *wujūd* a whatness is considered in such a way that it can be a subject or a predicate, or predicated of all or some, etc. These are features that a whatness can have only as a part of a compound meaning. This is certainly what Ibn Sīnā has in mind here, since in the parallel passage of *Mašriqiyyūn* he has 'meanings in the context of their being subject to composition' (*ma^{c}ānī min ḥayṯu hiya mawḍū^{c}atun lil-ta'līf*, *Mašriqiyyūn* 10.15).

So what Ibn Sīnā is telling us with his references to second *wujūd* and being subject to composition is that in the truths of logic, meanings are described in terms of how they fit into compound meanings. A glance at the examples (4.3)–(4.6), (5.4) will confirm that this is absolutely correct for the examples of truths of logic that we have examined so far. The compound meanings are the meanings of propositions.

We turn to the second added point in Ibn Sīnā's description of the subject individuals of logic. This second point is that the truths of logic are in aid of making available new information either by definition in terms of known meanings or by deduction from known meanings (*Mašriqiyyūn* 10.15f, *Madḵal* 15.11f, *Ilāhiyyāt* 10.18 'in the context of how they bring about a progression from [already] known (*ma*^{c}*lūm*) things to [previously] unknown (*majhūl*) things' (*min jihati kayfiyyati mā yatawaṣṣalu bihā min ma*^{c}*lūmin ilā majhūlin*)). So not any true proposition about meanings as parts of compound meanings counts as a truth of logic. There is a further requirement that the proposition is a help towards the aim of gaining new information in either of the two mentioned ways.

In several places Ibn Sīnā adds remakrs about the kinds of accident or feature that can be 'attached to' the subject individuals in a truth of logic. There is a list at *Madḵal* 15.5f:

(4.7) being a subject, being a predicate, being predicated of all or some, being essential, being accidental, and some other things that you will learn about.

This list is given in an explanation of second *wujūd*, so it might be meant just as an explanation of things that can be said about a component of a proposition. But a later list at *Madḵal* 22.10–12 is specifically said to be about what properties are ascribed to simple meanings in the context of the art of logic:

(4.8) whether one of these whatnesses is a predicate, or a subject, or a universal, or a particular, etc.

Further lists appear in *Ta*^{c}*līqāt* 502.4–505.12 (and I assume we can count at least this part of *Ta*^{c}*līqāt* as the authentic words of Ibn Sīnā himself):

(4.9) being universal (*kullī*), being existential (*juz'ī*), being singular (*šaḵṣī*), ... being necessary (*wājib*), being absolute (*muṭlaq*), being possible (*mumkin*), ..., being affirmative (*mūjib*), being negative (*sālib*), ..., being contradictory (*tunāqiḍu*), being a premise (*muqaddama*).

Note that the items in these lists are not themselves subject individuals of logic; they are 'essential accidents' (*lawāzim*, singular *lāzim*, *Ta*^{c}*līqāt* 503.3) or 'features' (*aḥwāl*, singular *ḥāl*, *Madḵal* 15.16, *Ta*^{c}*līqāt* 507.4) of the individuals. Hence they are items that appear not as subject terms or subject individuals of truths of logic, but as ingredients of the *predicates* of truths of logic. Again a glance at the concrete examples in (4.3)–(4.6), (5.4) will confirm that it has to be this way round.

The passage in *Ta^{c}līqāt* comments on some of the items listed, that although they can be used in logic, they are established (*tuṯbatu*, e.g. *Ta^{c}līqāt* 504.11) in metaphysics or First Philosophy. Thus

(4.10) being a genus (*jinsiyya*), being a differentia (*faṣliyya*) and being a species (*naw^{c}iyya*)

are used as accidents of things in logic, but are established in First Philosophy (*Ta^{c}līqāt* 506.6f). A few lines later we read that

(4.11) genus, differentia, species, proprium (*ḵāṣṣ*) and accident (*^{c}araḍ*)

as 'features in the teaching of the existent as existent' are studied not in logic but in theory of the universal, i.e. in First Philosophy (*Ta^{c}līqāt* 506.9–11). Exactly what is intended here I am not sure, but it seems clear that Ibn Sīnā is somehow limiting the use of these notions in logic.

4.4 The boundaries of logic

The previous two subsections give us enough facts about the truths of logic to allow a comparison with the passages in which Ibn Sīnā says that something is not the concern of the logician. These passages are overwhelmingly in *Maqūlāt* (5.1–8.15, 29.11, 38.3–5, 62.11, 87.2, 106.3, 118.15, 143.15, 152.13). We can note straight away that there are no category words of any kind in the lists (4.7)–(4.10), except for 'accidental' in (4.7). If 'accidental' is in (4.7) as one of the features of the subject individuals of logic, and not just part of the explanation of second *wujūd*, then we should note that it is contrasted there with 'essential' and not with 'substantial'; this is not a category distinction. In (4.11) 'accident' appears; but this is with a list of predicables, not categories; and in any case it is not described here as playing any role in logic.

In the opening pages of *Maqūlāt* Ibn Sīnā says forwards and backwards and sideways that the categories are no use for logic. (Thus *Maqūlāt* 3.13–4.1 not all features of the components of compound expressions used in logic are themselves helpful for logic, since some are not relevant to reaching new concepts or information; *Maqūlāt* 5.1–9 the student of logic, as opposed to First Philosophy, never needs to learn the ten categories; see Gutas [15] 300–303.) In fact he seems to say too much here, suggesting that the notions of genus and species are useless for logic (*Maqūlāt* 5.7–9), in contrast to (4.10). It could be that at *Maqūlāt* 5.7–9 he is saying just that the contrast between genus and species is irrelevant to logic. Perhaps more likely, he is concentrating on the central part of logic that studies syllogisms; the notion of genus is not needed here, though it certainly is needed in the theory of definitions.

The other references in *Maqūlāt* point out specific issues that don't concern the logician. These are most of them things that we would be unlikely to have thought of putting into laws of logic. One case worth noting is *Maqūlāt* 143.15, where Ibn Sīnā says that it is no business of the logician to establish the theory of relations. Today we regard the theory of relations as an integral part of logic. It could be that Ibn Sīnā's notion of logic is less inclusive than ours, or alternatively that his concept of establishing the theory of relations is semantic and linguistic rather than logical. At any rate nothing like 'relation' (*'iḍāfa*) appears in the lists (4.7)–(4.10).

At *Mašriqiyyūn* 82.13 Ibn Sīnā says that the truth of sentences 'as a fact of nature and not of necessity' is not a concern of logicians. Again we note that 'true' is not in the lists (4.7)–(4.10), though 'necessary' is. At *Išārāt* 94.16 Ibn Sīnā says that a logician examines a proposition without being concerned with whether the proposition is true.

At *Burhān* 87.10–12 he says that the question whether X is possible in the case of matter Y is a question that can't be dealt with in logic but has to be investigated in First Philosophy. Now 'possible' was one of the terms in the list (4.7)–(4.10), so Ibn Sīnā does accept that a truth of logic can talk in terms of whether a certain proposition is possible. His point here seems to be that truths of logic, even if they can use this notion, can't stipulate what is possible in medicine or biology (two fields he has been discussing).

Although Ibn Sīnā is adamant that categories play no role in the truths of logic, he is by no means so sure that they play no role in the *practice* of logic. For example we know, and (4.9) acknowledges this, that there are rules of logic about what is contradictory to what. But in several places Ibn Sīnā indicates that when we have a proposition ϕ and we want to find the contradictory negation of it, we should make sure that the contradictory negation carries the same 'additions' as ϕ, and he uses the categories as a check-list of what additions we might need to look for. Thus at ^{c}*Ibāra* 43.6–44.9 he mentions potential, place, time, relation. A similar list at *Mašriqiyyūn* 48.6f mentions relation, time, place, quality, dimension, act, passion, potential, act. In the discussion of the subject term of logic at *Mašriqiyyūn* 10.15–19, Ibn Sīnā remarks that while there is no requirement that the subject individuals of logic should be 'substances or quantities or qualities or the like', a logician may pay attention to these features when looking for expressions that are 'suitable to form parts of an explanatory phrase or an inference'. Presumably features like these will play some role in ensuring that the meanings being used are well-defined.

In the light of the facts above, what can Ibn Sīnā have meant when he said that the difference between permanent and necessary is not one of concern to the logician as logician? The most straightforward reading is that he means that the laws of logic never need to refer to this distinction. And in fact we note that the lists (4.7)–(4.10) don't contain any temporal notion such as 'permanent', though they do contain alethic modal notions like 'necessary' and 'possible'. Again the most straightforward reading of this fact is that Ibn Sīnā thinks that the truths of logic can stipulate what holds in general for necessity and possibility, just from the meanings of these two words, but it is not any part of a logician's task, as logician, to determine what laws hold for any specific category of modality. (In view of 'essential' and 'accidental' in (4.7), the ontological modalities might be an allowed exception.)

This account leaves several possibilities open. For example Ibn Sīnā might still be able to point to some laws that hold for temporal modalities but not for the abstract alethic ones. We will find that in fact he doesn't, but this is something we will have to discover from his texts. We turn now to his temporal logic.

(Forgive me an aside here. I have suggested elsewhere that a modern reader can probably make best sense of Ibn Sīnā's notion of second *wujūd* by thinking of meanings in second *wujūd* as *occurrences* of meanings, by analogy with the difference between words and the occurrences of words in sentences. One shouldn't lose sight of another aspect of all this. For Ibn Sīnā the fact that we can handle meanings as parts of compound meanings is a criterial divide between humans and all other beings in the lower world. Hence for him it is reasonable to think of this ability as providing a guarantee of our personal immortality. The fact that Ibn Sīnā invests so much religious significance in a fairly abstruse point in the foundations of logic is both shocking and incisive. I suspect Ibn Sīnā and Lukasiewicz would have found they had a lot in common here.)

4.5 Where do necessity and possibility fit in?

In (4.9) above, Ibn Sīnā lists 'necessary' and 'possible' among those features of meanings that are studied in logic. Does this mean that the two concepts belong in logic rather than in metaphysics? That might seem paradoxical in view of the exalted place that the theory of the necessary existent has in

Ibn Sīnā's metaphysics. But it seems to be what Ibn Sīnā intends. In the chapter i.5 of *Ilāhiyyāt* where Ibn Sīnā introduces 'necessary' and 'possible', he refers back to 'the volumes on logic' (*funūn al-manṭiq*, *Ilāhiyyāt* 35.5). This reference is at least to *Qiyās* 168.12–170.13 where Ibn Sīnā points out that we need to avoid the circularity of defining 'possible' in terms of 'necessary' and then 'necessary' in terms of 'possible'.

In fact the two passages complement each other. In *Qiyās* Ibn Sīnā argues that we should define 'possible' in terms of 'necessary' and not the other way round, because necessity signifies 'firmness of *wujūd*' and possibility signifies the absence of this (i.e. non-firmness of non-*wujūd*, presumably). At *Ilāhiyyāt* 29.5 Ibn Sīnā explains that 'the necessary' is not definable in terms of anything better known than it, so it is one of those 'primary' (*awwalī*) concepts that are lodged in our souls directly. This bodes ill for finding any definition of 'necessary'.

At *Qiyās* 168.12–17 Ibn Sīnā says, in language reminiscent of *Qiyās* i.5, that 'possibility' (*'imkān*) is like *wujūd* and oneness (*waḥda*) and 'which' and 'what' and 'thing' (*šay'*) by having a family of meanings that apply to different categories, so that it has no single genus. In fact it is 'equivocal' (*mušakkak*), which means that it has a range of meanings that are held together by some common theme (cf. *Maqūlāt* 11.3–7, where 'healthy' is given as an example).

If I understand right, Ibn Sīnā is telling us here that 'possibly' is an accident of meanings, so that for example if attached to 'brown' it gives the meaning 'possibly brown' and if attached to 'large' it gives 'possibly large'. The same must hold for 'necessary', 'impossible' and 'contingent'. We would expect that 'possibly' is defined as 'not necessarily not', but in fact the definition that Ibn Sīnā offers at *Qiyās* 164.12 (and endorses at *Qiyās* 164.16) is less direct than this. First, it is a definition of 'contingent' rather than 'possible'; so 'possible' must be defined disjunctively as 'either contingent or necessary'. Then 'contingent' is defined as 'not necessary, but such that no impossibility results from assuming it'. We can unpack this, though without any help from Ibn Sīnā/ First, a thing is impossible if and only if some impossibility results from assuming it; so 'contingent' boils down to 'not necessary and not impossible'. Further, 'impossible' means 'necessarily not the case', and so 'contingently' is definable from 'necessarily' as 'not necessarily and not necessarily not'. If we wanted to get directly to 'possible' we could extract the first conjunct and write simply

(4.12) 'Possibly X' means 'not necessarily not X'.

The fact that Ibn Sīnā never gives this direct definition is partly explained by the fact that he lacks the notion of giving definitions that use variables. There may be other reasons, but to the best of my knowledge, none that are worth mentioning here. The formulation at *Qiyās* 49.12f comes close, explaining 'not necessarily' as 'possibly not'.

Since necessity is undefinable, the laws of necessity will have to be ones that we intuit directly from the notion of necessity itself. For the laws of necessity and possibility we have another option, namely to derive these laws from the definition of 'possible' in terms of 'necessary'. But we can see that there is not much that can be got from that definition on its own. If 'possibly' means 'not necessarily not', then 'not possibly not' means 'not not necessarily not not', i.e. 'necessarily'. (Cf. *Masā'il* 86.15 for cancelling double negations in this context.) So the same definition holds the other way round; the relationship between 'possible' and 'necessary' is completely symmetric. To break the symmetry we have to observe that 'necessary' implies 'possible', in other words, 'necessarily' and 'necessarily not' are incompatible. This is clearly a rather strong intuition that we have, and Ibn Sīnā should have flashed it up in neon lights. I haven't yet found a place where he says it explicitly, but it is so often implicit that it deserves a name; we can call it the 'Necessary implies possible' principle.

This leaves an open question: What sciences investigate the properties of specific categories of modality? The laws of time presumably come under physics. What about the laws of ontological necessity? What are these laws, and where does Ibn Sīnā investigate them? This is purely speculative, but perhaps Ibn Sīnā believes that any laws that apply to the concept of ontological necessity on its own are in fact included in the laws that apply to necessity in general, so that they are appropriately covered by formal logic. There could be some general truths relating necessity and cause, for example, but then these would be handled in whatever science studies causes; some material in *Burhan* might come under this head.

Then likewise any laws that relate the notions of necessity and existence, in particular those of necessary existence, would belong to the science of existence, namely metaphysics. And indeed this is exactly where Ibn Sīnā puts his discussions of the necessary existent.

Chapter 5

Logical procedures

5.1 Self-evident axioms

Our primary interest in these notes is the truths of logic which say that certain moods are valid. These truths include the statements of the moods, as discussed ABOVE. If these truths are not self-evident, then on Ibn Sīnā's scheme they need to be derived, either in logic or in some higher science. We will describe as *axioms of logic* those truths of logic that meet both of two conditions: (1) they are either statements of moods or are used in deriving statements of moods, and (2) they are not derived within the science of logic by internal proofs.

Some truths of logic are self-evident and need no further argument to justify them. Plausible candidates are the truths stating the first-figure moods. But caution: Aristotle said, of concrete syllogistic arguments in first figure, that it is self-evident that the conclusion follows from the premises. Does it follow that the sentence stating that all such syllogisms are valid is also self-evident? It's at a different level of generality. If we check Ibn Sīnā text on the point, we find—as often—that Ibn Sīnā has been here before us. In *Qiyās* 71.1 he defines the perfect premise-pairs as

(5.1) those that make clear through their forms the necessity of conceding the conclusion [that follows] from them (*hiya allatī tuẓhiru li-ṣūratihā luzūma al-natījati ᶜanhā*).

So for Ibn Sīnā the self-evidence is a property of the form rather than of the individual concrete syllogism. What is self-evident is that a certain form has a certain logical property, and this is exactly what the relevant truth of logic expresses.

In practice Ibn Sīnā complicates the situation a little by using two different criteria for perfectness of an inference rule. We can call them *naturalness* and *immediacy*.

The naturalness criterion is that it's natural for us to think like that. For example Ibn Sīnā defends the perfectness of the principle 'What is possibly possible is possible' (i.e. that we can infer from 'possibly possible' to 'possible') by saying

(5.2) In our nature (*al-ṭab*^{c}) the thought of 'being possibly possible' is close to the thought of 'being possible'.

This is at *Išārāt* 143.15f; in *Qiyās* 190.13f he says

(5.3) For this case, the mind rapidly determines that what is possibly possilble is possible.

A comment of Marmura on another passage is relevant here. He [42] p. 339 remarks that 'the question here is whether *al-ṭab*^{c} refers to philosophy or to a disposition in the individual doing philosophy'. He comes down strongly on the side of the latter, citing 'the Avicenna doctrine that the rational soul in its natural state, that is, when it is free from bodily concerns and potentiality, knows things as they truly are'. Of course in our passage Ibn Sīnā is talking about logical principles available to us here and now, not when we are 'free from bodily concerns and potentiality'. But something carries over: if a movement of thought comes to us naturally, this creates a presumption that the thoughts are correct, and may even induce a sense of certainty.

The immediacy criterion is that we don't need to do any work in order to be convinced. For example at *Qiyās* 185.15–17 he explains that a certain principle of reasoning is not perfect because it 'is not known except through study (*naẓar*); if [it] had been known from the given data, then we wouldn't have had to do any work (^{c}*amal*) to prove it'. A little earlier (*Qiyās* 183.4) he has defended the perfectness of the 'Possibly possible' principle by saying 'There is no proof that would make this clear statement any clearer than it already is'; so our certainty of it is direct and can't owe anything to a further argument.

These two criteria could come apart. There could be a principle that rests on nothing but itself and is less than wholly intuitive. There could be a principle that strikes us as entirely natural, but only when one has taken the trouble to paraphrase it into a certain form. A priori it seems that either of these faults would make the principle less than perfect, so we will assume that Ibn Sīnā requires both criteria to be met.

Besides the statements of first-figure moods, the main other candidates for axioms of logic are theorems stating that a-conversion holds, or that e-conversion holds, or that i-conversion holds, or that a sentence can be expanded in a certain way for ecthesis, or (if we accept that this is used) that ecthetic *Darapti* holds. These are truths of logic that are used in deriving second- or third-figure moods. As we saw, Aristotle himself seems to derive some of these truths from others in a circular way. To beg as few questions as possible, we will regard all these truths of logic, and their analogues in other logics that Ibn Sīnā studies, as prima facie axioms. They all need to be either shown to be self-evident, or derived either from other axioms of logic, or perhaps derivable from principles of First Philosophy.

A further class of truths of logic consists of those truths which say that a certain formal sentence is the contradictory negation of another formal sentence. The role of these will need to be established. We saw that Ibn Sīnā has set up assertoric logic in such a way that he never needs to use contraposition, and one corollary of this is that he never needs to use contradictory negations either. But for modal logic it may be different. We will see in fact that his use of contradictory negations of broad absoluteness sentences is quite different from his use of contradictory negations of possibility sentences.

There is a tiresome terminological point that could cause problems further down the line if we don't address it. A logician who is assembling principles can justify some of them by giving proofs of them. For self-evident principles a proof is not appropriate, but justifications of another kind might be. For example it may be in order just to point out that the principle is self-evident. In other cases (and there are many examples of this in Ibn Sīnā's logic) the logician might clarify some of the concepts involved, because it can happen that a principle is self-evident when the concepts in it are taken one way, but plain false if the concepts are taken another way. In practice Ibn Sīnā usually speaks of a justification consisting of a proof as a *bayān*; so self-evident principles don't need a *bayān*. (At *Qiyās* 13.5 he speaks of propositions whose *bayān* is just to be posited; this is not his normal usage.) For the more general type of justification that includes not only proofs but also clarifications, his most common word seems to be *taḥqīq* 'verification'. The word is particularly common in *Muḵtaṣar* and *Najāt*, and in *Muḵtaṣar* we also meet *qawl muḥaqqiq* 'verificatory statement' (*Muḵtaṣar* 54a6, 60a1). Gutas [15] pp. 214–7 calls attention to this notion of *taḥqīq*; but note that Ibn Sīnā's use of the term in logic is a good deal wider than Gutas suggests, and is not restricted to validating argument by putting them into syllogistic form.

In sum: the logician will want to *verify* all the principles of logic, but only the one that are not self-evident can be given *proofs*. A corollary is that self-evident principles are something of a dead end in formal logic. They don't allow any kind of formal justification. In formal proofs they can only be used as starting-points.

5.2 Ectheses

Neither Aristotle nor Ibn Sīnā suggests that arguments by ecthesis should be validated by any other principle of logic. So the rule of proof that they represent must be self-evident. What form does this rule take? The answers below anticipate some questions that we will come to in later sections; but I hope it makes sense at least to raise the relevant questions at this stage.

In (3.5) we took the ecthesis rule for the proof of *Bocardo* to say the following:

(5.4) If A, B and D are meanings, and D is the meaning 'B and not A', then from the sentence $(o)(B, A)$ we can conclude both $(a)(D, B)$ and $(e)(D, A)$.

My Persian is creaky, but this looks to me very close to the statement that Ibn Sīnā himself gives at *Dānešnāmeh* 78.4f. Besides transposing A and C, the main difference is that Ibn Sīnā states $(e)(D, A)$ but leaves $(a)(D, B)$ to be drawn out later. (It also matches the parallel passages at *Muḵtaṣar* 51b3f, *Najāt* 61.11f, *Išārāt* 148.4f; *Qiyās* 116.10f is slightly more ambiguous.)

Thom [53] p. 169f, working from this same text in *Dānešnāmeh*, finds that Ibn Sīnā reasons 'in accordance with the rule'

(5.5)
$$\frac{Q \;\; \Pi N^e \;\; N\Sigma^i}{q} \quad \longrightarrow \quad \frac{Q \;\; \Pi\Sigma^o}{q}$$

(where N does not occur in Q or q)

This is a rule of the same general kind as Gentzen's natural deduction rule for elimination of $\exists$; N in the subsidiary derivation on the left is eliminated in the main derivation on the right. Examples in propositional logic show that Ibn Sīnā was well capable of formulating rules that involve subsidiary derivations. However, Thom's rule is an order of magnitude more complicated than (5.4), and though it could be formulated with an initial quantifier over meanings, one would be hard pressed to describe the resulting rule as self-evident. The crucial difference is that D in (5.4) is not a free variable that will need eliminating; it is definable in terms of A and B, and that definition forms part of the statement of the rule.

Ibn Sīnā doesn't say much about the definition of ecthesis or the ideas behind it. Three references are *Najāt* 52.2f, *Qiyās* 77.14–78.3, *Qiyās* 90.7–9. In all three of these passages, Ibn Sīnā offers *ta*^{c}*ayyun* or *ta*^{c}*yīn* as alternative names for ecthesis; these names both mean 'making determinate' or 'identifying uniquely'. In ecthesis we are given meanings, say A and B, and we define or specify uniquely a new meaning D in such a way that certain sentences involving D and A, or D and B, are true. So the new meaning is unique (*wāḥid*) and determinate (*mu*^{c}*ayyan*). This is the language that Ibn Sīnā uses to explain ecthesis, and his consistent practice is that when he uses ecthesis, he says what the new meaning D is.

We will see BELOW that in two-dimensional logic his descriptions of D are not as specific as they should be. But we will also see a reason, namely that he lacked a sound methodology for defining relational meanings. In short, Ibn Sīnā's theory and practice of ecthesis do support a formulation like that in (5.4).

A little more should be said. At *Qiyās* 78.1 Ibn Sīnā remarks that the new meaning can be determined 'either by perception (*ḥiss*) or by the intellect (^{c}*aql*)'. Perception certainly plays no role in the applications of ecthesis that Ibn Sīnā makes in his logic, though he is entitled to claim that any available information could be used to make the specification of D. He presumably mentions perception in recognition of Aristotle's remark at *Prior Analytics* i.41, 50a2, that ecthesis can be 'by perception' (*tô*$_i$ *aisthánesthai*). (Our text of Theodorus' translation omits this phrase of Aristotle, but Ibn Sīnā could also have read the commentary of Alexander of Aphrodisias which expands on this phrase.) Also the passage at *Qiyās* 90.7–9 reads best as saying that a certain individual (not a meaning) can be specified by D. But this probably doesn't indicate a different form of ecthesis; for Ibn Sīnā the individual would have to enter the propositions through its individual essence anyway, and an individual essence is a kind of meaning. The phrase *šay' wāḥid* at *Qiyās* 77.14), literally 'single thing', is most naturally read as 'single meaning' rather than 'single individual', given that *šay'* 'thing' is Ibn Sīnā's normal word for meanings. (Among many examples take *al-šay' al-wāḥid* at *Qiyās* 205.4, and *Maqūlāt* 246.5 where *šay' wujūdī* refers back to *ma*^{c}*nā wujūdī*. For the broader context see Wisnovsky [56], particularly Chapters

7 to 9 on the relationship between *šay'* and essence.)

Street [49] p. 140f finds two different kinds of ecthesis in Ibn Sīnā, and he associates the identifying of 'a particular thing' only with the first or 'perceptual' kind. For the second kind, which is 'used in syllogistic proofs', Street offers a formulation where the term D is under an existential quantifier, for example (translating one case of Street's (1.3.2) to our notation)

(5.6) If A and B are such that $(o)(B, A)$, then there is a D such that $(e)(D, A)$ and $(a)(D, B)$.

I think to get this formulation to work in a proof, we would need to adopt something along the lines of Thom's subsidiary derivation. Ibn Sīnā's own account coheres rather better than this; but Street acknowledges that his report of Ibn Sīnā's ectheses is based partly on the commentary of Ṭūsī.

5.3 Types of argument

It should be clear by now that both Ibn Sīnā and we need to have some understanding of the kinds of argument that are available for verifying the axioms of logic. This includes those axioms that are derived within logic from principles of First Philosophy. Four kinds of verification are worth identifying at this stage.

(1) Logical derivations from principles of First Philosophy

If Ibn Sīnā has at his disposable a principle of First Philosophy and some logical rules for deriving a further truth from it, then there is no reason why he shouldn't apply the logical rules and make the derivation. There are two main limitations on this kind of argument.

The first is that if Ibn Sīnā is justifying a rule of logic, it looks bad if he is going to use that same rule in order to justify it. This problem is going to crop up most often in the very early stages of setting up the science of logic; so for example we would expect it to be more of a problem for assertoric logic than for the modal logic, which can to some extent ride on the back of the assertoric.

The second limitation is that it doesn't make sense to use a formal derivation if the premises are in too crude a form. First Philosophy has to be set up without using the technical vocabulary of logic, and so some of its principles may first appear in a form that is too ambiguous or ill-defined to allow precise deductions from it. Or as Ibn Sīnā himself might put it, a principle may be delivered straight from the estimative faculty (the *wahm*) so that it or its parts are insufficiently intellected (*ma^{c}qūl*) to be counted as subject individuals for the science of logic.

There are concrete examples of this. First Philosophy delivers a principle of excluded middle, in the form

(5.7) There is no intermediate between affirmation and denial. (*Ilāhiyyāt* 48.15)

For logic one needs more precise statements about compatibility between propositions or meaningful sentences, taking into account phenomena like borderline cases (*mutawassiṭ*) or incongruence of concepts (*ğayr qābil*) or things that are only potential (*bil quwwa*) or empty concepts (*ma^{c}dūm*). Ibn Sīnā sets out some of these more precise statements in ^{c}*Ibāra* ii.2; for example these four troublesome phenomena are mentioned at ^{c}*Ibāra* 90.1f. Ibn Sīnā makes no attempt at a logical derivation of the statements in ^{c}*Ibāra* from the principle as stated in *Ilāhiyyāt*. In fact the principle itself is rather in limbo. Ibn Sīnā says in *Ilāhiyyāt* that it is explained in *Burhān*, but the formulation in *Ilāhiyyāt* doesn't occur in *Burhān*. The nearest thing in *Burhān* is a principle which is said to be 'absolutely general, applying to all sciences', namely

(5.8) Either affirming [a thing of a thing], or denying it, is true. (*Burhān* 155.15f)

Given Ibn Sīnā's usage with 'Either ... or', this could be a statement of non-contradiction together with excluded middle. In fact the further discussion at *Ilāhiyyāt* might be conflating excluded middle with non-contradiction, whereas ^{c}*Ibāra* is very clear about the difference.

So some looser kinds of argument will certainly be needed.

(2) Hand-waving arguments

A hand-waving argument is one that is not logically precise but seems to have the potential to be turned into a logically precise argument. Caveat emptor—until you have done the work you can't be sure that there really is a precise argument to be found. But life being what it is, arguments of this kind are often necessary and quite often convincing.

In the early twentieth century, workers in the foundations of logic and mathematics made a concerted effort to clean up the handwaving arguments that were the staple diet of this area. The result was to create new areas of logic; model theory in particular was the result of formalising previously loose notions of truth and definability. A modern logician reading

Ibn Sīnā has to be aware that before around 1900 there was no reason for anybody to expect that the bulk of arguments in the foundations of logic could be made precise and rigorous.

(3) Accommodation to the audience

Ibn Sīnā himself brings us into this territory with some remarks in the prologue to *Mašriqiyyūn*, describing how Ibn Sīnā had proceeded in his earlier writings (*Mašriqiyyūn*3.12–4.3, following Gutas' translation [15] pp. 38–40):

> Now since those who are occupied with Philosophy are forcefully asserting their descent from the Peripatetics among the Greeks, we were loath to create schisms and disagree with the majority of the people. We thus joined their ranks and Adhered in a Partisan spirit to the Peripatetics, since they were the sect among them most worthy of such an Adherence. We perfected what they meant to say but fell short of doing, never reaching their aim in it; and we pretended not to see what they were mistaken about, devising reasons for it and pretexts, while we were conscious of its real nature and aware of its defect. If ever we spoke out openly our disagreement with them, then it concerned matters which it was impossible to tolerate; the greater part [of these matters], however, we concealed with the veils of feigned neglect: ... in many matters with whose difficulty we were fully acquainted, we followed a course of accommodation [with the Peripatetics] rather than one of disputation, although with regard to what was disclosed to us from the moment when we first applied ourselves to this field, we would expressly reconsider our position and examine anew whatever we thought repeatedly demanded closer scrutiny because an opinion was confusing to us and doubt crept into our beliefs, and we said "perhaps" and "maybe". (5.9)

Before we throw up our hands in moral outrage, I should point out that every experienced teacher knows that you have to take your audience with you. At the very least this often means that some difficulties have to be swept under the carpet, and some alternative possibilities left unmentioned because they are likely to cause more confusion than enlightenment. That could cover 'feigned neglect'. (See Peter Donnelly [11] for an illuminating real-life example that involved explaining Bayes' Theorem to a jury.)

But Ibn Sīnā's 'devising reasons and pretexts' is harder to defend. What is perhaps least acceptable in Ibn Sīnā's apology is the suggestion that he actively defended positions that he believed were not just superficial but plain wrong.

Gutas's choice of 'accommodation' to translate *musā*^{c}*ada* seems well justified in context. But Wehr's dictionary offers for this word the slightly more positive translations 'support, backing, aid, help, assistance, enouragement'. In *Mašriqiyyūn* 10.14 Ibn Sīnā says that the *Šifā'* as a whole is written as a '*musā*^{c}*ada* to my Peripatetic colleagues'.

(4) Preparation

In *Qiyās* i.2 reviews some kinds of teaching that are appropriate in logic. One kind is where the student is brought to acquisition (*kasb*, *iktisāb*) of new knowledge through logical deduction from known premises. Case (1) above belongs here. For when this is not possible, Ibn Sīnā sketches two other approaches that the teacher can take, called 'reminder' (*taḏkīr*) and 'preparation' (*'i*^{c}*dād*). Reminder is what the name implies: the teacher brings to the front of the student's mind things that the student had come across before but had forgotten. Preparation is more interesting. This is where the teacher brings into the student's mind two things together; one is what the student needs to learn, while the other is something that provides no information on its own, but when put alongside the proposition to be learned, it acts as a catalyst to produce the required new knowledge. (*Qiyās* 16.2–7)

Ibn Sīnā gives no concrete examples of preparation, but he does indicate where it is likely to be useful in the teaching of logic. There is some reminder and preparation in ^{c}*Ibāra* (*De Interpretatione*), he tells us, but also some deductive reasoning. 'In what comes next' (*Qiyās* 17.1–8), i.e. in *Qiyās*, the part that is taught by logical deduction and acquisition is the part where there are few differences of opinion, and it relies on a part that is taught by reminder and preparation. This fits the pattern that we sketched earlier if the part that is taught by logical deduction consists of the internal proofs of assertoric logic, and preparation and reminder carry the task of teaching the axioms.

If this is right, then we can see Ibn Sīnā dividing the teaching of logic roughly into three levels. The most fundamental level, after the main concepts have been introduced, will be to verify the axioms. This part must be mainly pre-syllogistic, for reasons discussed above; so we can expect handwaving and preparation, garniished with a suitable amount of accommodation to the student. The next level will consist of the internal proofs of assertoric logic and of other logics that behave in an 'orderly and integrated' (*muttaṣil muttasiq*, *Qiyās* 16.9) way like assertoric logic. At this level we proceed by logical deduction. And finally there is a third and more advanced level, where the material being taught is controversial ('a sign of this being the large amount of difference of opinion', *Qiyās* 16.11). This part of logic is more exploratory and will have to rely on empirical investigation by student and teacher. So Ibn Sīnā mentions testing (*imtiḥān*) in connection with modal logic (*Qiyās* 193.1, 204.11, 208.6, 479.11) or in connection with mixed *muttaṣil* and *munfaṣil* sentences in propositional logic (*Qiyās* vi.2 309.8), and experiment (*tajriba*) and working out (*istiḵrāj*) in connection with modal logic (*Najāt* 75.15) and propositional logic (*Masā'il* 103.12).

One curious feature of the writing style of *Qiyās* is the large number of rhetorical questions. These are heavily concentrated in the parts that discuss logical principles rather than formal development. Examples are *Qiyās* 92.1, 95.15, 96.3, 97.4, 143.14, 152.15, 174.5, 183.8, 210.16, 214.8, 218.12. There are other uses of rhetorical questions, though they are rare; for example *Qiyās* 222.2 quotes a rhetorical question raised by Peripatetic writers, 225.7 is a stylistic variant of a legitimate formal argument, and 101.11 is to introduce a discussion rather than close it. There are also rhetorical questions in *Muḵtaṣar* (e.g. 47b8), *Najāt* (49.12) and *Masā'il*(101.7), but these are very much fewer. The significance of this use of rhetorical questions in *Qiyās* is at the very least that it marks a distinctive style of argument for dealing with issues of logical principle. Prima facie the effect is to replace cogency by bare assertion.

5.4 Conclusions so far

Conclusion 5.1 Ibn Sīnā understands ecthesis (*iftirāḍ*), at least as it is applied in the theory of syllogisms, to be a form of reasoning in which a sentence with terms A and B is used as a premise to derive two other sentences, one with terms A and C and the other with terms B and C, where C is a term defined by means of A and B.

Chapter 6

Two-dimensional logic

6.1 Ibn Sīnā's introduction of two-dimensional logic

At the close of section i.7 of the *Prior Analytics*, where Aristotle rounds off his survey of the assertoric syllogisms, the Paris manuscript of the Arabic translation has a rubric:

(6.1) It was an innovation of the Alexandrians to read only this far in the book; they refer to what follows it in the book as 'the part that is not read'. This [part] is the discussion of syllogisms composed of premises that have modalities. ([38] pp. 210f.)

Whether or not Ibn Sīnā had this rubric in his text of the *Prior Analytics*, he certainly wasn't discouraged from reading on. In fact this 'part that is not read', and perhaps even more the discussions of it by Theophrastus, Alexander of Aphrodisias and Themistius, had a profound effect on Ibn Sīnā's understanding of logic.

The texts of Theophrastus, Alexander and Themistius that Ibn Sīnā refers to (for example at *Najāt* 39.10f) are now mostly lost—though the relatively recent publication of a Hebrew paraphrase of a relevant work of Themistius [48] gives hope that more of this material may yet turn up. But this is not so important for us, because our main concern is not how Ibn Sīnā treated his sources, but the conclusions that he came to himself after reading those sources.

Immediately after the rubric just quoted, the Arabic Aristotle proceeds:

(6.2) Because the *muṭlaq*, the *ḍarūrī* and the *mumkin* premises differ from each other … ([38] 29b29)

This translates a passage in which the Greek Aristotle says that there is a difference between being something, necessarily being something and possibly being something. Here the Arabic *ḍarūrī* means 'necessary' and the Arabic *mumkin* means 'possible' (with some nuances to be discussed below). The Arabic *muṭlaq*, normally translated 'absolute', means 'not qualified' or 'not subject to any condition', which is not an item in the Greek original. The Arabic translator has taken what in Aristotle's Greek are three different things that a sentence might express, and has converted them into three kinds of sentence; in the process he has invented a new kind of sentence, the 'absolute' sentence. (See Lameer [40] 55–59 on how this innovation might have crept in as the translation passed through Syriac.)

Ibn Sīnā, reading the Arabic Aristotle, thought that in the part of the *Prior Analyics* 'that is not read', Aristotle was discussing the logical properties of sentences with one or other of three modes, 'necessary', 'absolute' and *mumkin*. He was aware that the Arabic Aristotle's *mumkin* could mean either 'possible' (i.e. not necessarily not the case) or 'contingent' (i.e. not necessarily the case and not necessarily not the case). In cases of ambiguity like this, Ibn Sīnā distinguishes between a 'broad' (*ᶜāmm*) or more inclusive sense, and a 'narrow' (*k̲āṣṣ*) or 'strict' (*ḥaqīqī*) or less inclusive sense. So we find in Ibn Sīnā frequent references to 'broad *mumkin*' and 'narrow (or strict) *mumkin*', which are different though closely related modes. The modes 'necessary', 'absolute', 'broad *mumkin*' and 'narrow *mumkin*' together form the main modes that Ibn Sīnā finds studied in Aristotle; we will call them the *alethic modes*.

The Arabic Aristotle adds these modes to assertoric sentences. Thus we find sentences like 'A is with necessity found in some B' ([38] 34b23) which Ibn Sīnā would normally write as 'Some B is an A, with necessity'. Adapting the notation $(i)(B, A)$, we can abbreviate this to

(6.3) $$(i\text{-}nec)(B, A).$$

Similarly we have sentence forms $(a\text{-}pos)(C, B)$, $(i\text{-}con)(C, A)$, $(o\text{-}abs)(A, D)$ with *pos*, *con* and *abs* for necessary, possible, contingent and absolute. Often Aristotle and Ibn Sīnā are unclear about whether they intend possible or contingent, so we will sometimes need to write such things as $(a\text{-}mum)(B, A)$ with *mum* for *mumkin*.

While we are about terminology, we should do something about the Peripatetic habit, which Ibn Sīnā follows, of using 'necessary' both of sentences that are necessarily true, and of sentences that state that something is necessarily the case. The 'necessary' sentences of alethic modal logic are of the second kind, not the first. A similar point applies to 'possible', 'contingent', 'absolute'. We will follow what has become a standard convention, that a sentence stating that something is necessarily the case is a *necessity* sentence. Likewise we speak of *absoluteness* sentences, *contingency* sentences etc. For abbreviation a necessity statement will be described as having the modality nec, a possibility statement as having the modality pos, and likewise con for contingency and abs for absoluteness. For the ambiguous possibility/contingency form we will continue to say *mumkin*, abbreviated to *mum*.

The ambiguity between *pos* and *con* was pretty blatant, but Ibn Sīnā believed that he could find in Aristotle, Theophrastus, Alexander and Themistius discussions of ambiguities in *nec* and *abs* too. As we pass from Ibn Sīnā's *Muḵtaṣar* through *Najāt* and *Qiyās* to *Mašriqiyyūn*, we can sense a steady progression. In *Muḵtaṣar* Ibn Sīnā is concerned to set out the views of these earlier logicians, and to give some of his own reactions. By the time we reach *Mašriqiyyūn*, his reactions have settled into a collection of new sentence forms that amount to a new form of logic, and he no longer mentions the earlier logicians. It will become clear below that the effects of this new form of logic were already well entrenched in Ibn Sīnā's account of modal syllogisms in *Muḵtaṣar*. So probably the progression from *Muḵtaṣar* to *Mašriqiyyūn* marks an improvement in presentation rather than a change of content. The account in *Išārāt* is if anything a step backwards from *Mašriqiyyūn*, since it is less clear about the range of new sentence forms. Probably this is the result of the extreme brevity of the discussions in *Išārāt*.

For example Ibn Sīnā believed that Theophrastus and Themistius on the one side, and Alexander on the other side, disagreed about what is expressed by an absolute sentence. (Possibly this should read 'an absoluteness sentence'. But in these writers the distinction is not always clear.) For Alexander, an absolute sentence always expresses that all or some of the things that are Bs are sometimes As and sometimes not As. The other two logicians thought that an absolute sentence could express that all or some of the things that are Bs are sometimes As, without ruling out that some of these things might always be As. From Ibn Sīnā's discussions it is not at all clear (at least not to me) whether Ibn Sīnā thinks these earlier logicians are disagreeing about the meaning of the word translated as 'absolute', or whether they agree about the sense of the word but disagree about how one should interpret the sentences that fall under it; and if the latter, whether he thinks this is a disagreement about how these sentences are normally used, or a disagreement about how logicians should use them. Maybe he thinks these authors were themselves unclear about which of these they meant.

But at least by the time of *Qiyās* and *Mašriqiyyūn*, Ibn Sīnā is clear in his own mind: the view he attributes to Alexander should be read as a description of a particular type of sentence, which he calls *wujūdī*. A *wujūdī* sentence is one which expresses something of the form

(6.4) Every (or some) B is sometimes an A and sometimes not an A.

(Cf. *Mašriqiyyūn* 65.13f.) Ibn Sīnā also refers to sentences of this kind as 'the kind after the broad absolute', where a broad absolute sentence is one which expresses something of the form

(6.5) Every (or some) B is sometimes an A.

(E.g. *Mašriqiyyūn* 77.1–6, 79.1–3.) Ibn Sīnā also refers to these *wujūdī* sentences as 'narrow absolute' (e.g. at *Qiyās* 130.4, 162.8, *Išārāt* 145.1), in analogy with the distinction between broad and narrow *mumkin*. Ibn Sīnā believes that both broad and narrow absolute sentences occur regularly in normal scientific discourse. (He also believes that there is a particular problem about how negative universal broad absolute sentences are expressed, at least in Arabic; but I say no more about this here.)

Ibn Sīnā also believed that in Theophrastus he coul find a speculation about three different ways in which a sentence 'Every B is an A' can be read as expressing a necessary truth. Here I skip over the historical evidence ([13] p. 187ff, [48], Ibn Sīnā *Qiyās* i.5, 41.5–13) and concentrate on what Ibn Sīnā took from it. It seems that Ibn Sīnā had in front of him a claim that 'Every B is an A' can be read as expressing a necessity in the following three ways:

(6.6)
(a) Unconditionally.
(b) Under a condition that the subject is *mawjūd*.
(c) Under a condition that the predicate is *mawjūd*.

Here *mawjūd* could mean either 'existing' or 'true', and the subject could be either the subject term or the subject individual; so there is multiple ambiguity. Ibn Sīnā picked out two readings of (b) that he found significant, namely

(6.7) Every B is an A throughout the time while its [individual] essence is satisfied (i.e. while the individual exists).

and the second as

(6.8) Every B is an A throughout the time during which it is a B (i.e. while the subject term is true of the individual).

Setting out the paraphrases (6.7) and (6.8) in *Mašriqiyyūn*, Ibn Sīnā proposes for (6.7) the name *ḍarūrī*, i.e. 'necessary'; for (6.8) he proposes the name *lāzim*, which could be read as 'adherent'. Although both of these sentences express a kind of conditional necessity, Ibn Sīnā also calls the adherent sentences the 'adherent absolutes' (*Mašriqiyyūn* 79.14f). They also appear with names that only make sense in context, like 'this kind of absolute' (*Qiyās* 40.16, 128.14).

One can speculate about why Ibn Sīnā mentions essences in sentences like (6.7). But from his examples and comments it is clear that he just intends 'throughout the time while the individual exists'. Often he drops the mention of essence and just says 'while it continues to exist', as at *Qiyās* 77.3 and 91.2 and at *Mašriqiyyūn* 71.14f.

6.2 Features of two-dimensional sentences

By the end of these reflections, Ibn Sīnā has managed to transform Aristotle's alethic modal sentences, and some early reflections on how these sentences should be understood, into a whole raft of new sentence forms. These new forms have several things in common.

First, they contain no alethic modes, and no alethic modes are used in defining them.

This is implicitly denied by Thom [55] p. 74, who includes the word 'necessarily' in his definitions of both (d) and (ℓ). This must be a misunderstanding between Thom and his informant, because there is no textual evidence to support it. We have seen at (2.8) above that Ibn Sīnā allows that a thing can be permanent without being necessary. Ibn Sīnā does describe (d) sentences as 'necessary' (*ḍarūrī*), but this surely means that he counts permanence as a kind of necessity, not that necessity has to be read into the definition.

In this context it is perhaps unhelpful that a number of published works refer to Ibn Sīnā's sentences (6.7) as 'substantial', apparently mistranslating

Ibn Sīnā's word *ḏāt* 'essence' as 'substance'. It's hard to see how this came about. Al-Fārābī does say that *jawhar* (the normal Arabic word for 'substance') is sometimes used to mean essence (*Ḥurūf* [12] 63.9), and Ibn Sīnā confirms this at *Ḥudūd* Definition 15 ([25] p. 23) and at *Qiyās* 22.3. But if Ibn Sīnā ever goes the other way and uses *ḏāt* to mean substance—and Goichon [14] records no cases where he does—it would need an extremely strong argument to show that Ibn Sīnā has this in mind when he uses the word *ḏāt* in the sentences (6.7). Goichon [14] pp. 134, 136 describes the translation of *ḏāt* by 'substantia' as an unfortunate and confusing error, and I can only agree.

Second, these new sentence forms all contain a reference to time. In fact nearly all of them contain, besides the usual Aristotelian quantifier which we can now call the *object quantifier*, a second quantification over *times*. Because of this double quantification I will refer to these new sentences as *two-dimensional sentences*, borrowing this name from Oscar Mitchell who in the early 1880s independently began to develop Aristotle's assertoric logic in a similar direction [44]. As Ibn Sīnā must have observed from the outset, these two-dimensional sentences have logical relationships between them. And so we can refer to the logical study of these sentences as *two-dimensional logic*.

Third, these sentence forms, at least in Ibn Sīnā's mature account of them, come in four flavours like the four kinds of assertoric sentence: (a), (e), (i) and (o), and at least the main forms have contradictory negations that Ibn Sīnā describes. For example the contradictory negation of

(6.9) Every B is an A for as long as it exists.

is the (o) sentence

(6.10) Some B is, at some time during its existence, not an A.

So the two-dimensional forms include existential time quantifications that are dual to the universal ones in (6.7) and (6.8). The form

(6.11) Every B is, at some time during its existence, an A.

is one we can recognise as the form that Ibn Sīnā thought he found in Theophrastus and Themistius, which he called 'broad absolute'. (Cf. (6.5) above and *Mašriqiyyūn* 68.3–5.)

I add a remark that will play only a marginal role in this paper, but it may help for orientation. Another development that we owe to Ibn Sīnā is his extension of the classes of *muttaṣil* and *munfaṣil* propositional compound sentences to (a), (e), (i) and (o) forms. I believe this development took place within the framework of Ibn Sīnā's two-dimensional logic, broadly as follows. He reversed the relative scopes of the object and time quantifiers in two-dimensional sentences, and this gave him sentence forms that could be regarded as propositional compounds, generalising the propositional compound forms discussed by earlier Peripatetic logicians. In doing so he noticed that the *muttaṣil* sentences can be presented as exactly analogous to the assertoric sentences. The resulting propositional syllogisms obey exactly the same formalism as the assertoric ones: same moods, same justifications, but with time quantification in place of object quantification. Ibn Sīnā presents this result in *Qiyās* vi.1, spelling out the syllogisms with almost exactly the same order and commentary that he had used for the assertoric syllogisms in *Qiyās* ii.4.

A fourth feature of these two-dimensional sentences is that their truth-conditions are completely clear and unambiguous, at least after one has navigated a path through Ibn Sīnā's confusing explanations. This is partly the result of his removing the modal expressions 'necessary', 'possible' etc. from the sentences—the first feature above. But Ibn Sīnā takes a further step to guard against a possible ambiguity in the quantifiers. Some Peripatetic logicians had noted that a quantification over 'all Bs' can be over things that are actually Bs, or it can be over things that could possibly be Bs. Ibn Sīnā tells us frequently that he restricts these quantifications to things that are 'actually' (*bil fi*^{c}*l*) Bs. (Thus *Muḵtaṣar* 40a10–44a10; the phrase *bil fi*^{c}*l* occurs thirty-three times in this passage, always with reference to this point about the quantification. Also *Mašriqiyyūn* 68.3, 6f; this last is with reference to a 'necessary' sentence, blocking the suggestion sometimes made, that Ibn Sīnā's quantification over actual Bs might not apply to modalised propositions.)

This feature has also been denied by Thom. At [52] p. 362 Thom quotes

Inati's translation of *Išārāt* ([34] 93.10–12, [22] p. 99), and comments

(6.12) [Avicenna] takes the subject-term of an absolute or modal proposition to apply to whatever falls under the term, "be it so qualified in a mental assumption or in external existence, and be it so qualified always or not always, i.e., in just any manner". This formulation self-consciously rejects the idea that the subject-term of an absolute or modal proposition applies just to what actually exists.

If Thom is right then Ibn Sīnā in *Išārāt* has abandoned one of his most cherished positions in his earlier logical writings; we would certainly not be entitled to read his new position back into the logic of *Najāt* or *Qiyās*, as Thom goes on to do. Thom doesn't say what features of the quoted passage he takes as evidence for his conclusion, but let me guess that they are any or all of the following three: (a) the reference to 'mental assumption' as opposed to 'external existence', (b) the phrase 'in just any manner', and (c) the absence of any qualifying phrase 'in actuality' (*bil fi*^{c}*l*).

As to (a): Ibn Sīnā has forestalled this reading at *Qiyās* 21.6–10, where he spells out that for him, existence in thought counts as actual. He wants to be able to say that mathematical objects like the icosahedron are actual though they are not in the material world.

As to (b): the phrase 'in just any manner' (*kayfa ittafaqa*) is a stylistic variant of the more usual *kayfa kāna* 'however it is'. It certainly doesn't rule out a requirement for things to be actual; for example at *Išārāt* [34] 143.10 ([22] p. 136) Ibn Sīnā writes 'Every C is a B in act, in any way' (*bil fi*^{c}*l, kayfa kāna*).

As to (c): in reading Ibn Sīnā it is always dangerous to infer anything from the absence of a phrase in one passage when the phrase occurs in other parallel passages. This is particularly true of *Išārāt*, which was written in a telegraphic style. Even in *Muḵtaṣar*, where Ibn Sīnā leaves us in no doubt about the requirement of actuality, he sometimes doesn't mention this requirement. An example is at *Muḵtaṣar* 40a5, explaining 'Every B is an A', where incidentally he also says 'however it is described, permanently or not permanently, we don't know when'.

Although Thom's particular piece of evidence doesn't hold up, there are two other reasons why he is right to be cautious.

The first is that although Ibn Sīnā consistently says that he intends the object quantification to be over actuals, he never says the same for the time quantification. In fact some of his examples suggest that he must be including times that never were or will be actual, for example at *Qiyās* 30.10 'imagine a time when there are no animals except humans', or at *Qiyās* 134.11 'some time when nothing is coloured white or red'. I believe these passages occur only with wide time scope, which puts them outside the range of most of the passages discussed in this paper. I also have an impression that they are partly a hangover from earlier Peripatetic speculations about reducing propositional logic to predicate logic. But a complete account will need to say something about them.

The second reason for caution is that Ibn Sīnā, when he discusses possibility, accepts i-conversion from 'Some B can be an A' to 'Some A can be a B'. There are obvious counterexamples to this conversion if we require that the quantification in the second sentence is only over actual As. For example it seems entirely possible that there never was and never will be a purple cow, though some accident of biology could turn a cow purple. In this case some cow can be purple; but things that aren't cows don't have the potential to become cows, and it is not true that any actual purple thing ever was or will be a cow, so it is false that some actual purple thing can be a cow. The problem is not that Ibn Sīnā disowns his statements about quantifying over actuals when he comes to discuss possibility—he doesn't. Rather it is that the things that he says in different places seem not to be compatible.

This is not the kind of problem that has a quick fix. It need not trouble us until we come to consider in general how Ibn Sīnā deals with statements of possibility. But we must come back to the problem when we have a better broad perspective on what Ibn Sīnā is trying to do in the alethic logic of possibility.

One last point to be mentioned here is that in his *Physics* Ibn Sīnā defines time in terms of possibility (*Al-samā*^{c} *al-ṭabī*^{c}*ī* ii.11, 155–159). One might be tempted to say that as a result the laws of time must be no better known than the laws of possibility. But this is false. The facts that we need to know about time in order to check the truth or falsehood of sentences in the language of two-dimensional logic are very rudimentary, and none of them depends in the least bit on questions about the definition of time in terms of possibility. For example it is completely irrelevant whether or not time is discrete or continuous.

Chapter 7

Formalities

7.1 Formalising two-dimensional logic

Most of the two-dimensional sentence forms that Ibn Sīnā introduces are clearly enough described to allow formalisation in a two-sorted first-order language with an object sort and a time sort. We use lower case latin letters for the object variables and greek letters for the time variables. The relations all take the form $Rx\tau$, meaning that the object x is an R at time τ. There is one distinguished relation $Ex\tau$, which means that x exists (or as Ibn Sīnā would prefer, the essence of x is satisfied) at time τ.

We can reach most of the relevant sentences by starting with the assertoric sentence forms as in (3.3) above and making some replacements as in the following example. We have the assertoric formal (a) sentence

(7.1) $$(a)(B, A), \quad \text{i.e.} \quad (\forall x(Bx \to Ax) \wedge \exists x Bx).$$

We also have a *modality* (d) as follows:

(7.2) $$\forall \tau(Ex\tau \to Ax\tau)$$

expressing that x is an A throughout the time while x exists. We combine these two ingredients by putting the modality in place of Ax, and then replacing Bx by $\exists \tau Bx\tau$. This gives the formal sentence

(7.3) $$(\forall x(\exists \tau Bx\tau \to \forall \tau(Ex\tau \to Ax\tau)) \wedge \exists x \exists \tau Bx\tau).$$

Since this sentence comes from combining $(a)(B, A)$ with the modality (d), we call it

(7.4) $$(a\text{-}d)(B, A).$$

The same recipe works if we start from the (e), (i) or (o) forms, with a suitable twist on the augment of the (o) form:

$$\begin{array}{lll} (e\text{-}d)(B,A) & : & \forall x(\exists\tau Bx\tau \to \forall\tau(Ex\tau \to \neg Ax\tau)) \\ (i\text{-}d)(B,A) & : & \exists x(\exists\tau Bx\tau \wedge \forall\tau(Ex\tau \to Ax\tau)) \\ (o\text{-}d)(B,A) & : & (\exists x(\exists\tau Bx\tau \wedge \forall\tau(Ex\tau \to \neg Ax\tau)) \vee \forall x\forall\tau\neg Bx\tau) \end{array} \tag{7.5}$$

Note that in the negative cases (e) and (o) we replace $\neg Ax$ by the modality with the negation immediately in front of A.

Three other modalities behave the same way, namely the modalities (ℓ), (m) and (t):

$$\begin{array}{lll} (\ell) & : & \forall\tau(Bx\tau \to Ax\tau) \\ (m) & : & \exists\tau(Bx\tau \wedge Ax\tau) \\ (t) & : & \exists\tau(Ex\tau \wedge Ax\tau). \end{array} \tag{7.6}$$

(The letters are taken from the descriptions in *Qiyās* and *Mašriqiyyūn*; see [20].) For example we have

$$(o\text{-}m)(B,A) \quad : \quad (\exists x(\exists\tau Bx\tau \wedge \exists\tau(Bx\tau \wedge \neg Ax\tau)) \vee \forall x\forall\tau\neg Bx\tau) \tag{7.7}$$

which says that some sometimes-B is, at some time while it is a B, not an A.

Ibn Sīnā's general assumptions ([18]) allow us to add that nothing has a positive property at any time when the thing doesn't exist; in a phrase, nonexistents have no positive properties. So for every relation R,

$$\forall x\forall\tau(Rx\tau \to Ex\tau). \tag{7.8}$$

Also we quantify only over things that exist at some time:

$$\forall x\exists\tau Ex\tau. \tag{7.9}$$

We call the sentences (7.8) and (7.9) the *theory of* E, and we adopt them as background assumptions (or meaning postulates) whenever we are dealing with two-dimensional logic. Under these assumptions, each of the sentences in the following list entails all the sentences after it:

$$(g\text{-}d)(B,A), \;\; (g\text{-}\ell)(B,A), \;\; (g\text{-}m)(B,A), \;\; (g\text{-}t)(B,A) \tag{7.10}$$

where g is any of a, e, i, o. So we count d as stronger than ℓ, which is stronger than m, which is stronger than t.

We call a, e, i and o the *aristotelian forms*, and we call d, ℓ, m and t the *core avicennan forms*. The sentence forms $(g\text{-}h)(B, A)$, where g is an aristotelian form and h is a core avicennan form, and R and S are any two distinct relation symbols, will be called the *core two-dimensional forms*. Ibn Sīnā himself doesn't distinguish them by a name, but they are the leading forms in his account in *Qiyās* i.3 and *Mašriqiyyūn*, and they allow us to build a sensible logical theory around them.

In fact Ibn Sīnā uses other forms besides these core two-dimensional forms. He often calls attention to the *wujūdī* sentences which express that something is sometimes an A and sometimes not an A. Formally these are most smoothly handled by applying the modality (t) to the following four fictitious assertoric forms:

(7.11)
$$\begin{array}{lcl} (\ddot{a})(B,A) & : & \forall x(Bx \to (Ax \wedge \neg Ax)) \\ (\ddot{e})(B,A) & : & \forall x(Bx \to (\neg Ax \wedge Ax)) \\ (\ddot{\imath})(B,A) & : & \forall x(Bx \wedge (Ax \wedge \neg Ax)) \\ (\ddot{o})(B,A) & : & \forall x(Bx \wedge (\neg Ax \wedge Ax)) \end{array}$$

The modality (t) is applied separately to both Ax and $\neg Ax$. So for example we have

(7.12)
$$\begin{array}{lcl} (\ddot{a}\text{-}t)(B,A) & : & \forall x(\exists\tau Bx\tau \to \\ & & (\exists\tau(Ex\tau \wedge Ax\tau) \wedge \exists\tau(Ex\tau \wedge \neg Ax\tau))) \end{array}$$

We will call these forms the *double-dot forms*, and the process of passing from a form g to $\ddot{g}$ will be called *double-dotting*. In *Mašriqiyyūn* 80.14–20 Ibn Sīnā also discusses the corresponding forms with (m) in place of (t), but we will not need to consider these.

Passing from $(\ddot{a})$ to $(\ddot{e})$, or from $(\ddot{\imath})$ to $(\ddot{o})$, is called *reduction to the negative* (*rujū*ᶜ ᶜ*alā sālibih*, *Qiyās* iii.5 174.16), and the move in the opposite direction is *conversion to the affirmative* (ᶜ*aks* ᶜ*alā 'ījābih*, *Qiyās* iv.4 208.17). Since Ibn Sīnā allows the moves in both directions, it seems that he regards $(a\text{-}t)(B, A)$ as logically equivalent to $(e\text{-}t)(B, A)$, and likewise for the existential forms. Our formalisations reflect this. But it follows that Ibn Sīnā adds the augments in both affirmative and negative cases, or in neither. For simplicity we assume neither, though I suspect he plays it by ear. The only moods that it affects are *Darapti* and *Felapton*.

Ibn Sīnā also considers sentences got by fixing the time to a particular moment or interval α, for example

(7.13)
$$(a\text{-}z)(B,A) \quad : \quad \forall x(Bx\alpha \to (Ex\alpha \to Ax\alpha))$$

Here z abbreviates Ibn Sīnā's name for these, *zamānī* or 'temporal'. If α is the present then these forms correspond to the Latin *ut nunc* sentences.

If ϕ is any one of these new sentence forms, then we can uniquely recover from ϕ the assertoric sentence form that gave rise to it. We write $\pi_o\phi$ for this assertoric sentence form, and we call it the *assertoric projection* of ϕ.

Among these various forms, the ones that will chiefly concern us are those where the avicennan form is d or t. The reason for this is that Ibn Sīnā associates these two forms with the alethic modes of necessary, broad absolute and possible. Most of the other forms above he groups together as other forms of absolute. We will refer to the two-dimensional sentences with avicennan form d or t as the (dt) *fragment*.

Tying these various sentence forms to Ibn Sīnā's text is not always straightforward. In *Qiyās* for example, the sentences spelt out in the introductory sections are mostly two-dimensional, but when Ibn Sīnā comes to study rules of inference he switches mainly to alethic modal forms. However, he often sprinkles temporal words over these alethic forms, and he sometimes switches back to straightforwardly two-dimensional forms in order to discuss a particular point. So when he writes an alethic modal form, the reader has to ask whether it should be read straightforwardly as an alethic modal form, or whether it is really a disguise for a two-dimensional form. Unfortunately these could both be the case together; Ibn Sīnā is not above writing things that are intended to be read in two different ways simultaneously. See [17] p. 374f for a case in point, from *Qiyās* ix.6.

7.2 Metatheorems of two-dimensional logic

We assemble here some facts about the validity of inferences in two-dimensional logic. Mathematical proofs are given in [20]. Ibn Sīnā himself will have verified as many as he cared to by the kind of *istikrāj* that we saw him applying to propositional logic in (4.1).

Contradictory negations

Fact 7.2.1 *To find the contradictory negation of a core two-dimensional sentence $(g\text{-}h)(B, A)$, where g is an aristotelian form and h is an avicennan form, apply the* *following swaps to g and h:*

$$\begin{array}{ccc} a & \leftrightarrow & o \\ e & \leftrightarrow & i \\ d & \leftrightarrow & t \\ \ell & \leftrightarrow & m. \end{array}$$

Conversions and other one-premise inferences

Fact 7.2.2 *The following entailments hold between pairs of two-dimensional sentences with a given subject relation symbol and a given predicate relation symbol.*

$$\begin{array}{ccccccc} (a\text{-}d) & \Rightarrow & (a\text{-}\ell) & \Rightarrow & (a\text{-}m) & \Rightarrow & (a\text{-}t) \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ (i\text{-}d) & \Rightarrow & (i\text{-}\ell) & \Rightarrow & (i\text{-}m) & \Rightarrow & (i\text{-}t) \\ & & & & & & \\ (e\text{-}d) & \Rightarrow & (e\text{-}\ell) & \Rightarrow & (e\text{-}m) & \Rightarrow & (e\text{-}t) \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ (o\text{-}d) & \Rightarrow & (o\text{-}\ell) & \Rightarrow & (o\text{-}m) & \Rightarrow & (o\text{-}t) \\ & & & & & & \\ (\ddot{a}\text{-}t) & \Leftrightarrow & (\ddot{e}\text{-}t) & & (\ddot{i}\text{-}t) & \Leftrightarrow & (\ddot{o}\text{-}t) \\ & & \Downarrow & & \Downarrow & & \Downarrow \\ & & (e\text{-}t) & & (i\text{-}t) & & (o\text{-}t) \end{array} \tag{7.14}$$

Fact 7.2.3 *The following, and their immediate consequences by Fact 7.2.2 above, are the only conversions that hold between core two-dimensional sentences:*

$$\begin{array}{llcl} (a\text{-}t)\textit{-conversion:} & (a\text{-}t)(B,A) & \Rightarrow & (i\text{-}t)(A,B) \\ (e\text{-}d)\textit{-conversion:} & (e\text{-}d)(B,A) & \Leftrightarrow & (e\text{-}d)(A,B) \\ (e\text{-}\ell)\textit{-conversion:} & (e\text{-}\ell)(B,A) & \Leftrightarrow & (e\text{-}\ell)(A,B) \\ (i\text{-}m)\textit{-conversion:} & (i\text{-}m)(B,A) & \Leftrightarrow & (i\text{-}m)(A,B) \\ (i\text{-}t)\textit{-conversion:} & (i\text{-}t)(B,A) & \Leftrightarrow & (i\text{-}t)(A,B) \end{array} \tag{7.15}$$

Valid moods

We write a mood with premise-pair (ϕ, ψ) and conclusion χ as (ϕ, ψ, χ). This mood is *optimal* in a given figure, if it is valid, but if either we weaken a premise or we strengthen the conclusion, staying within that figure, then the resulting triple is not a valid mood. The *assertoric projection* of (ϕ, ψ, χ) is the triple $(\pi_o\phi, \pi_0\psi, \pi_o\chi)$ of assertoric projections of the three sentences. Likewise the *avicennan form* of the triple (ϕ, ψ, χ) is the triple (h_1, h_2, h_3)

where h_1 is the avicennan form of ϕ, h_2 is the avicennan form of ψ and h_3 is the avicennan form of χ.

Fact 7.2.4 *Suppose $(\phi(C,B), \psi(B,A))$ is a premise-pair and $\chi(C,A)$ is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for $(\phi(C,B), \psi(B,A), \chi(C,A))$ to be optimal in first figure:*

(a) The assertoric projection of $(\phi(C,B), \psi(B,A), \chi(C,A))$ is optimal in assertoric logic.

(b) The avicennan form of (ϕ, ψ, χ) is one of the following five triples:

$$(t,t,t),\ \ (t,d,d),\ \ (d,\ell,d),\ \ (\ell,\ell,\ell),\ \ (m,\ell,m).$$

Fact 7.2.5 *Suppose $(\phi(C,B), \psi(A,B))$ is a premise-pair and $\chi(C,A)$ is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for $(\phi(C,B), \psi(B,A), \chi(C,A))$ to be optimal in second figure:*

(a) The assertoric projection of $(\phi(C,B), \psi(B,A), \chi(C,A))$ is optimal in assertoric logic.

(b) The avicennan form of (ϕ, ψ, χ) is one of the following five triples:

$$(t,d,d),\ \ (d,t,d),\ \ (\ell,\ell,\ell),\ \ (m,\ell,m),\ \ (t,\ell,t),.$$

Fact 7.2.6 *Suppose $(\phi(B,C), \psi(B,A))$ is a premise-pair and $\chi(C,A)$ is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for $(\phi(C,B), \psi(B,A), \chi(C,A))$ to be optimal in third figure:*

(a) The assertoric projection of $(\phi(C,B), \psi(B,A), \chi(C,A))$ is optimal in assertoric logic.

(b) The avicennan form of (ϕ, ψ, χ) is one of the following five triples:

$$(t,t,t),\ \ (t,d,d),\ \ (d,t,m),\ \ (\ell,m,m),\ \ (m,\ell,m).$$

Fact 7.2.7 *If a mood in core two-dimensional logic is valid, then it remains valid if one or both of the premises are double-dotted, unless the mood is* Darapti *or* Felapton *and both premises are double-dotted. Double-dotting the second premise in first or third figure allows the conclusion to be double-dotted too.*

As noted in Subsection 3.3 above, Ibn Sīnā lists those moods that are conclusion-optimal, i.e. they are valid, but they become invalid if the conclusion is strengthened. In each figure, the conclusion-optimal moods (ϕ, ψ, χ) can be found from a list S of the optimal moods as follows. First, we check that the premise-pair (ϕ, ψ) is productive by checking that

(7.16) there is a triple (ϕ', ψ', χ) in S where ϕ is or entails ϕ' and ψ is or entails ψ'.

Then if the pair is productive, the strongest conclusion is the strongest sentence χ such that there is a triple (ϕ', ψ', χ) as in (7.16).

A metaprinciple

Fact 7.2.8 (Orthogonality) *For each triple (ϕ, ψ, χ) of core two-dimensional sentences in one of the three figures, the necessary and sufficient conditions for this triple to be a valid conclusion-optimal mood consist of two conditions, one of which says that the assertoric projection is valid and conclusion-optimal in assertoric logic, and the other refers only to the avicennan form of the triple.*

There is enough evidence that Ibn Sīnā was well aware of this principle, at least as a heuristic.

By the Orthogonality principle, every (valid, conclusion-optimal) two-dimensional mood has an assertoric projection that is an assertoric mood. We can name the two-dimensional mood by naming its assertoric projection and then listing the avicennan forms of its sentences. Thus for example *Barbara(t,d,d)*, which is a two-dimensional mood by Fact 7.2.4 above, is *Barbara* with a (t) first premise, a (d) second premise and a (d) conclusion.

Internal proofs

Fact 7.2.9 *There are four valid two-dimensional moods in the (dt) fragment where the internal proof of their assertoric projection by conversion or ecthesis doesn't lift to the two-dimensional case. They are as follows:*

- *In second figure,*

$$Cesare(d, t, d), \;\; Camestres(t, d, d), \;\; Festino(d, t, d).$$

- *In third figure,*

$$Disamis(t, d, d).$$

Fact 7.2.9 implies that the internal proofs using ecthesis can all be lifted to the (dt) fragment. There are four such cases:

(7.17) $Baroco(t,d,d),\ \ Baroco(d,t,d),\ \ Bocardo(t,t,t),\ \ Bocardo(t,d,d).$

An ecthetic argument that Ibn Sīnā cites at BELOW needs a further ecthesis for $(i\text{-}t)$-sentences. The next Fact assures us of the ectheses needed in these five cases.

Fact 7.2.10 *We have the following ectheses:*

$$\begin{array}{ll}
(1) & (o\text{-}t)(C,B) \vdash (i\text{-}t)(C,D), (e\text{-}\ell)(B,D) \\
 & \textit{where } Dx\tau \ \equiv\ (\exists\sigma(Ex\sigma \wedge Cx\sigma) \wedge \neg Bx\tau) \\
(2) & (o\text{-}d)(C,B) \vdash (i\text{-}d)(C,D), (e\text{-}d)(B,D) \\
 & \textit{where } Dx\tau \ \equiv\ (Cx\tau \wedge \forall\sigma(Ex\sigma \to \neg Bx\sigma)) \\
(3) & (o\text{-}t)(B,A) \vdash (a\text{-}t)(D,B), (e\text{-}t)(D,A) \\
 & \textit{where } Dx\tau \ \equiv\ (Bx\tau \wedge \forall\sigma(Ex\sigma \to \neg Ax\sigma)) \\
(4) & (o\text{-}d)(B,A) \vdash (a\text{-}t)(D,B), (e\text{-}d)(D,A) \\
 & \textit{where } Dx\tau \ \equiv\ (Bx\tau \wedge \exists\sigma(Ex\sigma \wedge \neg Ax\sigma)) \\
(5) & (i\text{-}t)(B,A) \vdash (a\text{-}t)(D,B), (a\text{-}t)(D,A) \\
 & \textit{where } Dx\tau \ \equiv\ (Bx\tau \wedge Ax\tau)
\end{array}$$

Fact 7.2.10 deserves three remarks. First, these ectheses are not all easy to find and check; they are as good examples as you can find of inference rules that are *not* self-evident.

Second, note the ℓ in (1). By Fact 7.2.4 this ℓ is needed for $Celarent(d,\ell,d)$; so we see that even operating the (dt) fragment sometimes requires us to use ℓ sentences. This is not the only example of this phenomenon. Ibn Sīnā himself cites another at *Išārāt* 145.5–11.

Third, Street [49] p. 152 doubts that $Baroco(abs,nec,nec)$ can be proved by ecthesis. If $Baroco(abs,nec,nec)$ is read as $Baroco(t,d,d)$ then (1) of Fact 7.2.10 shows how the proof goes. But in Street's paper $Baroco(abs,nec,nec)$ is treated as an alethic mood in an unspecified modal system, and in that setting I am not sure that the question whether this mood is provable by ecthesis need have a determinate answer.

7.3 The (dt) reduction

All the moods in the (dt) fragment can be derived by a reduction to assertoric logic. The reduction proceeds by changing the terms so that they include the references to time. We can call this method *incorporation*, in the sense that the terms are expanded to incorporate extra material. This is a move that Ibn Sīnā recognises and refers to quite often as 'making (a modality) a part of the predicate' (*ju^c ila juz'an min al-maḥmūl*), for example at *Muḵtaṣar* 44b7, *Najāt* 37.1f, *Qiyās* 42.4f, 86.4, 130.11, *Išārāt* 98.13f. He speaks less often of making a modality a part of the subject; but this may be because he includes a time reference in the subject by default, reading 'Every B' as 'Everything that was, is or will be a B at some time'.

To apply incorporation to the (dt) fragment, we introduce for every term, say A, two new terms 'always A' and 'sometimes A', in symbols A^+ and A^-. Formal definitions are

$$\begin{array}{lcl} A^+x & : & \forall\tau(Ex\tau \to Ax\tau) \\ A^-x & : & \exists\tau(Ex\tau \wedge Ax\tau). \end{array} \tag{7.18}$$

With these new terms we can translate any sentence of the (dt) fragment into an assertoric sentence, as follows:

(7.19)

dt sentence	:	assertoric translation
$(a\text{-}d)(B, A)$	:	$(a)(B^-, A^+)$
$(a\text{-}t)(B, A)$	:	$(a)(B^-, A^-)$
$(e\text{-}d)(B, A)$	:	$(e)(B^-, A^-)$
$(e\text{-}t)(B, A)$	:	$(e)(B^-, A^+)$
$(i\text{-}d)(B, A)$	:	$(i)(B^-, A^+)$
$(i\text{-}t)(B, A)$	:	$(i)(B^-, A^-)$
$(o\text{-}d)(B, A)$	:	$(o)(B^-, A^-)$
$(o\text{-}t)(B, A)$	:	$(o)(B^-, A^+)$

Together with these translations, we write Th($\pm$) (the theory of plus and minus) for the set of all sentences of the form

$$\forall x(A^+x \to A^-x). \tag{7.20}$$

These sentences are provable from the theory of E. Note that this reduction to assertoric logic is quite different from the assertoric projection.

One can show:

Fact 7.3.1 *(a) The valid moods in the (dt) fragment are exactly those whose translations are provable (by compound syllogisms) in assertoric logic if we allow Th($\pm$) as added premises.*

(b) The optimal valid moods are exactly those whose translations are valid syllogisms in assertoric logic.

(c) The conclusion-optimal valid moods are exactly those whose translations are provable in assertoric logic if we allow as added premises the sentences of Th($\pm$) for the terms which are predicates in the premises.

Fact 7.3.1 has a consequence that might be important for understanding Ibn Sīnā. By the Fact, the laws of the (dt) fragment will apply to any other logical system that translates down into assertoric logic in the same way. So we should look at the reduction and see what it presupposes. Each term A comes in two forms, a strong one A^+ and a weak one A^-; the strong implies the weak. In every sentence of the logic being reduced, the subject term is in the weak form. That's all. In particular the reduction doesn't assume anything along the lines that A^- is the De Morgan dual of A^+ (as for example that 'sometimes' means 'not always not').

So Ibn Sīnā would get exactly the same valid moods as in the (dt) fragment if he replaced 'sometimes' by 'throughout every Tuesday' and 'always' by 'throughout every Tuesday and Thursday', and then read his quantifiers as 'Everything (or something) that is a B throughout every Tuesday'. Or coming closer to Ibn Sīnā's metaphysical interests, he could read A^-x as 'x is a consistent meaning that is compatible with A' and A^+x as 'x is a consistent meaning that is incompatible with not-A', and again he would get the same laws as those of the (dt) fragment.

(Temporary note: At present [20] has A^+ and A^- the other way round. I had reckoned that the one with E positive should be A^+. But I now think it's more intuitive the other way round. Sorry; this will be repaired.)

Chapter 8

Ibn Sīnā reports the (dt) fragment

8.1 Ibn Sīnā lists the moods

So now we have two kinds of 'necessary' sentence and two kinds of 'broad absolute' sentence. One kind is the alethic sentences with modality either nec or abs, except where Ibn Sīnā indicates that he means some other kind of absoluteness. We will refer to this class of alethic sentences as the (nec/abs) *fragment* of alethic modal logic. The other kind is the two-dimensional sentences with avicennan form (d) or (t); these are the ones that Ibn Sīnā himself refers to as 'necessary' or 'broad absolute'. What is the relationship between the alethic and the two-dimensional versions?

We are going to do an experiment. First we will list, as list **A**, all the conclusion-optimal moods in the (dt) fragment. Then quite separately from this, we will list, as list **B**, all the moods in the alethic (nec/abs) fragment that Ibn Sīnā himself accepts. Then we will compare the two lists.

List A. By the Orthogonality principle (Fact 7.2.8), the list **A** need only list the avicennan forms, since the assertoric forms that go with them are determined by assertoric logic.

We take each figure in turn. For each figure we consider the four pairs (d,d), (d,t), (t,d) and (t,t). For each such pair (h_1,h_2) we can check from the appropriate one of Facts 7.2.4–7.2.6 whether the pair is productive, by looking to see whether there is a listed triple (k_1,k_2,k_3) with $h_1 \geqslant k_1$ and $h_2 \geqslant k_2$. If there is such a triple, we look for the strongest value of k_3 among such triples, and we call it h_3. Whenever (h_1,h_2) is productive, we count the triple (h_1, h_2, h_3) as validated and we put it into List **A**.

First Figure:

(8.1)

premise-pair	productive	strongest conc	validated triple
(d, d)	Yes	d	(d, d, d)
(d, t)	Yes	t	(d, t, t)
(t, d)	Yes	d	(t, d, d)
(t, t)	Yes	t	(t, t, t)

Second Figure:

(8.2)

premise-pair	productive	strongest conc	validated triple
(d, d)	Yes	d	(d, d, d)
(d, t)	Yes	d	(d, t, d)
(t, d)	Yes	d	(t, d, d)
(t, t)	No		

Third Figure:

(8.3)

premise-pair	productive	strongest conc	validated triple
(d, d)	Yes	d	(d, d, d)
(d, t)	Yes	m	(d, t, m)
(t, d)	Yes	d	(t, d, d)
(t, t)	Yes	t	(t, t, t)

The m in Third Figure looks like a misprint. But we can check it with any third figure mood, say *Datisi*:

(8.4) Some sometime-B is a C throughout its existence.
Every sometime-B is sometimes an A.

What is the strongest core two-dimensional conclusion we can get in this figure, i.e. with subject C and predicate A? Answer:

(8.5) Some sometime-C is an A sometime while it's a C.

The ideal conclusion would be that some sometime-A is always a C; but to get into third figure we need to convert this, and by Fact 7.2.3 the best conversion available is $(i\text{-}m)$-conversion. It will be interesting to see what Ibn Sīnā does with this case.

List B. Appendix B of [20] will give full references to the relevant passages of *Muḵtaṣar*, *Najāt*, *Qiyās* and *Išārāt*. I checked Ibn Sīnā's text myself and then compared with Street's list on page 160 of his paper [49]. Since our lists agreed in every detail, Street's published list will serve here. Street puts the major premise before the minor, in the Latin style, so we need to reverse these two. He writes L for nec and X for abs. Translating across into our present notation, we reach:

List **B**:

	First figure:	(abs, abs, abs), (nec, abs, abs), (abs, nec, nec), (nec, nec, nec).
(8.6)	Second figure	(nec, nec, nec), (nec, abs, nec), (abs, nec, nec).
	Third figure	(abs, abs, abs), (nec, nec, nec), (abs, nec, nec), (nec, abs, abs).

Results. Under the mapping $nec \mapsto d$ and $abs \mapsto t$, the lists are identical except for the third figure case where List **A** has (d, t, m). This discrepancy is completely accounted for if we suppose that Ibn Sīnā is working within the (dt) fragment, so that he is looking not for strongest conclusions but for strongest (dt) conclusions. There is a reason to expect him to do this, namely the genetic hypothesis. By that hypothesis one should expect that the modality of the strongest conclusion is a modality of one of the premises. The triple (d, t, m) is the only counterexample to that expectation.

With that proviso, the result of our experiment is that the two lists are a hundred per cent identical.

Readers of Street's [49] will see that his listing of valid moods includes two other items; we should check that they don't disturb the pattern. One is that he describes two of the first figure moods as 'imperfect'. This refers not to their validity but to the justification that Ibn Sīnā gives for them; we will return to this below.

The other is that Street includes two further moods with a sentence form that he labels A; this form is what on his page 136 he describes as 'perpetual (*al-dā'ima*)'. This form is a figment. Ibn Sīnā has no such form; he does label some sentences as *dā'im*, but these are the same sentences that he calls 'necessary', and very often he uses both labels together. The class of 'perpetual' sentences as a separate class was introduced a century and a half later by Rāzī (e.g. *Mulaḵḵas* 184.2), as a conscious departure from Ibn Sīnā's logic. Readers with no Arabic can confirm the point from Street's own translations. On his page 146 the A sentence is in a proof which he says is 'not given in Avicenna'. On the next page he has an A sentence in a proof said to be from *Najāt*; his translation at 2.2.2 on page 159 has no mention of perpetuity, and in fact the passage comes from a place where Ibn Sīnā is reporting Aristotle's assertoric syllogisms (*Najāt* 63.9–11).

Review The first point to make is that these results are highly significant. The two lists were compiled from completely different data sets. List **A** was calculated from the semantics of a class of sentences described by Ibn Sīnā in the early parts of *Qiyās* and *Mašriqiyyūn*. List **B** records Ibn Sīnā's verdicts on alethic modal moods in other parts of Ibn Sīnā's texts. Compare for example with tables of moods accepted by Latin scholastic logicians (such as Buridan, cf. [8] pp. 41–44). In those cases known to me, the Latin logicians present their material proof-theoretically, and the main thing that one can check is that they have followed their own proof rules correctly. This is not our situation, because the information in List **A** makes no appeal of any kind to Ibn Sīnā's proof procedures. In this respect our results are more like, say, finding the right gear ratios in the Antikithera mechanism—though admittedly less startling than that case.

Nor are there any symmetries or obvious patterns in List **A** that could have led Ibn Sīnā to the information in List **B** by a happy accident.

Prima facie the results give strong support to the view that Ibn Sīnā, when he lists valid moods in the (nec/abs) fragment of alethic modal logic, is in fact reporting what is true in the (dt) fragment. But in view of the results of Subsection 7.3 above, we need to phrase this carefully. Ibn Sīnā is clearly working from some source that gives exactly the same valid moods as in the (dt) fragment. But are there other possible sources with this property?

The answer is certainly Yes. For example we would have the same List **B** in front of us if Ibn Sīnā was using modal predicate logic and reading A^+x as $\Box Ax$ and A^-x as $\Diamond Ax$. For the moment this particular suggestion is idle. Ibn Sīnā has already told us what sentences he is working with, namely the two-dimensional ones; and his insistence that he is quantifying only over actuals is hard to reconcile with the idea that his subject terms all take the form $\Diamond A$. But it's best not to close this door before seeing more evidence.

Our second slice of evidence will consist of the internal proofs that Ibn Sīnā offers for (nec/abs) moods in second and third figures. Do these agree with what is reported in Facts 7.2.9 and 7.2.10 about what methods of internal proof are available for the (dt) fragment? Do they throw any other light on the kind of sentences that he thinks he is dealing with?

We should note two other conclusions that can be drawn from the precise agreement of Lists **A** and **B**. One is the mundane but reassuring point that Ibn Sīnā is indeed considering only conclusion-optimal moods, as we have been supposing. This is reassuring because he never says explicitly that this is what he is doing.

The other conclusion is that Ibn Sīnā was capable of sustained and accurate work in formal logic, including work in areas that had not been considered before. (This conclusion will have to lapse if it turns out that Ibn Sīnā knew a work in which Galen had already described the (dt) fragment, but I don't suppose anybody expects this.) The results create a presumption that Ibn Sīnā's other claims in formal logic should also be taken seriously.

8.2 Ibn Sīnā checks the internal proofs

We review the justifications that Ibn Sīnā gives for second- and third-figure syllogisms in the (nec/abs) fragment. The passages in question are *Muḵtaṣar* 54a1–55a14, *Najāt* 67.1–68.9, *Qiyās* 130.4–159.16 and sections of *Išārāt* 147.10–153.2.

Some of the material in these sections is irrelevant to our purpose. There are sections that report and discuss what is in Aristotle and his commentators. There are sections that simply list what moods Ibn Sīnā accepts; we took these into account in the previous section. *Qiyās* iii.1 and iii.3 contain long digressions on sentences with wide time scope; for the present I am not counting these as part of the (nec/abs) fragment. In *Išārāt* the things that we are looking for are mixed up with some material on other modalities.

When these irrelevances are removed, virtually all of what remains falls into three groups:

(i) Discussion of proofs of *Baroco* and *Bocardo* by ecthesis or contraposition. This occupies *Muḵtaṣar* 54a17–54b7 (Baroco) and 55a10–13 (Bocardo); *Najāt* 69.9–12 (Bocardo); *Qiyās* 159.6 (Bocardo); *Išārāt* 152.10–153.2 (Bocardo).

(ii) Discussion of the proof of $Disamis(abs, nec, nec)$. This occupies *Muḵtaṣar* 55a14; *Najāt* 69.12f; *Qiyās* 158.3; *Išārāt* 152.1–4.

(iii) Discussion of the proofs of *Cesare*, *Camestres* and *Festino* where one premise is nec and the other is abs. This occupies *Muḵtaṣar* 54a8–16; *Najāt* 67.8–68.9; *Qiyās* 130.10–132.14.

In some cases *wujūdī* sentences are mentioned too.

We note at once that by Fact 7.2.9 and (7.17) these are exactly the places where the justifications in the assertoric case don't carry over straightforwardly to the (dt) fragment. Anybody who wants to claim that Ibn Sīnā is doing something other than reporting the situation with the (dt) fragment will need to show that these three topics are also an appropriate choice of topics for Ibn Sīnā to discuss in relation to that something other. For example if Ibn Sīnā is following not (dt) but its reduction to assertoric logic, then none of these three topics will need special discussion, because the assertoric proofs are already adequate. If he is following some version of modal predicate logic, then we need to be shown what kinds of rule he is using, and how these rules produce the same problems as the adaptations of the assertoric rules to the (dt) fragment.

Proofs of *Baroco* and *Bocardo* by ecthesis

Ibn Sīnā is already using ecthesis for these cases in assertoric logic. The ecthetic proofs adapt to the cases listed at (7.17). What is not routine is to find ectheses that work in the (dt) case, as in Fact 7.2.10. In fact Ibn Sīnā never spells out the ectheses for all four cases. He outlines the proofs for *Baroco(nec,abs,nec)* at *Muḵtaṣar* 54a17–54b3 using alethic modal language, and for $Bocardo(abs, nec, nec)$ at *Muḵtaṣar* 55a10–12, again in alethic modal language. The alethic language is not well set up for specifying the ecthetic term: for example at *Muḵtaṣar* 54b1f he says that D is what is a C and not an A; but he needs 'what is a C and necessarily not an A'. For the *Bocardo* case he doesn't even attempt to give a full description of D. The proof of $Bocardo(abs, nec, nec)$ at *Najāt* 69.8–13 likewise gives an inadequate explanation of D. The treatment of $Bocardo(abs, nec, nec)$ at *Qiyās* 159.6f doesn't even attempt a description of the proof. At *Išārāt* 153.1f there is an incomplete description of D, followed by an instruction to the reader to complete the argument. In none of these texts does Ibn Sīnā attempt a description of D for $Baroco(abs, nec, nec)$, the case where ℓ is needed.

I suspect that the reason why Ibn Sīnā is not more forthcoming about these ecthetic terms is that he didn't know how to be more precise. Our descriptions of them use two variables, but variables in this style were not part of Ibn Sīnā's tool-kit. He was after all the first logician to work with a logic where every sentence has two quantifications. He could reasonably reckon that a description ike 'what is a C and not an A', so far as it goes, is self-evidently in the right area to make the proof work, and he had no formal apparatus for taking the question any further. But see also what he does in the proof of *Disamis* below.

Proof of *Disamis(abs,nec,nec)*

Ibn Sīnā mentions this case in all four sources as something needing special treatment. The statement in *Muḵtaṣar* is very brief and barely says more than that ecthesis will give us what we want. The account in *Išārāt* 152.1–5 is fuller, essentially as follows:

(8.7)

$$\begin{array}{ccccc}
(a\text{-}t)(B,C) & & & (i\text{-}d)(B,A) & \\
 & & & \text{(ecthesis)} & \\
 & & (a\text{-}t)(D,B) & & (a\text{-}d)(D,A) \\
 & \text{(Barbara(t,t,t))} & & & \\
 & (a\text{-}t)(D,C) & & & \\
 & \text{(conversion)} & & & \\
 & (i\text{-}t)(C,D) & & & \\
 & & \text{(Darii(t,d,d))} & & \\
 & & (i\text{-}d)(C,A) & &
\end{array}$$

The proof is given with no modalities. As in the cases discussed just above, Ibn Sīnā specifies D with the inadequate description 'some B that is an A', but he adds at once that this should be adjusted so as to prove a conclusion with the same modality as the second premise. (In fact the definition

$$Dx\tau \equiv (Bx\tau \wedge \forall\sigma(Ex\sigma \rightarrow Ax\sigma)) \tag{8.8}$$

works here.)

Ibn Sīnā gives this same proof by ecthesis at *Qiyās* 118.7–9 in his treatment of assertorics and absolutes; it is needed there to cope with the case where the second premise is *wujūdī*. He refers back to this proof at *Qiyās* 226.16 for *Disamis(mum,nec,nec)*. Aristotle had already mentioned that there is a proof of *Disamis* by ecthesis, but we don't know what he used it for.

Cesare*, *Celarent* and *Festino

This is the most interesting case. Street [49] p. 148 comments that Ibn Sīnā finds that these moods have a necessary conclusion 'without however giving the proofs'. To my eye this is not correct; in *Qiyās* Ibn Sīnā gives two proofs for this result. But both proofs are odd, and one can see how they might be missed.

The first proof that Ibn Sīnā proposes for these moods is by incorporation, *Qiyās* 130.11–131.3. The proof that he points to is as follows for $Festino(nec, abs, nec)$:

(8.9)
$$
\dfrac{\begin{array}{c} (i\text{-}nec)(C,B) \\ (i)(C^-,B^+) \end{array} \qquad \begin{array}{c} (e\text{-}abs)(A^-,B^+) \\ (e)(A^-,B^+) \\ (e)(B^+,A^-) \end{array}}{\begin{array}{c} (o)(C^-,A^-) \\ (o\text{-}nec)(C,A) \end{array}}
$$

Yes, that works. So do the other two. Ibn Sīnā's description concentrates on the process of incorporating; some of his text here is incoherent and looks like rough notes to be sorted out later. Possibly Ibn Sīnā had it in mind to put a better account into his *Appendices* (*Qiyās* 139.1), which as far as we know were never written.

Although this proof is watertight and completely solves the problem of these second-figure moods, there may be reasons why Ibn Sīnā would be reluctant to rest with this use of incorporation. There are several places where he indicates that he dislikes justifying a logic by reduction to another logic. We quoted one at (4.1) above. Also the method is not in Aristotle—though one might cite *Prior Analytics* i.35, 48a29–39 to show that Aristotle was aware of the possibility. And third, Ibn Sīnā has no formal procedure to regulate the kinds of paraphrase that can be used. Over eight hundred years later, Frege was to condemn this lacuna as one of the major faults of the old Aristotelian logic.

The second proof is described in all three of *Muḵtaṣar*, *Najāt* and *Qiyās*. It has a different character from all the proofs above; in the language of Section 5.3 above, it is at best a proof by hand-waving. It doesn't describe what you need to write on the page to reach the required conclusion. Instead it proposes a way in which the student can look at the data, with a hope that this will convince the student. (If the student isn't convinced, he is instructed to try other books [sic] where Ibn Sīnā goes into more detail, *Najāt* 68.3f.) As Ibn Sīnā expresses it in *Muḵtaṣar* 54a13, if every C is a B with necessity, and no A is a B, then there is an 'essential distance' (*bawn ḏātī*) between the natures of C and A. There is no more content in this argument than there was in the incorporation argument, but the reference to essence and nature is a cue to the reader that we might here be formulating a principle on the basis of First Philosophy. Let me call this principle the Essential Distance principle and leave it there for the moment. (If I said any more, it would be that the principle looks a very plausible origin for Suhrawardi's Illuminationist Second Figure Principle [51] 23.6ff.)

A further point to mention is that a large part of the discussion in this part of *Qiyās* is explicitly in the temporal language of two-dimensional logic. For example this applies to the whole of the discussion at *Qiyās* 131f where Ibn Sīnā introduces the problem about adapting the assertoric justification to *Cesare(d,t,d)*.

Failed moods

We should note what Ibn Sīnā says about the failed moods when all the sentences are absolute. For example at *Muḵtaṣar* 51b15 he says that the ecthetic proof in second figure doesn't work because the proof 'reduces to proof through the same figure'. The only place where he used ecthesis in second figure in the assertoric case was to prove *Baroco*. We can verify from (3.4) above that if we try to copy this proof for *Baroco(t,t,t)*, then the ecthesis rule works only in the form

$$(o\text{-}t)(C,B) \ \vdash \ (i\text{-}t)(C,D) \text{ (or } (i\text{-}t)(D,C)),\ (e\text{-}t)(D,B) \tag{8.10}$$

so that the next step would be to maka a deduction from

$$(e\text{-}t)(D,B),\ (a\text{-}t)(A,B) \tag{8.11}$$

which is again in second figure. He also remarks that the proof by contraposition fails because no contradiction is found. For *Baroco(t,t,t)* the argument by contraposition would draw a conclusion from the second premise $(a\text{-}t)(A,B)$ and the contradictory negation of the conclusion, i.e. $(a\text{-}d)(C,A)$. But the optimal conclusion from this premise-pair in two-dimensional logic is $(a\text{-}t)(C,B)$, which doesn't contradict the first premise $(o\text{-}t)(C,B)$. So his brief remark in *Muḵtaṣar* is an exact description of the failure of two methods for proving *Baroco(t,t,t)*. (There is no corresponding remark in the treatment at *Najāt* 60.10–61.4, or at *Qiyās* 116.7–12.)

8.3 Stocktaking: the $(nec/abs) \to (dt)$ mapping

We have used a mapping from sentences of the (nec/abs) fragment to sentences of the (dt) fragment. I will call this mapping the *mapping from* (nec/abs)

to (dt), or more briefly the $(nec/abs) \to (dt)$ *mapping*. We say that this mapping *preserves validity* if whenever an argument in the (nec/abs) fragment is valid, then the corresponding argument in the (dt) fragment is valid too. We say that it *reflects validity* if the same happens but in the other direction, from (dt) to (nec/abs). This is standard logical terminology, and we will carry it over in the obvious way to other mappings and other things that might be preserved or reflected.

For example we have made no claim that Ibn Sīnā regards the $(nec/abs) \to (dt)$ mapping as preserving meanings. Evidence will emerge later that he almost certainly doesn't regard it as preserving meanings. (To anticipate: there are at least two kinds of reason for doubting that Ibn Sīnā regards the mapping as preserving meanings. First, he has a similar mapping from (nec/mum) to (dt), but he doesn't regard mum and abs sentences as synonymous. Second, the kinds of argument that he uses for establishing truths about alethic modal sentences are in general very different from those that he uses with two-dimensional sentences; with the alethic sentences he uses a great deal more hand-waving and accommodation.) This is why we use the word 'mapping', rather than 'translation' or 'paraphrase' which do imply that meanings are preserved. An example of a mapping that does preserve meanings is incorporation.

In a perceptive paper on Ibn Sīnā's modal logic, Asad Q. Ahmed [2] observes that Ibn Sīnā discusses alethic modal sentences in the Aristotle style, and contrasts these sentences with what he calls Ibn Sīnā's 'peculiar manner of reading' some of these sentences (p. 21). This peculiar manner turns out to be what we have been calling the $(a\text{-}d)$ and $(e\text{-}d)$ sentences. In a footnote on the same page, Ahmed refers to Ibn Sīnā's 'several different ways of looking at a proposition'. These two-dimensional sentences are surely not just ways of reading alethic sentences; they are sentences in their own right. This should be clear from a study of *Qiyās* 21–23, where Ibn Sīnā provides a number of scientific statements (some taken from biology, geography, physics and astronomy) as illustrations of the two-dimensional forms. From this passage the presumption should be that Ibn Sīnā selects the two-dimensional forms because they illustrate logically significant features found in normal scientific discourse. If these 'peculiar manners of reading' are recognised as sentences in their own right, then Ahmed's account falls into line with our account of the contrast between alethic and two-dimensional sentences.

Ahmed says on his opening page (p. 3) that Ibn Sīnā is 'trying to find an interpretation of the theory [of modal syllogisms] amenable to Aristotle's conclusions'. This is said before any evidence is presented, and it may be one of the assumptions with which Ahmed has approached the text. Certainly people have made such an assumption. But our evidence so far has to count against this assumption. In every case where Ibn Sīnā notes a difference between his own views and those of Aristotle, his own views coincide with what is true in the (dt) fragment. There is never any contest; the (dt) fragment wins and Aristotle loses every single time. We can conclude that Ibn Sīnā's account of the alethic modal logic of necessary and broad absolute is not an attempt to interpret or accommodate Aristotle.

Ahmed also remarks (p. 22): 'As there are different manners of construing a premise, the same syllogism will sometimes yield one conclusion, sometimes another'. This is a very interesting remark, because it points to the dog that didn't bark in the night. We have seen no single case where Ibn Sīnā presents a syllogism in necessity and broad absoluteness sentences, and finds that its correctly deduced conclusion is different from the conclusion of the corresponding (dt) syllogism. That suggests the following, which at the moment is only a conjecture about Ibn Sīnā:

(8.12) (Conjecture) For Ibn Sīnā, the logical truths of the (nec/abs) fragment are equivalent to those of the (dt) fragment under the mapping $nec \mapsto d$ and $abs \mapsto t$.

If Ibn Sīnā wanted to prove that the logical truths of the two fragments are equivalent in this way, how would we expect him to go about it, given that we are not saddling him with any belief that the $(nec/abs) \to (dt)$ mapping preserves meanings?

Our earlier discussion of the structure of logic as a science suggests an answer: Ibn Sīnā would prove that the axioms of the (dt) fragment are also true in the (nec/abs) fragment. Since all the affirmative truths of the (dt) fragment are derivable from the axioms by internal proofs, it would follow that the same holds for the (nec/abs) fragment; so every valid mood of the (dt) fragment would correspond to a valid mood of the (nec/abs) fragment. Strictly we should ask for an argument in the other direction too, to eliminate the possibility that there are valid inferences in the (nec/abs) fragment that don't correspond to anything valid in the (dt) fragment. But Ibn Sīnā is not strong on questions of invalidity, and he might well decide to take a rest after establishing that validity is reflected by the $(nec/abs) \to (dt)$ mapping.

In any case, exactly what would Ibn Sīnā need to establish about the (nec/abs) fragment in order to carry this argument forward? We can read off the answer from what we have covered so far. The axioms of the two-dimensional (dt) fragment are as follows:

(a) The eight first-figure moods, which by the Orthogonality principle are the four assertoric modes with appropriate avicennan modes attached:

$$Barbara(t,t,t), Celarent(t,t,t), Darii(t,t,t), Ferio(t,t,t);$$
$$Barbara(t,d,d), Celarent(t,d,d), Darii(t,d,d), Ferio(t,d,d).$$

(b) The valid modalised forms of a-conversion, e-conversion and i-conversion that lie within the (dt) fragment, namely:

$(e\text{-}d)$-conversion, $(i\text{-}t)$-conversion and $(a\text{-}t)$-conversion.

(c) The five forms of ecthesis listed in Fact 7.2.10.

All of these except the form of ecthesis that uses ℓ have counterparts in the (nec/abs) fragment via the $(nec/abs) \to (dt)$ fragment. If Ibn Sīnā is to prove that the truths of the (dt) fragment correspond to truths of the (nec/abs) fragment, then we expect to find him validating the axioms listed above, both in the (dt) fragment and in the (nec/abs) fragment. Chapter 10 will investigate how far this is what we find.

Note that the list of axioms of the (dt) fragment makes no mention of contradictory negations. This is because Ibn Sīnā's internal proofs for the (dt) fragment make no use of contraposition, and hence no use of contradictory negations either. We saw earlier that Ibn Sīnā has adopted a form of ecthesis that allows him to deduce all the assertoric moods without any use of contraposition. He has managed to do the same for the (dt) fragment of two-dimensional logic. The fact that this is possible in principle is a consequence of Fact 7.3.1 for the (dt) reduction, though Ibn Sīnā presumably didn't know that fact.

8.4 Conclusions so far

Conclusion 8.1 Ibn Sīnā in his treatment of alethic modal logic works with two classes of sentence, though they are not always clearly distinguished. One is alethic modal sentences in the style of the Arabic Aristotle, and the other is Ibn Sīnā's own two-dimensional sentences.

Conclusion 8.2 In his reports of the valid syllogisms of the (nec/abs) fragment of alethic modal logic, and his justifications of the second- and third-figure syllogisms in this fragment, Ibn Sīnā is taking his information from the corresponding facts about the (dt) fragment of two-dimensional logic.

Conclusion 8.3 Ibn Sīnā's account of the alethic modal logic of necessary and broad absolute is not an attempt to interpret or accommodate Aristotle.

Chapter 9

A critique of Aristotle

9.1 Deconstruction of a metarule

In *Qiyās* 140.8–141.2 [28] Ibn Sīnā reports an argument used by Aristotle in *Prior Analytics* i.10, 30b18–31, to show that in *Cesare* and *Camestres* in second figure, if the affirmative premise is a necessity statement and the negative premise is not, then the conclusion can't validly be taken to be a necessity statement. Ibn Sīnā makes a brief reply in *Qiyās* 141.3–9, and a more substantial one in *Qiyās* 142.15–144.5.

Ibn Sīnā changes the lettering to his usual convention: C minor term, B middle term, A major term (or in Arabic j, b, a). Here is a brief exposition of Aristotle's argument, with the lettering as in Ibn Sīnā's version.

We have a valid syllogism in *Camestres*,

(9.1)
No C is a B.
Every A is a B, with necessity.
Therefore no C is an A.

Aristotle claims to show as follows that the mood got by adding 'with necessity' to the conclusion is not valid. He argues: Suppose it is valid. Then we would have

(9.2) No C is an A, with necessity.

By e-conversion of necessity sentences we infer

(9.3) No A is a C, with necessity.

But also by conversion of the second premise

(9.4) Some B is an A, with necessity.

These last two sentences yield

(9.5) Some B is not a C, with necessity.

But this can't be right, because 'nothing prevents us choosing' the matter of the first premise in such a way that every B is a C, with possibility. In other words we can choose B and C so that in fact no Cs are Bs, but every B could be a C. If we choose the matter of the syllogism in this way, then we have succeeded in deducing a falsehood from true premises.

Aristotle's argument is a challenge to Ibn Sīnā, because of Conclusion 8.2 above. The rejected syllogism, namely (9.1) with its conclusion upgraded to necessary, maps to the two-dimensional syllogism

(9.6) $(e\text{-}t)(C,B),\ (a\text{-}d)(B,A).$ Therefore $(e\text{-}d)(C,A)$

or spelled out in natural language

(9.7) Every sometimes-C is sometimes not a B.
Every sometimes-A is always a B.
So every sometimes-C is never an A.

which is a valid conclusion-optimal syllogism. So if Aristotle is right, the mapping from nec and abs to d and t must fail somewhere in the argument.

Accordingly Ibn Sīnā tracks Aristotle's argument, checking its image under the mapping. At least up to (9.11) below, Ibn Sīnā states the sentences using only alethic modalities, so you might easily miss the connection to two-dimensional logic. But wait for the finale.

Aristotle supposed for contradiction that the syllogism (9.1) is valid with a necessary conclusion. This syllogism maps to (9.7). Aristotle opened his attack by applying e-conversion to the conclusion, deriving (9.3). Under Ibn Sīnā's mapping this conversion is valid and gives

(9.8) Every sometimes-A is never a C. (This maps *Qiyās* 140.11.)

Next Aristotle applied e-conversion to the second premise, getting (9.4) which under Ibn Sīnā's mapping yields

(9.9) Some sometimes-B is always an A.

But in two-dimensional logic (9.9) doesn't follow from the second premise of (9.7). That might be the end of the matter, but Ibn Sīnā persists and writes down what *does* follow in two-dimensional logic, though he writes it in alethic language:

(9.10) Some sometimes-B is sometimes an A. (This maps the first sentence of *Qiyās* 140.12.)

Never mind: Ibn Sīnā will note at *Qiyās* 204.1f, and we can easily confirm it directly, that (9.8) and (9.10) together entail

(9.11) Some sometimes-B is never a C. (This maps the second sentence of *Qiyās* 140.12.)

And (9.11), or rather the alethic statement of it, is exactly Aristotle's conclusion. Moreover Aristotle is clearly right if we understand him as saying that we can find B and C so that every sometimes-C is at least once not a B (which represents the first sentence of *Qiyās* 141.13), but also every sometimes-B is at least once a C (which represents the negation of the second sentence of *Qiyās* 141.13, the sentence that Aristotle says we can make false).

So we can choose a matter that exactly fits the (dt) mapping of Aristotle's claim.

The next comments are mine, not Ibn Sīnā's.

We have reached a strange situation. Aristotle claims to have proved a metatheorem. Ibn Sīnā has shown that the metatheorem is false, and also that the proof of the metatheorem is correct. There has to be a mistake somewhere. If this were purely in alethic modal logic, we could put the problem down to some obscurity in the basic notions, some twist in the concept of necessity maybe. But in two-dimensional logic we can't do that. The logic is as robust as standard predicate logic is today, and nobody who works with that logic thinks that it harbours formal paradoxes. So somebody somewhere has made an open-and-shut mistake.

But also the two-dimensional logic guarantees we can find the mistake. We only need take concrete examples, check whether they verify Aristotle's metatheorem, and if they don't, check at what point Aristotle's reasoning deduces something false from something true.

This is exactly what Ibn Sīnā proceeds to do. Aristotle has said that 'nothing prevents us choosing' a certain kind of matter. So we choose such a matter and see what happens.

We return to Ibn Sīnā's text, at the point where he declares 'respondeo' (*naqūlu*, *Qiyās* 143.1). Aristotle had said that nothing prevents us from choosing B and C so that no C is a B but every B could be a C. Ibn Sīnā interprets the task as choosing B and C so that

(9.12) Every B is at least once not a C;
and every C is at least once a B.

He gives two examples. His first choice, at *Qiyās* 143.2, is

(9.13) B = human, C = laughing (actually, not just potentially).

Every human is at least once not laughing ('while he is not laughing', *^{c}indamā lā yaḍḥaku*, note the switch from alethic to temporal vocabulary). But everything that laughs is human; Ibn Sīnā thinks he has shown already that only humans genuinely laugh, because laughter involves a capacity to be surprised, which in turn implies rationality (*Madkal* 46.4f, 75.1f).

Given B and C as above, we add to them the other premise of the original syllogism:

(9.14) Every A laughs, with necessity.

(*Qiyās* 143.3.) Ibn Sīnā understands this as implying that every A is always laughing, or at least that no A has the potential to be not laughing. Note that Ibn Sīnā doesn't bother to find an interpretation for A; it will be irrelevant.

Now by (9.14) and the second sentence of (9.12), every A is at least once human (*Qiyās* 143.4). Therefore by the first sentence of (9.12), every A is at least once not laughing (*Qiyās* 143.6). This contradicts (9.14). The outcome is that 'nothing prevents us from choosing' B and C so as to satisfy (9.12), but once we have done that, logic does prevent us from also assuming the other premise of *Camestres* with necessity. (*Qiyās* 144.5.)

With this brief and inconspicuous argument Ibn Sīnā has lobbed in two bombshells. First, he has uncovered a subtle but definite mistake in Aristotle's modal reasoning. Of course Aristotle may have meant something different from what we and Ibn Sīnā are taking him to mean. That possibility always hovers in the background when one reads Aristotle. But it seems that in the West nobody noticed the mistake in Aristotle's (supposed) reasoning until Paul Thom pointed it out in his [54] p. 125 in 1996. Several standard references expound Aristotle's argument and make exactly the same mistake as Aristotle; one even praises Aristotle for the sophistication and accuracy of the argument. It was apparently a very difficult mistake to detect.

The second bombshell is Ibn Sīnā's analysis of what has gone wrong in Aristotle's argument. Aristotle had assumed that if two formal sentences ϕ and ψ with terms B and C are consistent with each other, and then we take a third formal sentence χ whose terms are C and some other term A, then the set $\{\phi, \psi, \chi\}$ would also be consistent. This assumption is provably true for assertoric sentences (though probably both Aristotle and Ibn Sīnā could only have proved it by running through all possible cases). But it is false for two-dimensional logic, and so by Conclusion One it should be false for alethic modal logic too.

The significance of this result becomes clearer if we paraphrase it and relate it to Aristotle's definition of a syllogism, which we cited in Subsection 3.1 above as 'a piece of discourse in which when two or more sentences are proposed, something else follows of necessity from their being true …'. Suppose two sentences are given, respectively with terms C, B and terms B, A. Then Aristotle, along with virtually all other logicians until the nineteenth century, supposed that if 'something else' with two terms follows from these two, then this something else must be a sentence with terms C, A. Ibn Sīnā's counterexample shows that this is in general not true. The premises of modal *Camestres*, as in (9.1) above, entail that some B is with necessity never a C.

This is another place where if Ibn Sīnā and his successors had pursued his lead, they could have altered the history of logic radically. We could have seen proof rules like those of Peirce or even Gentzen several hundred years earlier than we did. But it never happened; perhaps even Ibn Sīnā was too beholden to Aristotle's procedures. Nevertheless Ibn Sīnā certainly understood what he had proved here about the possible forms of arguments. He repeated the point, with an argument of a different form, in *Išārāt* 145.5–9, [22] p. 399f. The form that he gave in *Išārāt* is different from the one above, in that it needs a sentence of avicennan form (ℓ); Ibn Sīnā knew this and said it.

These two discoveries, namely that Aristotle's argument about *Camestres* was mistaken, and that a certain basic assumption about the possible shapes of arguments was mistaken, could of course have been made using only alethic modal logic. This is certain for the first discovery, since Thom himself made it using only alethic modal logic. But by that route it took a thousand years longer to discover, and it eluded most of the best brains in Aristotelian scholarship. As far as I know, nobody ever made the second and more far-reaching discovery using only alethic modal logic.

In this respect, but perhaps not in any other respect, two-dimensional logic stands to alethic modal logic as Kripke structures stand to modern modal systems. It provides a robust and essentially set-theoretic semantics, so that we can say with certainty what is the case and what is not the case. Ibn Sīnā could even have claimed as Boole did:

(9.15) There is even a remarkable exemplification, in its general theorems, of that species of excellence which consists in freedom from exception. And this is observed where, in the corresponding cases of the received mathematics, such a character is by no means apparent. ([7] p. 7)

(For 'received mathematics' read 'received alethic modal logic'.)

I call attention again to three details. First, when at *Qiyās* 140.12 Ibn Sīnā quotes (9.4) (which in the Arabic Aristotle at 30b27 has 'with necessity'), he leaves out the necessity. This could be taken for carelessness, until we note that in two-dimensional logic the argument works without the necessity, but not with it. Second, when at *Qiyās* 143.2f Ibn Sīnā states his first counterexample, he doesn't say how he interprets A. Again this might be taken for sloppiness, until we register that the whole point of the example is that A can be any term whatever, as long as (9.14) comes out true. And third, we noted that when Ibn Sīnā returns to this theme in *Išārāt*, he correctly notes that with the rearranged example that he gives there, an (ℓ) sentence is needed. These are all small items, but they sum up to a picture of a logician very much in control of the fine details of his arguments. If later we find this same logician using specious arguments, it will demand an explanation.

See [19] for a closer examination of this passage of *Qiyās*.

But that's enough raving about Ibn Sīnā's achievements. We must get back to the serious business of studying the relationship between the (dt) fragment of two-dimensional logic and the (nec/abs) fragment of alethic modal logic.

9.2 Conclusions so far

Ibn Sīnā makes a number of claims and assumptions in this passage, and they need some unpicking. In the first place he indicates that Aristotle is wrong in rejecting *Camestres(abs,nec,nec)*, and in the second place he traces to its source the error in Aristotle's argument against this mood.

So the entire passage starts from a claim that *Camestres(abs,nec,nec)* is valid. But Ibn Sīnā gives no new arguments for this claim within this passage itself. So presumably the claim rests on Ibn Sīnā's more general conclusion that the mapping from (nec/abs) to d/t reflects affirmative logical laws; cf. Conclusion 8.2 above.

Conclusion 9.1 Ibn Sīnā shows by examples that the following is not true:

(9.16) If ϕ and ψ are two alethic modal sentences, both with terms A and B, and χ is a third alethic modal sentence with terms B and C, where A, B and C are all distinct, and ϕ is consistent with ψ, then the set consisting of ϕ, ψ and χ is also consistent.

The examples are given in a vocabulary that is partly temporal and partly alethic. In practice this hardly matters, because the examples are clearly counterexamples to the two-dimensional version of (9.16), and highly plausible counterexamples to the alethic version.

In general Ibn Sīnā's use of counterexamples is not the high point of his logic, so it's pleasing to be able to report a place where he definitely gets them right. But the really significant point here is that until Ibn Sīnā raised the matter, the truth of (9.16) seems never to have been an issue. Thom remarks (Syllogism REF).

Conclusion 9.2 Ibn Sīnā claims that Aristotle mistakenly believed that (9.16) above is true, and that Aristotle's supposed demonstration of the invalidity of *Camestres(abs,nec,nec)* rests on this false belief.

These two claims are again highly plausible given what Aristotle himself says.

Most of Ibn Sīnā's argument in this section is a display of how to find the error in a faulty argument. He pursues as much of Aristotle's argument as he can within the laws of the (dt) fragment, and even shows that (dt) rules can be used to bridge gaps in Aristotle's argument. By doing this he squeezes down the area within which Aristotle's mistake must occur, until he is left only with the false claim (9.16), which has to be where the mistake lies.

But there are two different ways of reading this set of moves, because of the ambiguous signals that Ibn Sīnā sends about the sentences involved. Route A is to shift Aristotle's argument from alethic modal logic (Aristotle's own setting) to two-dimensional logic, and use the proven laws of two-dimensional logic to perform the squeeze. Route B is to stay within alethic modal logic and use the mapping from (nec/abs) to d/t to reflect the laws of the (dt) fragment back into alethic modal logic. If Ibn Sīnā was convinced enough that the laws of the (dt) fragment are the same as those of the (nec/abs) fragment, it wouldn't make much difference which route he took. But as a piece of objective logical research Route A has to be preferred (which is why I tend to take Route A in talks on this passage).

Conclusion 9.3 In this section Ibn Sīnā proves a previously unrealised fact about the possible forms of inferences in two-dimensional logic, which distinguishes this logic from assertoric logic. It can be read, via the mapping, as a fact about alethic modal logic too; though the examples that Ibn Sīnā give allow us to infer it for alethic logic without going via the mapping. But Ibn Sīnā himself discovered it via the mapping, and it seems very probable that the mapping was what enabled Ibn Sīnā to uncover this fact, which evaded almost all other scholars of Aristotelian logic until recent years.

Chapter 10

The axioms of the (dt) and (nec/abs) fragments

FROM HERE ON, PRELIMINARY NOTES

Be prepared for a marked change of speed and comfort in this chapter. We are leaving behind us the level planes of formal calculation and moving into the much bumpier terrain of axiomatics.

10.1 The axioms considered

We listed in Section 8.3 the axioms that Ibn Sīnā needs to validate for both the (nec/abs) fragment and the (dt) fragment if he is to make a case that both fragments satisfy the same truths of logic. In this section we consider how far this agrees with Ibn Sīnā's own idea of what he needs to validate. The relevant texts are *Muḵtaṣar*, *Najāt*, *Qiyās* and *Išārāt*.

The four first-figure moods with modality *(abs,abs,abs)* are listed at *Muḵtaṣar* 49b10–50a3, *Najāt* 57.5–58.1, *Qiyās* 109.16–110.2, *Išārāt* 143.3–9.

The four first-figure moods with modality *(abs,nec,nec)* are discussed at *Muḵtaṣar* 53b4–54a1, *Najāt* 66.2–67.1, *Qiyās* iii.1 125.6–130.3 and *Išārāt* 143.3–9 (again).

The conversions of absolute sentences are discussed at *Muḵtaṣar* 46a3–48a12, *Najāt* 45.1–48.2, *Qiyās* 75.1–94.9 and *Išārāt* i.5.3, 114.1–117.4. The conversions of necessary sentences are discussed at *Muḵtaṣar* 48a12–48b17, *Najāt* 48.3–49.8, *Qiyās* 95.1–105.14 and *Išārāt* 117.5–118.10. Most of what Ibn Sīnā has to say about ecthesis is included in his comments on conversions.

This list agrees well with the list of axioms in Section 8.3. The main items that are not listed in Section 8.3 but are discussed by Ibn Sīnā are moods including other kinds of absoluteness sentence; see Section BELOW. Along with these there is some discussion of sentences with wide time scope. Ibn Sīnā never suggests that these items are needed for validating the truths of the (nec/abs) fragment; rather they represent arguments that are outside the nec/abs fragment.

We will see that in his discussion of e-conversions of necessity sentences, Ibn Sīnā introduces a detour through possibility sentences.

Išārāt i.5.5 118.2 says that $(a\text{-}nec)$ converts to $(i\text{-}mum)$. This is puzzling because $(a\text{-}nec)$ should surely entail $(i\text{-}abs)$, which (as *Išārāt* acknowledges at 116.7f) converts to broad absolute. But in fact all of *Muḵtaṣar*, *Najāt* and *Qiyās* are clear that $(a\text{-}nec)$ converts to $(i\text{-}abs)$, so *Išārāt* has given us a rogue statement at 118.2. At BELOW I offer an explanation: Ibn Sīnā's rearrangement of his material in *Išārāt* has resulted in a confusion between two different questions. It's never safe to rely on a single quotation from *Išārāt* without checking the point in Ibn Sīnā's other writings.

The main item that is listed in Section 8.3 but is not adequately discussed by Ibn Sīnā is ecthesis. Outside assertoric logic, he never (and I emphasise never) adequately specifies the required ecthetic term. Within two-dimensional logic the reason for this is very clear: the ecthetic term expresses a binary relation, and Ibn Sīnā has no methodology for defining binary relations. Our modern understanding of how to define them goes only back as far as Frege's *Grundgesetze* in 1892. (For Ibn Sīnā's struggles with definitions of binary relations, see for example *Išārāt* i.2.11, 67.3–9 on how to define 'father', and *Ḵiṭāba* 135.12f on how to define 'companion'.)

In sum, the fit with Section 8.3 is good.

10.2 Validating the first figure moods

The discussion of the condition of productivity for first figure absolute moods in *Qiyās* and *Išārāt* does more than list the moods. It contains what might be the makings of a general verification of the first figure moods with universal minor premise. As *Išārāt* 142.14 puts it,

(10.1) The minor term is included in the middle term. (*yadḵulu 'aṣḡaruhu fi al-'awsaṭ*; *Qiyās* 108.13f is similar.)

So anything that holds of the individuals under the minor term will hold of those under the middle term too. This is hardly a proof of *Barbara* and

Celarent from anything more general than them. But it does achieve two things. First, it indicates something that we should look for when generalising *Barbara* and *Celarent* to other versions of them, namely that the individuals falling under the minor term are asserted to fall under the middle term too. And second, it gives a formulation that embraces both *Barbara* and *Celarent*, and this is a step towards an integrated account of the whole syllogistic.

On the other hand this formulation fails for *Darii* and *Ferio*, because in neither of these is the minor term said to be included in the middle term. I didn't find any similar formulation in Ibn Sīnā that covers these two moods. But he certainly regards them as perfect; apparently he is happy to let them take care of themselves.

If we follow Ibn Sīnā's lead and confine ourselves to the first figure moods with universal minor premise, are these all covered by the formulation (10.1)? For the alethic moods it seems they are; in all of these the minor premise takes the form 'Every C is a B', which on the face of it says exactly what (10.1) says. This holds both for the (abs, abs, abs) case and for the (abs, nec, nec) case.

For the two-dimensional moods it is not quite so obvious. The subject term C doesn't name a class of individuals; if anything it names a class of pairs consisting of individual and time. The premise $(a\text{-}t)(C, B)$ doesn't say that every such pair under C is also under B; that would need the stronger statement $(a\text{-}\ell)$. Earlier Ibn Sīnā has given an example to show that the times can't be assumed the same in both subject and predicate:

(10.2) Everything that breathes in breathes out. (*Qiyās* 23.5)

But another route gets us home, namely incorporation. Incorporating the times in the subject and predicate terms gives us the premise 'Every sometimes-C is a sometimes-B'. Incorporation also translates the subject term of the major premise into 'sometimes-B'. So the effect of incorporation is to turn the moods into assertoric *Barbara* or *Celarent*.

Does Ibn Sīnā himself follow this route? He shouldn't, because the incorporation step should prevent him counting the moods as perfect. But in the analogous case of moods with 'necessary' and 'contingent', where he is more on the defensive and feels he has to say more, we will find him giving justifications that do look as if they involve incorporation. There is also an explicit reference to incorporation at *Qiyās* 127.3f, but with reference to a possible misunderstanding of the major premise when it is taken as necessary.

In short, the validations that he gives can be read as applying both to the alethic moods and to the two-dimensional ones. There are a few explicit references to permanence or non-permanence in this passage of *Qiyās*, but they never play a role in the justifications offered. Several are to distinguish broad absoluteness from the overtly temporal form (ℓ), as we will note below.

None of the four texts introduces, in connection with these first-figure moods, any argument that applies to one category of modalities rather than another (in the sense of Section 2.3). There is barely enough material here to justify classifying it under the heads considered in Section 5.3, but if there is an implicit reference to incorporation then we could count it as hand-waving.

10.3 Validating the conversions

Ibn Sīnā's arguments for or against various conversions are set against what he found in the Arabic Aristotle. We noted earlier that Aristotle seems to justify e-conversion of assertoric propositions by i-conversion of assertoric propositions, and vice versa; and that Ibn Sīnā accepts the escape route from this circle that Alexander offered, namely to interpret a remark of Aristotle as pointing to a proof of i-conversion by ecthesis. Ibn Sīnā takes Aristotle's arguments for assertorics as templates for arguments for absolute propositions, and so he proposes to justify $(i\text{-}abs)$-conversion by ecthesis. Chapter ABOVE suggested that he intends the same as the ecthetic proof that he mentions for assertoric *Darapti*.

The Arabic Aristotle, when he moves on from assertoric conversions to conversions of modal sentences, justifies $(e\text{-}nec)$-conversion by reference to $(i\text{-}pos)$-conversion and vice versa REF. Ibn Sīnā is aware of this circularity and proposes to deal with it in the same way as with the assertoric case. We will come to possibility sentences later. But we must note here that there is an ambiguity in the reduction of $(e\text{-}nec)$-conversion in the Arabic Aristotle, because some features of his text suggest that he might be reducing not to $(i\text{-}pos)$ but to $(i\text{-}abs)$.

The first of these features is the wording of Aristotle's initial statement at 25a30f:

(10.3) If it was that with necessity no B is an A, then with necessity no A is a B; because if it could be (*jāza*) that some A is a B then it could be that some B is an A.

The question here is whether 'could be' (*jāza*) is meant as a part of the sentences under discussion. If it is, then the sentences are possibility sentences. But another possible reading would understand Aristotle's sentence as:

(10.4) For all choices of B and A, if it was that with necessity no B is an A, then with necessity no A is a B. This is because, if there could be B and A such that [with necessity no B is an A but] some A is a B, then there could be B and A such that [with necessity no B is an A but] some B is an A, [which is absurd].

On this second reading, Aristotle's argument proves that 'Necessarily no A is a B' entails the falsehood of 'Some A is B', i.e. it entails 'No A is a B', and Aristotle has muddled this confusion with 'Necessarily no A is a B'.

That might seem forced. But the Arabic Aristotle goes on immediately to claim that 'With necessity every B is an A' entails 'With necessity some A is a B', and his proof of this reads (25a33):

(10.5) If it was that some A is a B without necessity, then some B is an A without necessity'.

Here there is no mention of possibility at all, and Ibn Sīnā would very reasonably read the argument as a reduction of $(a\text{-}nec)$-conversion to $(i\text{-}abs)$-conversion.

In both *Najāt* and *Išārāt*, Ibn Sīnā repeats in his own words the argument for $(i\text{-}abs)$-conversion by ecthesis. There is no surprise here; the argument is correct both for the assertoric case and for $(i\text{-}t)$, though perhaps redundant in both cases. What is more surprising is that he repeats in his own words Aristotle's argument for $(e\text{-}nec)$-conversion, including both of the puzzling features just mentioned. (Note for example the reference at *Išārāt* 117.10 to an unexplained 'requirement of absoluteness'.) We have to suppose that this is deliberate. Ibn Sīnā can spot an ambiguity as well as the next man; he must have some reason for maintaining this level of obscurity. The passage, in both *Najāt* and *Išārāt*, seems a prime candidate for labelling as accommodation to his Peripatetic readership.

There is further evidence of accommodation in the corresponding passage of *Qiyās* at 95.11–96.4:

(10.6) What other people say is better, namely that if it's possible that some B is a C then the assumption of it is not an impossibiility. ... So if it's assumed that 'Some B is a C' is true (*mawjūd*), then in that case 'Some C is a B' si true, and hence is—as you know—false but not an impossibility. But you have already said that 'Necessarily no C is a B'; so how could the sentence 'Some C is a B' not be impossible?

Recall our earlier remarks about rhetorical questions in *Qiyās* (REF ABOVE).

From its position in Ibn Sīnā's overall presentation, this passage seems to be intended to justify an argument that converts questions about possibility into questions about absoluteness. It presents the idea that if ϕ entails ψ then 'Possibly ϕ' entails 'It is not impossible that ψ'. Since 'not an impossibility' (*ğayr muḥāl*) should mean the same as 'possible', this boils down to the rather plausible principle

(10.7) If ϕ entails ψ then 'possibly ϕ' entails 'possibly ϕ' (or as Ibn Sīnā might phrase this conclusion, '*in kāna* ϕ bil 'imkān, *fa* ψ *bil 'imkān*'.

We will meet this principle again. We can call it the *Possibility preserves entailment* principle. Ibn Sīnā has taken it from the Arabic Aristotle, cf. 34a25–32.

The principle looks like a sequent rule:

(10.8)
$$\frac{\phi \;\vdash\; \psi}{\Diamond\phi \;\vdash\; \Diamond\psi}$$

Ibn Sīnā did use sequent rules, chiefly in his propositional logic (REF [20]. But this particular one needs to be held at arm's length. Adding *bil 'imkān* to a sentence need not have the effect of adding a possibility operator whose scope is the whole sentence. It can instead have the effect of adding 'possibly' to the copula or the predicate. For example if ϕ is the sentence 'Some B is an A' and ψ is the sentence 'Some A is a B', then an application of the principle could be read as yielding

(10.9) 'Some B can be an A' entails 'Some A can be a B'.

Justifying $(i\text{-}pos)$ by this route looks too much like theft rather than honest toil (to borrow a phrase from Bertrand Russell). If this is the argument that Ibn Sīnā wants us to accept at *Qiyās* 95.11–96.4, then we have to mark it down as a bad case of accommodation to his Peripatetic audience.

However, in *Najāt* and *Išārāt* Ibn Sīnā doesn't invoke the principle that possibility preserves entailment—at least not as a justification of $(i\text{-}pos)$-conversion and hence of $(e\text{-}nec)$-conversion. Instead, as we saw, he calls on ecthetic *Darapti*. But we also saw ABOVE that he doesn't have the methodological tools needed for stating the new defined term correctly. At *Išārāt* 117.13 he says to the reader 'It's for you to prove ths by ecthesis; so take that 'some' to be D'. His comment here is inadequate for the same reason as in the cases we examined before. The statement at *Najāt* 48.5f is also inadequate: 'Otherwise it is possible that some A is a B, so let that be C'. (The question needs an answer, whether or not the 'possible' is part of the sentence here.)

In sum, Ibn Sīnā has set out to convince the reader that $(e\text{-}nec)$ converts. In *Najāt* and *Išārāt* he has tried to do this by first paraphrasing an argument of Aristotle that claims to reduce the question to the convertibility of $(i\text{-}mum)$, and then following this with a hint he takes from Aristotle, that this convertibility can be proved by an ecthetic argument. This argument is incompletely described and may in fact apply to $(i\text{-}abs)$ rather than $(i\text{-}mum)$. One could try to defend Ibn Sīnā as follows. He believes that the question of the convertibility of $(e\text{-}nec)$ hits bedrock with the ecthetic argument, and so he believes that we can in principle reconstruct exactly what conversion is proved, and of exactly what sentences, by working backwards from the ecthetic proof. Unfortunately he lacks the tools to make the ecthetic proof precise enough to carry through this proposal, so he has to leave us the raw materials for us to do the best we can for ourselves.

We simply don't know how close this is to Ibn Sīnā's own assessment of the position. But three things are clear: (1) that Ibn Sīnā believes that some form of $(e\text{-}nec)$-conversion holds, (2) that he has no cogent argument for any plausible precise statement of this conversion rule, and (3) that the materials which he offers the reader in support of the conversion advance only an infinitesimal distance, if at all, beyond what Ibn Sīnā found in the Arabic Aristotle and the Arabic Alexander of Aphrodisias.

The lack of any visible forward movement from Aristotle and Alexander is frustrating. Street [49] p. 144 claims to detect in Ibn Sīnā's argument at *Išārāt* 117.8–13 a feature that seems not to be in Aristotle or Alexander, namely that Ibn Sīnā makes the move of 'supposing a possible actual'. The snag is that, as far as I can see, this move doesn't occur in Ibn Sīnā either.

At the step where Street sees the move from possible to actual, all Ibn Sīnā says is that we should use ecthesis on the preceding (i) sentence (*wa-farḍa ḏālika*, *Išārāt* 117.9). There is no mention of actuality (*bil fi*^{c}*l*) in this passage.

Besides the above on conversion of necessary universal negatives, Ibn Sīnā launches into an attack on the view that $(e\text{-}abs)$ always converts. His argument uses the temporal reading $(e\text{-}t)$, and he shows with an example that $(e\text{-}t)(B, A)$ doesn't entail $(e\text{-}t)(A, B)$. The example is taken from the text of Aristotle: put B = horse and A = sleeps. Every sometime-horse is sometimes not sleeping, but it doesn't follow that every sometimes-sleeping thing is sometimes not a horse. Ibn Sīnā goes on to show (as in BELOW) that there are other readings of 'absolute' under which (e) sentences do convert, but his implication is that there is no presumption that an $(e\text{-}abs)$ sentence will convert, and if the reasoner wants to use some special kind of absolute (e)-sentence that does convert, then that's up to the reasoner.

So Ibn Sīnā has claimed to prove everything required to show that the conversion axioms hold both for t and for abs. But we have noted that he does it without assuming that contradictory negation takes abs to nec.

10.4 Other kinds of sentence

In all four works, Ibn Sīnā devotes some time to checking which of the axioms hold for forms of absoluteness sentence that are distinct from (t). In *Išārāt* he adopts a name for these other forms of absoluteness: he calls them *ḥiyal*, the plural of *ḥīla*. This word most commonly means trickery, with overtones of dishonesty and deception. But these overtones are completely irrelevant here, and we should look instead at other places where Ibn Sīnā uses the word quite neutrally for 'devices' of various kinds. In *Burhān* 205.19 and 206.7 he speaks of the 'science of *ḥiyal*', and this is probably the same as the science of 'moving *ḥiyal*' that he mentions in *Aqsām al-*^{c}*ulūm* 112.7 as using information derived from mathematics. He presumably means the science of mechanical devices with moving parts. In *Qānūn* REF he refers to various implements with medical applications, for example a device for keeping patients from getting wet, or an instrument for extracting ???. So I translate *ḥiyal* in this logical sense as 'devices'. Possibly Ibn Sīnā thinks of the intended purpose of these devices as the identification of some forms of sentence that have useful logical properties, for example conversions.

There are some other kinds of sentence that Ibn Sīnā counts as absolute, and we should check whether the first figure moods are valid, or self-evidently valid, for these too. One example is the two-dimensional (z) sentences that specify a particular time. Provided the time specified is the same in both premises (a point that Ibn Sīnā himself indicates, REF), all these cases can be proved by paraphrase into assertorics too, and the paraphrase is simpler than it was with (t).

Another case to consider is the sentences with wide time scope, thus:

(10.10) It's true at some time that some C at that time is a B at that time.
It's true at some time that no B at that time is an A at that time.
Therefore it's true at some time that some C at that time is not an A at that time.

This is clearly invalid.

10.5 Conclusions so far

Conclusion 10.1 The valid moods and the internal proofs are read off from the (dt) case.

2. The first-figure moods are checked for all cases, not just the (dt) cases (which are rather sidelined at this point).

3. There is no evidence of the first-figure moods being given any argument specifically for one category of modality.

4. The evidence taken together implies that Ibn Sīnā is describing the (dt) situation and carrying it over to some abstract form of nec/abs alethic modes, which are constrained by e.g. not being wide time-scope and by having their modalities attached to their predicates.

Bring in the Ahmed here. Right that there is a mapping from nec/abs to (dt), but wrong that the mapping changes what is valid. Wrong also that this is an attempt to accommodate Aristotle's modalities.

Look forwards to handling of possibility. Note that by the genetic hypothesis the laws each hold for at most two modalities at a time.

Conclusion 10.2 Ibn Sīnā arranges his material so as to prove the correspondence $nec/abs \mapsto d/t$ without assuming that this correspondence is a paraphrase, and in fact without assuming that abs is the De Morgan dual of nec.

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