Ibn Sīnā’s alethic modal logic

Wilfrid Hodges
Herons Brook, Sticklepath, Okehampton Devon EX20 2PY, England
wilfrid.hodges@btinternet.com

DRAFT March 2015

THIS IS AN INCOMPLETE DRAFT OF A PAPER, BUT IT IS COMPLETE UP TO THE POINT WHERE WE FIRST START DRAWING CONCLUSIONS FROM THE EVIDENCE. I HAVE TO PUT IT ON ONE SIDE NOW IN ORDER TO FINISH TWO COMMISSIONED PAPERS, BUT I HOPE TO HAVE IT COMPLETE IN A COUPLE OF MONTHS. 22 March 2015.

1 Introduction

This paper is a report on Ibn Sīnā’s logic of necessary, possible, contingent and absolute, which we will refer to as his alethic modal logic. We describe what he did and why he did it. Two new features of our account are, first, a description of the logical properties of the two-dimensional (temporal) logic which he sets out in Qiyās i.3 and Mašriqiyyūn, and second, a review of his account of logic as a science. The two-dimensional logic was a major innovation in its own right, and it had the potential to revolutionise logic if Ibn Sīnā’s successors had recognised it for what it was. The account of logic as a science and the logic itself have generally been treated in isolation from each other, but in fact neither makes full sense without being closely tied to the other.

We separate Ibn Sīnā’s treatment of alethic modal logic into three parts: first the listing of valid moods, second the justification of these moods where the justification is internal to the science of logic, and third the justification where it relies on drawing principles from First Philosophy. The first and second parts are a highly accurate report of the facts of two-dimensional logic. The third part is much less convincing as logic. It is best accounted for as an attempt to derive axioms for an alethic modal logic which is abstract in the sense that it applies to modalities of any category (including
both temporal and ontological); and it is illuminated by Ibn Sinā’s own remarks about how the science of logic can proceed in such cases.

In the literature on Ibn Sinā’s alethic modal logic, much has been said about Ibn Sinā’s attitude to Aristotle’s modal logic, and about the relationship of Ibn Sinā’s logic to that of his successors from the late twelfth century onwards. Both these enquiries should be based on an account of what Ibn Sinā’s alethic modal logic consists of in its own right. His references to Aristotle fall broadly into two groups. Firstly there are attempts to mine Aristotle and the Peripatetic literature for intuitions and heuristics to support finding axioms. Secondly there are a number of passages which are criticisms of Aristotle or other Peripatetic writers, but these criticisms are not essential to the alethic logic itself. One could delete them without altering the logical content, as Ibn Sinā himself does in Ḣarāṭ.

There is one overriding difference between Ibn Sinā’s work and that of his successors from Fakr-al-Dīn al-Rāzī (a century and a half after Ibn Sinā) onwards. Rāzī took the view that in practice it is impossible to do properly motivated work in modal logic if we don’t know precisely which modal category we are dealing with at any one time. He developed a new paradigm for modal logic which allows most of Ibn Sinā’s work to be included, but only with reference to two modalities, which are always clearly distinguished: temporal and ontological. Myself I am strongly in sympathy with Rāzī here. Maybe Ibn Sinā’s abstract modal logic was always a will o’ the wisp, though as often with Ibn Sinā it raises original and deep questions. The flood of original research that followed Rāzī’s proposals is in sharp contrast with the lack of progress in the period between Ibn Sinā and Rāzī.

The broad outlines of this paper were obtained in January 2014 and circulated to a number of people; I thank Zia Movahed and Saloua Chatti in particular for their responses. But it has taken all the time since then to fill in details, and that process continues. Part of the problem is that there is not yet a published body of sifted data about Ibn Sinā’s logic that one can refer back to. As it is, the paper makes several demands on the reader’s acquiescence. But it became tiresome not to be able to give people an account of the matter. The paper is still only a draft and will sit on my website.
2 Remarks on modalities

We will need some notions that I had thought were common currency. But reading around and talking to some people has convinced me that they are not, so it would be better to be explicit about them. (My thanks to Yde Venema for a useful discussion, but as always, don’t blame him if anything is incoherent.)

Georg Henrik von Wright on page 2 of his classic work [48] on modal logic presents a table:

<table>
<thead>
<tr>
<th>alethic</th>
<th>epistemic</th>
<th>deontic</th>
<th>existential</th>
</tr>
</thead>
<tbody>
<tr>
<td>necessary</td>
<td>verified</td>
<td>obligatory</td>
<td>universal</td>
</tr>
<tr>
<td>possible</td>
<td>permitted</td>
<td>existing</td>
<td></td>
</tr>
<tr>
<td>contingent</td>
<td>undecided</td>
<td>indifferent</td>
<td></td>
</tr>
<tr>
<td>impossible</td>
<td>falsified</td>
<td>forbidden</td>
<td>empty</td>
</tr>
</tbody>
</table>

The individual items under each of the heads are, in his terminology, modes or modalities; this seems to be standard usage. He says that the columns represent four modal categories. This is his own usage, and it has not fared so well. Perhaps no modal logicians use the term ‘category’ this way today.

The term is found among linguists. For example

Modal logic has to do with the notions of possibility and necessity, and its categories epistemic and deontic concern themselves with these notions in two different domains. (Bybee and Fleischman [6] p. 4)

But notice the difference from von Wright: the categories of epistemic and deontic are now kinds of necessity or possibility. The alethic modes don’t form a category; rather they are words that (as some other linguists tell us) can be used to express items in modal categories. On this view the alethic modes do have a meaning of a sort, because for example ‘possibly’ has to be incompatible with ‘necessarily not’, and both ‘necessary’ and ‘contingent’ have to imply ‘possible’. So at least they have enough meaning to carry some logical relationships between them.

In what follows I will go with von Wright to the extent of using ‘category’ for a family of modal notions that provide a necessity notion, a matching possibility notion etc. Von Wright’s epistemic and deontic are two standard examples. Ibn Sīnā would surely add temporal and ontolog-
or something similar. Von Wright’s alethic modes don’t form a category in this sense. They play more the role of abstract place-holders, but they do have meanings of a kind.

Often in this paper we will find Ibn Sinā treating ‘necessary’ and ‘permanent’ as in some way equivalent notions. So we should set on record at once that he was perfectly capable of distinguishing between them. We give two quotations to show this; both of them will be useful to us later. The first is from Ḥ iṣārat:

An example of that which endures and is non-necessary is something like the affirmation or negation, applicable to an individual [of a quality] accompanying him in a non-necessary manner as long as he exists: as you may correctly say that some human beings have white complexions as long as their essence [[is satisfied]], even though that is not necessary.

He who believes that non-necessary predication is [[not]] found in universal propositions has committed an error. For it is possible that universal propositions have that which is applicable, affirmatively or negatively, to every individual subsumed under them … at a determined time as that of the rising and the setting of the [[planets]] and that of the eclipse of the sun and the moon; or at an undetermined time as that which belongs to every born human being such as respiration … (Ḥ iṣārat [29] 89f; trans. Inati [17] pp. 93f)

The double brackets are my emendations of Inati. Her missing ‘not’ may mean she is translating from a different Arabic text, but the sense surely requires ‘not’ here.

And the second quotation is from Qiyās:

… being permanent is not the same as being necessary. [A thing is] necessarily what it is by its nature, and this requires that if it is false of an individual then it is permanently false of that individual; while [a thing is] permanent either by its nature or because it just happens to be. But it is not for the logician as logician to know the truth about this. (Qiyās 48.14–17)
So for Ibn Sinā there is a difference between being permanent and being necessary, but this is not a difference for the ‘logician as logician’. What can he mean?

Below we will find an answer in Ibn Sinā’s own account of what logicians do as logicians. He is very articulate about this. But a prerequisite for understanding what he says is that we know some of the art of logic itself; so we begin with a section on assertoric logic, what Ibn Sinā learned from Aristotle and how he adjusted it for his own use.

3 Assertoric logic

3.1 What Ibn Sinā inherited

Ibn Sinā knew Aristotle from the Arabic translations of his works. Most of the classical Arabic translations of Aristotle were due to a team of Syriac-speaking translators associated with Ḥunayn b. Ishāq and his son Ishāq b. Ḥunayn in the 9th century. One translation of the Prior Analytics has come down to us from this period; we have it in two manuscripts, one in Paris and the other in Istanbul. Lameer ([35] pp. 3f) assembles evidence that points to this translation being the work of one Theodorus, a member of the Syriac team somewhere around the mid 9th century. Very likely the translation was made first into Syriac and then into Arabic. The default assumption must be that Ibn Sinā worked from a version of Theodorus’ translation; though I know of no research to confirm this, and it might be difficult given Ibn Sinā’s preference for saying everything in his own words. I will cite the translation as ‘the Arabic Aristotle’, using the edition of Jabre [33] but giving references to the Greek original.

According to the Arabic Aristotle (24a14) there are three kinds of sentence, namely universal (kullı), particular (juz’ı) and unquantified (muhmal). The name juz’ı covers both existentially quantified sentences and singular sentences about a named individual. Ibn Sinā will distinguish these and refer to the first kind as existential (again juz’ı) and the second as singular (šakṣī or makṣūs). Ibn Sinā maintains that within formal logic the singular sentences behave as if they were universal and the unquantified sentences behave as if they were existential (Qiyās 109.11–13), so we can save paper by concentrating below on the universal and the existential. Ibn Sinā will refer to the properties of being universal or existential as the ‘quantity’ (kamm, kammiyya) of a sentence; in the Arabic Aristotle this expression is found only in the chapter headings, which may have been added later.
The Arabic Aristotle (24a20) also distinguishes between sentences in which something is said of something and sentences in which something is ‘not said’ (i.e. is denied) of something. Ibn Sīnā will read this as a distinction between affirmative (mūjib) and negative (sālib) sentences. Being affirmative or negative is the ‘quality’ (kayfa, kayfiyya) of the sentence; kayfiyya is found already in the text of the Arabic Aristotle.

So there are four kinds of sentence:

(a) : ‘Every B is an A’.
(e) : ‘No B is an A’.
(i) : ‘Some B is an A’.
(o) : ‘Not every B is an A’.

At least this is how Ibn Sīnā read the Arabic Aristotle. Aristotle himself rarely spelt out the sentences, and when he did he usually used a technical vocabulary that put the A before the B. This accounts for the backwards ordering of the letters in Ibn Sīnā and other Arabic logicians. The labels (a) for ‘universal affirmative’, (e) for ‘universal negative’, (i) for ‘existential affirmative’ and (o) for ‘existential negative’ are a later Latin invention, but they give us a useful shorthand.

Every sentence has a ‘contradictory negation’ (naqīd, 34b29) that denies what the sentence affirms, or vice versa. The contradictory negation of ‘Every B is an A’ is ‘Not every B is an A’, and conversely; the contradictory negation of ‘No B is an A’ is ‘Some B is an A’, and conversely.

We will call the sentence forms above the assertoric sentence forms, and their logic will be assertoric logic. The Arabic Aristotle has no distinguishing name for them; Ibn Sīnā sometimes refers to them as the ‘standard’ (mašūr) forms. In the Arabic Aristotle it is not clear whether the schemas above are themselves objects of interest, or whether they are regarded as shorthand for longer sentences that have vernacular text in place of the letters A, B. We will need to make this distinction; I will refer to the schemas as formal sentences as opposed to the concrete sentences that are got by putting text in place of the letters. The Arabic Aristotle describes B as the ‘subject’ (mawdā‘) of the sentences (e.g. at 24b29) and A as their ‘predicate’ (mahmūl, e.g. at 24a27); these names may refer either to the letter or to the text understood to replace the letter, or to the meaning of the text. The subject and the predicate are referred to as ‘terms’ (ḥudūd, singular ḥadd, 24b17).

Although the Arabic Aristotle seems to be consistent in applying the expression mawdā‘ to a term of sentences, Peripatetic logicians developed a habit of using it to refer to the individuals that fall under the subject term. For example the sentence ‘Every horse sleeps’ has the subject term ‘horse’, but
one says also that horses are subjects of it. To avoid this confusion I will speak of the horses as the subject individuals, as opposed to the subject term. Ibn Sinā has his own ways of resolving this ambiguity.

The Arabic Aristotle defines a ‘syllogism’ (qiyāṣ) as a piece of discourse in which when two or more sentences are proposed, something else follows from their being true, of necessity and intrinsically (24b29f). The proposed sentences are called ‘premise’ (muqaddama, 24a23). The something else that follows is called ‘conclusion’ (natiJa, 30a5) or occasionally ‘goal’ (maṭlūb, 42a40). In practice he limits himself to syllogisms with just two premises, at least in the part of the Prior Analytics that concerns us here.

In sections i.4–6 (25b27–29a17) the Arabic Aristotle runs through a list of all the syllogisms; the syllogisms are expressed using formal assertoric sentences or paraphrases of them, and they are classified by ‘figure’ (šakl 26a14). There are three figures. For a conclusion with subject C and predicate A, the first figure has a premise with subject C and predicate B, and a premise with subject B and predicate A; the second figure has premises with B the predicate in both; the third figure has premises with B the subject in both. It will be helpful to speak of a formal syllogism, expressed with formal assertoric sentences, as a mood, and a pair of formal sentences as a premise-pair; Ibn Sinā will use ḍarb for ‘mood’ and qarīnā for ‘premise-pair’. When a premise-pair fails to produce a conclusion in a given figure, the Arabic Aristotle says that it is ‘not a syllogism’; it will be helpful if we adopt a term used by Ibn Sinā and say that the premise-pair is productive (muntij) if it does yield a conclusion in the given figure.

In this context the Arabic Aristotle describes the term C as the ‘minor extreme’, the term B as the ‘middle’ and the term A as the ‘major extreme’ (25b35, 26a19). (‘Extreme’ is ra’s, literally ‘head’; ‘minor’ is šagīr; ‘middle’ is awsat and ‘major’ is kābīr. Variants later in the Arabic text are taraf for extreme, aṣgār for minor and aṭkābār for major; these are the expressions that Ibn Sinā will normally use.) The premise containing C is the ‘minor premise’ (ṣuğrā) and the premise containing A is the ‘major premise’ (kubrā). Since the Arabic Aristotle rarely sets out concrete examples of syllogisms, there is room for reading either the minor premise or the major premise as the ‘first’ premise; in practice the Arabic logicians took the minor premise as first, the opposite way to the Latins.
The Arabic Aristotle tells us ([33] 24b24–28) that

A perfect (kāmil) syllogism is a syllogism which needs, for proving what must be the case given its premises, the use of something other than those premises. And a syllogism that is not perfect is one which needs—for proving what must be the case given its premises—the use of one thing, or a combination of things, which must be the case given the premises that compose the syllogism, but which has not been used in the premises.

Moreover all the first figure syllogisms are perfect ([33] 26b28), but none are perfect in the second ([33] 27a1) or third ([33] 28a5) figure. The syllogisms in second and third figure are ‘made perfect’ (yükmalu) by having certain things ‘attached’ (ulḥiqa) to them.

There are three kinds of attachment, as follows. One is ‘conversion’ (‘aks, 30a5), which consists of replacing a premise of the form ‘Every B is an A’ or ‘Some B is an A’ by the sentence ‘Some A is a B’, or a premise of the form ‘No B is an A’ by the sentence ‘No A is a B. This is done in such a way that the syllogism ‘reduces’ (raja‘a) to a syllogism in first figure. The conclusion of this second syllogism either is the conclusion needed from the original syllogism, or it entails that needed conclusion through a further conversion.

A second kind of attachment is ‘ecthesis’, where a new term is ‘posited’ (wudi‘a) or ‘stipulated’ (yufradu). In Aristotle this seems to cover more than one kind of argument. We will say more on it later.

A third kind of attachment is contraposition (literally ‘denying the statement’, ra‘f al-kalām, or ‘absurdity’, kalf in the spelling that Ibn Sinā favoured). When we use contraposition, we show that one of the premises of the syllogism, together with the contradictory negation of the conclusion, entails the contradictory negation of the other premise. This method can be used when the rearranged syllogism has already been shown correct, for example if it is in first figure.

So for every assertoric syllogistic mood, the Arabic Aristotle either states that it is self-evidently correct, or he proves the correctness by some kind of reduction to a mood whose correctness is self-evident. For premise-pairs that he regards as not a syllogism (i.e. not productive), he uses a method which he calls ‘terms’ (hudūd) to prove that no conclusion follows from them in their figure. The method is subtler than first appears, and there is evidence that Ibn Sinā struggled to understand it. But briefly, suppose the figure requires a conclusion with subject C and predicate A. Then the method consists in setting out two examples of concrete premise-pairs of
the given form, both consisting of true sentences, where in the first case the sentence ‘Every C is a B’ is true, and in the second case the sentence ‘No C is a B’ is true. The examples are specified by giving concrete terms for them—hence the name ‘terms’. As the Arabic Aristotle says at [33] 26b19f, ‘It is clear that when there are terms fitting this description, then there is not a syllogism’.

At [33] 27b38 the Arabic Aristotle offers the same set of terms to eliminate several different formal premise-pairs at the same time. Aristotle may have intended nothing more than saving a little effort, but we will see that this move had a significant effect on Ibn Sīnā.

These are by no means the only surviving writings in which Ibn Sīnā discusses logic. See the index of citations at the end of this paper for some other examples.

3.2 Ibn Sīnā’s logical writings

Among the works of Ibn Sīnā’s maturity that have come down to us, six are particularly relevant to formal logic. I summarise briefly what they are, with references to the Inventory of Avicenna’s Works in Gutas [11].

*Muktaṣar* Gutas [11] p. 433 names this the *Middle Summary on Logic*. We have no precise dating, but a date around the early 1010s is plausible. The work has not been printed, and I thank Alexander Kalbarczyk for giving me access to the Nuruosmaniye manuscript.

*Najāt* This is an encyclopedia, called *The Salvation* in Gutas [11] p. 115. It was published soon after *Qiyās* below, but we know that its logic section is taken from an earlier work, the *Shorter Summary on Logic* from around 1014, with a few probable editorial changes. There is a translation of the logic section [3] by Asad Q. Ahmed.

Mašriqiyyūn  Gutas [11] p. 119 calls this work The Easterners, but with some misgivings about the title. The work was a survey of various areas of philosophy; from the logic section fewer than a hundred pages survive, roughly corresponding to the first of the nine books of Qiyās. But Ibn Sīnā advertises the work as more direct and less biased in favour of the Peripatetics than Qiyās, and this is borne out by the contents. Its main contributions are a full and integrated discussion of definitions, and the best-organised presentation of what below we will call Ibn Sīnā’s two-dimensional logic. Gutas [11] p. 132 dates it to 1027–8.

Dānešnāmeh  This work, the Philosophy for ʿAlāʾ-ad-Dawla, is a relatively elementary summary of philosophy written in Persian at some time between 1023 and Ibn Sīnā’s death in 1037 (Gutas [11] p. 118). The first section is on logic, treated from a practical point of view. There is a French translation of the whole work, [1].

Išārat  This late work is called Pointers and Reminders and dated to 1030–4 (Gutas [11] p. 155). It covers a range of philosophical topics, beginning with logic. In logic the differences from Qiyās and Mašriqiyyūn are very visible, but I believe they are mainly in presentation rather than content. One of them is extreme brevity, which made the work prime material for later commentators. There is a translation of the logic section [17] by Shams Inati.

3.3 What Ibn Sīnā added to Aristotle

One of Ibn Sīnā’s most important additions to Aristotle is most fully treated not in the works above, but in his ʿIbāra, the volume of the Šifā’ that comes immediately before Qiyās, corresponding to Aristotle’s De Interpretatione. (See ʿIbāra [22] 79.11–80.12 and the discussion in [14].) Here Ibn Sīnā explains the meanings of the assertoric sentence forms, in enough detail to justify the following translations into first-order sentences. In the diagram (8) the righthand column gives the first-order translations of the (a), (e), (i) and (o) sentences, and the lefthand column gives convenient abbreviations
of these formulas.

\[
\begin{align*}
(a)(B,A) & : (\forall x(Bx \rightarrow Ax) \land \exists x Bx) \\
(e)(B,A) & : \forall x(Bx \rightarrow \neg Ax) \\
(i)(B,A) & : \exists x(Bx \land Ax) \\
(o)(B,A) & : (\exists x(Bx \land \neg Ax) \lor \forall x \neg Bx)
\end{align*}
\]

(8)

Given these meanings, one can check that the fourteen moods listed by Aristotle are exactly those where the premise-pairs are productive and the conclusion is the strongest conclusion (with the appropriate terms for the given figure) that can be deduced from the premises. We will refer to the clauses $\exists x Bx$ in the first sentence and $\forall x \neg Bx$ in the fourth as the *augments*, respectively the *existential augment* in the first and the *universal augment* in the fourth.

Maybe this is the best place to note that neither Aristotle nor Ibn Sinā operates with the modern notion of *validity* in dealing with syllogisms. For us an inference is *valid* if and only if its conclusion is a logical consequence of its premises. For both Aristotle and Ibn Sinā the operative notions are first that the premises are productive in a figure (i.e. there is a valid inference from them to a conclusion in that figure), second that a sentence follows validly in the given figure, and third that a sentence is the strongest that can be drawn in that figure. When Ibn Sinā writes out a syllogistic mood as one that he accepts, he is normally taking it to be *conclusion-optimal*, i.e. it is valid and its conclusion is the strongest that can be validly drawn in the relevant figure. (There is no requirement that the premises are the weakest that will allow that conclusion.) I will use the notion of validity because it is more versatile than these older notions; but one should be aware that this often involves some paraphrasing of the originals.

Ibn Sinā reports the contents of *Prior Analytics* i.4–6 in several places, most straightforwardly in *Muktasār* 49b9–53a6, *Najāt* 57.1–64.3, *Qiyāṣ* ii.4, 108.12–119.8 and *Dānešnāmeh* 67.5–80.2. Besides these four accounts, we also have a report in *Iṣārat* i.7, 142.10–153.9 ([17] 135–143) which is sketchier and mixed with modal material. In *Qiyāṣ* vi.4, 296.1–304.4 Ibn Sinā repeats the entire scheme in detail, but with a version of propositional logic in place of Aristotle’s assertoric sentences.

In all these accounts Ibn Sinā reports the same fourteen moods as Aristotle, in the same order (apart from some slight variation in *Iṣārat*). Moreover the justifications that he offers are almost exactly the same as Aristotle’s. (This is fully documented in Appendix A of [15].) In first figure he tells us, following Aristotle, that all the moods are perfect. In second and
third figures he repeats Aristotle’s justifications by conversion, ecthesis and contraposition, with only a very few variations, mostly insignificant.

In fact the only significant variation from Aristotle is that Ibn Sinā introduces a proof of second-figure Baroco by ecthesis. This proof appears in all six of his reports. As I read him, he intends a proof along the following lines, where the deductions are direct from top to bottom and $Dx$ is defined as $(Cx \land \neg Bx)$:

```
(9)  
  (o)(C, B)  (a)(A, B)  
    (i)(C, D)  (e)(B, D)  (Celarent)  
      (e)(A, D)  (conversion)  
        (e)(D, A)  (Ferio)  
          (o)(C, A)
```

Strictly the ecthesis is a non sequitur, because by (8), $(i)(C, D)$ implies that at least one thing is a $C$ and $(o)(C, B)$ doesn’t imply this. But the procedure can still be justified, as for example in [15].

Though Ibn Sinā never discusses the point, the introduction of this proof for Baroco has the effect that he can give justifications of all the second- and third-figure moods without ever invoking contraposition. Not that he objects to contraposition; he mentions it in all the cases where Aristotle did. But contraposition uses some propositional logic—as is particularly clear in his analysis of it in Qiyās viii.3—and this would certainly not be the only place where Ibn Sinā aims to set up the foundations of a logic without invoking other logics.

In fact there already is an ecthetic justification for third-figure Bocardo in the Arabic Aristotle (28b20f), with a remark that it makes contraposition
unnecessary. I believe Ibn Sīnā reads this argument as follows:

He takes $Dx$ to mean $(Bx \land \neg Ax)$, following the guidance of the Arabic Aristotle that it is ‘the some of $B$ taken from what is not in $A$’. His argument for Baroco is a straightforward rearrangement of this argument.

Ibn Sīnā makes some further changes in the general layout. He includes two items that were not in Aristotle, namely conditions of productivity (ṣarāʾīṭ al-ʿintāj) and rules of following. The conditions of productivity are necessary and sufficient conditions for a pair of sentences in a figure to be productive. As Ibn Sīnā presents them, there are a set of conditions that apply to all three figures, together with a further set of conditions that apply just to one figure. For example a condition applying to all three figures is that at most one of the premises is negative; for the second figure we have the stronger condition that exactly one of the premises is negative. Ibn Sīnā states the conditions precisely as they appear in Philoponus [41] 70.1–21, except that Ibn Sīnā usually includes a further condition applying to all three figures; this further condition is correct but redundant. From remarks in Philoponus it appears that the conditions were first assembled from Aristotle’s proofs of non-productivity, by noting where Aristotle handled a group of formal premise-pairs together (as we remarked at the end of Subsection 3.1 above).

The rules of following tell us, given a productive premise-pair in a particular figure, what is the strongest conclusion in that figure that can be drawn from the premises. The Peripatetic logicians had a tendency to assume that each logical property of the conclusion is inherited from one of the premises, and so the conclusion can be described by saying which of the premises it ‘follows’ (in Arabic yatbaʿu) for each of its logical properties. Ibn Sīnā also has a further piece of terminology, which as far as I know...
he introduced himself. In the case of modalities he says that the premise whose modality is inherited by the conclusion is the premise with the *cibra*. Because of the obvious analogy with genetics I translate *cibra* as *dominance*, and I refer to the Peripatetic assumption that the conclusion inherits its modality (and other features) from one or other premise as the *genetic hypothesis*. The link to genetics is not just a modern fancy; Ibn Sīnā uses *cibra* in this genetic sense in his biological essay *Hayawān* 159.7.

Ibn Sīnā states the rule of following for assertoric logic in several places, and nearly always in a form that is wrong for *Darapti* and *Felapton*, which don’t inherit their quantity from either parent. It’s hardly likely that he was unaware of this exception, and in fact he gets it right in *Uyūn al-hikma* [30] 50.2f, where he explains that there is an *cibra* for quality (but by implication not also for quantity). Probably the error is the result of a common tendency to be careless about minor counterexamples.

4 The science of logic

(This is a footnote that I hope to be able to remove sooner rather than later. In this section I am moving outside my comfort zone; my expertise is in mathematical logic and classical languages, not in epistemology or philosophy of science. So I would welcome any advice and corrections, but subject to two reasonable requirements. First, attempts to formulate a description of the science of logic are unlikely to be helpful if they are not informed by knowledge of the facts of logic, the relevant logic here being Ibn Sīnā’s logic. Second, attempts to establish Ibn Sīnā’s views on any topic are unlikely to be successful if they are based on an unrepresentative sample of his available writings on the topic. Of the published modern discussions of the issues raised in this section, I know of none that address the first requirement at all, and none that are fully satisfactory on the second. So we have here a real opportunity to increase our understanding.)

4.1 The structure of a science

Ibn Sīnā regards logic as a ‘theoretical art’ (*ṣīnāʿa nazariyya*, Najāt 8.8), and also as a ‘science’ (*ʿilm*, Qiyās 10.11f). Every science or theoretical art has ‘principles’ (*mabādiʿ*, singular *mabdaʾ*) and ‘theorems’ (*masāʾil*, singular *masʿala*, literally ‘question’). Both principles and theorems are propositions which the science guarantees to be true. The difference between them is that the theorems are demonstrated in the science using premises that are already
principles or theorems of the science; the principles are either proved using premises from a ‘higher’ science, or they are not proved at all because they are self-evidently true. (Burhan 155.1–7). I ignore here what Ibn Sinâ describes as the ‘rare’ case of principles proved in a lower science, though I am not sure what he is referring to.

Ibn Sinâ calls logic both a science (‘ilm) and an art (šinā‘a). There is a difference between these two descriptions. To learn a science, we learn a class of true propositions and we learn how to demonstrate their truth. To learn an art, we learn a skill that consists in acting according to certain ‘rules’ (qawānîn, singular qānūn, Jadâl [25] 21.11). The chief principles and theorems of any theoretical science are universally quantified (e.g. Qiyās 4.4, Burhan 220.8 and passim), since these sciences deal with causes and not with particular instances. But Ibn Sinâ also describes the rules of an art as ‘universal’ (kull, Jadâl 21.11), and at least in the case of logic it seems that he makes no consistent distinction between principles and theorems on the one hand, and rules on the other. At least for the case of logic, it will be helpful to lump together the principles, the theorems and the rules as the truths of logic.

As the mention of ‘higher’ sciences indicates, Ibn Sinâ puts the sciences into a hierarchy of higher and lower; a principle of a science, if it is not self-evident, is deduced using truths of a higher science. He also has a relation of inclusion between sciences at the same level, as for example anatomy is included in medicine and planar geometry is included in geometry.

Ibn Sinâ is clear that there is one science that is above all other sciences, namely the part of metaphysics that he describes as First Philosophy (al-falsafa al-‘ulâ). This science investigates the properties of basic meanings such as [EXISTS] and [ONE], as opposed to the more specific topics of the other sciences (Burhan 166.1f). Ibn Sinâ refers to it as ‘providing the principles of the other sciences’ (Ilâhiyyât 5.7f), and it is presumably the part of metaphysics that Ibn Sinâ describes at Aqsâm al-‘ulûm 112.15–17 as ‘investigat[ing] the bases and principles of such sciences as physics, mathematics and the science of logic, and refut[ing] false opinions about these’. In various places Ibn Sinâ talks about the borderline between First Philosophy and logic, often to say that some things which are commonly regarded as logic should be referred back into First Philosophy (e.g. Maqâlât 5.1–9, Qiyâs 13.6f, Burhan 188.8f). Nor does Ibn Sinâ ever suggest that there is any science intermediate between First Philosophy and logic. So we infer that logic lies directly below First Philosophy in the hierarchy.

But there is a complication. Although logic takes principles from First
Philosophy, First Philosophy has to rely on logic for the validation of its arguments. Ibn Sīnā is never in any doubt that First Philosophy is a rational discipline: it has ‘demonstrations’ (Burhān 179.12f) and ‘syllogisms’ (Burhān 188.8) and ‘proofs’ (Burhān 87.13). See also the wealth of references in Bertolacci [4] Chapter Six on the demonstrative content of metaphysics; some of these references must certainly refer to First Philosophy. But logic is the art which establishes the principles by which we test whether a demonstration does derive its conclusion from its premises.

This is not yet a paradox, but it does need some sorting. If the justification of the arguments of logic rests on the arguments of First Philosophy, and the justification of the arguments of First Philosophy rests on those of logic, then we have a vicious circle, and neither branch of science can claim that its arguments are properly justified. Ibn Sīnā’s response is that since we clearly do have justified arguments in both these sciences, there have to be some arguments that need no justification from other arguments; in fact there must be truths that are self-evidently true and not in need of any justification. ‘So it is clear that not all knowledge comes through demonstration, and that some of what is known is known through itself and directly’ (Burhān 118.18). Also we can know that a demonstration is valid without having to count a statement of its validity as one of its premises (Qiyās 11.11–12.2).

So Ibn Sīnā says enough to guard against threats of circularity. But we will see that there still are problems about the interrelation between logic and First Philosophy; modal logic will generate some.

There is a second complication. Presumably some of the universal sentences expressible in the language of a science will be false, and so their contradictory negations, which are existential sentences, will be true. For example in logic some formal premise-pairs will be unproductive, which is to say that there are counterexamples to various putative conclusions. Now for universal statements Ibn Sīnā makes a distinction between those which express accidental truths (for example that all the planets are in the ascendent today, cf. (4) above) and essential truths. Every essential truth φ has a cause, and it’s the task of the relevant science to locate that cause and feed it into a demonstration (burhān) of φ—to show not just that φ is true but also why φ is true. But Ibn Sīnā’s picture of science has no corresponding distinction for existential sentences. The fact that such-and-such a premise-pair is unproductive is no more or less scientific than the fact that all the planets will be in the ascendent tomorrow. This has to be reckoned a blind spot in Peripatetic scientific theory.
It is certainly not a coincidence that the one place where Ibn Sīnā can be convicted of significant formal errors of logic is in his treatment of non-productive premise-pairs in propositional logic. It never occurred to him that Aristotle’s method of terms needs a scientific justification. If he had tried to work out a justification, he would have realised at once that the method needs adjustment when one applies it to the propositional logic of munfaṣīl sentences. But he died before he realised this.

One might try hiding the quantifiers inside the definition of ‘productive’. But then for example the statement ‘No premise-pairs of such-and-such a form are productive’ is a negative statement, and Ibn Sīnā’s account of negative truths in science is hardly better than his account of existential ones.

There is a further point before we leave Ibn Sīnā’s general theory of the sciences. Ibn Sīnā certainly doesn’t believe that we learn new facts only by deriving them from already known principles. Often our first awareness of new facts comes from hands-on experience. (Here we touch on what is often referred to as Ibn Sīnā’s ‘empiricism’—see Gutas [12], McGinnis [37].) Mostly we learn from hands-on experience; key words in his accounts of this are tajriba (‘experience’ or ‘experiment’), intīḥān (‘testing’) and istikrāj (‘working out’). He applies all of these words both to medical and to logical discovery. For example he tells us:

As for us, without seeking any help we worked out (istakrajnā) all the syllogisms that yield propositional compound goals, and this without needing to reduce them to predicative syllogisms; and we enumerated all the propositional compound propositions. We invite those of our contemporaries who claim to practise the art of logic to do likewise, and to compare all of their findings with all of ours. (Masā’il [31] 103.12–14)

In Ibn Sīnā’s view, we have an intellectual facility for converting our experience of many and varied instances into concepts for describing what happens in these instances, in such a way that if the concepts are added to the foundations of a science, they allow us to deduce theorems that account for the instances. For him, this is how science advances. (But he has no conception of using experience to correct mistakes in the foundations of a science; you can’t correct what is known to be true. His sciences are pre-Galilean.)
4.2 The truths of logic

We can apply Ibn Sīnā’s notions of science to the truths of logic. On his account, these truths will fall into three classes: (1) those that are self-evident (bayyin bi-nafsih) and need no proof, (2) those that are proved wholly within the science of logic (we will say that these are internally justified), and (3) those whose demonstrations rely on one or more premises from First Philosophy. There may also be (4) truths of First Philosophy that logic takes over and uses.

We ought to be able to look at Ibn Sīnā’s logical writings and make some plausible guesses about what exactly he takes the truths of logic to be, and which of the classes (1)–(3) he puts them in. In fact I recommend this as a very healthy exercise. Not that we need to rely just on plausible guesses. Ibn Sīnā himself takes us some of the way. For example in Qiyās i.2 he discusses how logic helps the other sciences by providing rules that ‘measure’ whether inferences are sound or not. In his first example he says

\[(12)\quad [\text{Logic}] \text{ helps by being a measure which tells us that this premise-pair is productive. (Qiyās 11.17)}\]

The specific premise-pair that he mentions is a particular case; presumably logic provides a general rule which says that such-and-such premise-pairs are productive, and Ibn Sīnā’s example fits the conditions. So the rule is a condition of productivity. Ibn Sīnā’s next example illustrates how logic can confirm that a certain conclusion follows; here logic is providing a rule of following.

In both these examples the rules are being used affirmatively, to show that a given premise-pair is productive and that a given sentence follows from the pair. Generally Ibn Sīnā justifies the affirmative side of his conditions of productivity and rules of following by running through all the relevant moods and checking each mood. At Qiyās 108.10, after stating part of these rules for assertoric logic, he says ‘You will learn these things later as we consider the separate cases’. So the justification of the affirmative content of these rules rests on establishing, for each of the valid moods, that it is in fact valid.

What is the form of the statement that assertoric Barbara is valid? Using the definition of Barbara we can write it out:

\[(13)\quad \text{For all } C, B \text{ and } A, \text{ if it is posited that every } C \text{ is a } B \text{ and that every } B \text{ is an } A, \text{ then it follows that every } C \text{ is an } A.\]

So we have a universal truth, which quantifies over $C, B$ and $A$. What exactly is being quantified over? The fact that there are three variables here
is no worry for Ibn Sīnā; he regularly follows the advice of Alexander of Aphrodisias, that a triple of universal quantifiers can be read as a single universal quantifier over triples. But still the truth needs a subject term; it needs a value for $X$ in the paraphrased form

\[(a)\] Given any triple of $X$s, if the $(a)$ sentence with subject the first element of the triple and predicate the second, and the $(a)$ sentence with subject the second element and predicate the third, are both posited, then there follows the $(a)$ sentence with subject the first element and predicate the third.

(Cf. Qiyās 184.2f for this use of ‘first element’, ‘second element’, ‘third element’.)

We know Ibn Sīnā’s answer to this question, because he tells us in several places (Madkali 15.4–7, Ilāhiyyāt 10.17–11.2, Mašriqiyyān 10.15 among them). The subject term $X$ is ‘meaning’ (ma‘nā) or ‘whatness’ (māhiyya, the ‘quiddity’ or definitional core of a meaning); sometimes he adds ‘well-defined’ (ma‘qūl, literally ‘intellected’). In other words, the subject individuals of logic are well-defined meanings. (This is one place where we need to be clear about the difference between the subject term and the subject individuals.)

We can check that Ibn Sīnā’s description works for the affirmative side of the conditions of productivity and the rules of following:

\[(a)\] Given any triple of well-defined meanings, if the sentence with subject the first element and predicate the second, and the sentence with subject the second and predicate the third, satisfy the following conditions [namely those for first figure], then the premise-pair consisting of the first sentence and the second is productive.

\[(a)\] Given any triple of well-defined meanings [etc. as above], the sentence which has subject the first element and predicate the third, and has such-and-such a quantity and such-and-such a quality, is a consequence of the aforementioned sentences.

Strictly these sentences should be tightened up to restrict the meanings to ones of the appropriate type for assertoric logic; for example they should be of noun or verb type, not proposition or particle type. This kind of restriction on the subject term is a very good illustration of what Ibn Sīnā says
at Najât 135.12–136.3 about how the subject terms of the truths of a science adapt the subject term of the science as a whole.

In the next section we will dig deeper into Ibn Sīnā’s description of the subject term of logic. But already we have enough to fit assertoric logic into Ibn Sīnā’s scheme of a science. We use the numbering from the beginning of this subsection.

(1) Some truths of logic are self-evident and need no further argument to justify them. Plausible candidates are the truths stating the first-figure moods. But caution: Aristotle said, of concrete syllogistic arguments in first figure, that it is self-evident that the conclusion follows from the premises. Does it follow that the sentence stating that all such syllogisms are valid is also self-evident? It’s at a different level of generality. If we check Ibn Sīnā text on the point, we find—as often—that Ibn Sīnā has been here before us. In Qiṣṣās 71.1 he defines the perfect premise-pairs as

(17) those that make clear through their forms the necessity of conceding the conclusion [that follows] from them (hiya allatī tuẓhiru li-ṣūratīhā luzūma al-natījati ‘anhā).

So for Ibn Sīnā the self-evidence is a property of the form rather than of the individual concrete syllogism. What is self-evident is that a certain form has a certain logical property, and this is exactly what the relevant truth of logic expresses.

Other candidates are theorems stating that a sentence of a certain form converts in a certain way, or that a sentence can be expanded in a certain way for ecthesis. Aristotle presents these results without commenting on how they are known, so presumably he regards them as obvious. As far as we can tell, Ibn Sīnā follows suit; though bear in mind that here we are talking only about assertoric logic.

It may help to put meat on the bones if we spend a moment on the rules for ecthesis (iftirād). In (10) we took the ecthesis rule for the proof of Bocardo to say the following:

(18) If A, B and D are meanings, and D is the meaning ‘B and not A’, then from the sentence (o)(B, A) we can conclude both (a)(D, B) and (e)(D, A).

My Persian is creaky, but this looks to me very close to the statement that Ibn Sīnā himself gives at Dānešnāmeh 78.4f. Besides transposing A and C, the main difference is that Ibn Sīnā states (e)(D, A) but leaves (a)(D, B) to
be drawn out later. (It also matches the parallel passages at Muktaşar 51b3f, 
Najât 61.11f, Išārāt 148.4f; Qiyās 116.10f is slightly more ambiguous.) Thom 
[46] p. 169f, working from this same text in Dānešnāmeh, finds that Ibn Sinā 
reasons ‘in accordance with the rule’

\[
\begin{array}{c}
Q \Pi N^e \rightarrow Q \Pi \Sigma^o \\
\frac{\Pi N^e \rightarrow N \Sigma^i}{q} \quad \frac{Q \Pi \Sigma^o}{q}
\end{array}
\]

(where \(N\) does not occur in \(Q\) or \(q\))

This is a rule of the same general kind as Gentzen’s natural deduction rule 
for elimination of \(\exists\); \(N\) in the subsidiary derivation on the left is eliminated 
in the main derivation on the right. Examples in propositional logic show 
that Ibn Sinā was well capable of formulating rules that involve subsidiary 
derivations, though I think the free occurrences of \(N\) would probably have 
stumped him. However, Thom’s rule is an order of magnitude more com-
plicated than (18), and though it could be formulated with an initial quan-
tifier over meanings, one would be hard pressed to describe the resulting 
rule as self-evident. The crucial difference is that \(D\) in (18) is not a free 
variable that will need eliminating; it is determined (\(mu.say\), Ibn Sinā 
would say) by \(A\) and \(B\). In fact Ibn Sinā never offers any argument in justi-
ﬁcation of this ecthesis, and my own feeling is that (18) is straightforward 
ough to justify him in treating it as self-evident. The same will apply to 
all of Ibn Sinā’s applications of ecthesis in assertoric logic, and arguably in 
two-dimensional logic too.

Street [43] p. 140 ascribes to Ibn Sinā a different rule of ecthesis, which 
in our notation would be

\[
(20) \quad \text{If } A \text{ and } B \text{ are meanings such that } (o)(B, A), \text{ then there is a mean-
\text{ing } D \text{ such that } (e)(D, A) \text{ and } (a)(D, B).}
\]

with the \(D\) indeterminate (\(gây\r mu'ayyan\)). This form is simpler than Thom’s, 
but only because Thom has taken care of how an indeterminate element can 
be used in a valid proof. Possibly Ibn Sinā was unaware of such issues; but 
this need not concern us, because in fact Ibn Sinā does always say enough to 
determine what \(D\) is. To my eye the indeterminate form is less self-evident 
because it leaves us asking ‘What \(D\) would work for this?’.

(2) Some truths of logic are proved wholly within the science of logic. 
Given (1) it’s obvious what these truths are in assertoric logic; they are the
theorems expressing that the second- and third-figure moods are valid. As we saw, Aristotle proves them by using the validity of first-figure moods and of conversions and ectheses; and all of these can be written down as theorems of logic that were justified before the second- and third-figure moods.

(3) Are there truths of logic whose demonstrations rely on one or more premises from First Philosophy? It seems to me that no affirmative truths of this kind are needed in assertoric logic. To the extent that assertoric logic generates valid moods, everything is taken care of under (1) and (2). But there remain the negative results, which say that certain formal premise-pairs are not productive, or that a certain conclusion is the strongest that can be drawn in a given figure. These are proved by showing the existence of counterexamples.

4.3 The logician as logician

The main thing that logicians do as logicians is to formulate and apply the rules of logic. So we should in theory be able to reach a better understanding of what Ibn Sīnā means by the phrase ‘the logician as logician’ if we set alongside each other the places where he uses this or similar phrases, and the places where he explains what the rules of logic look like. This enterprise really deserves a paper of its own, or perhaps several, since it ties in closely with Ibn Sīnā’s general notion of a science. The present section is a holding operation.

Most of what Ibn Sīnā tells us about the form of the truths of logic is wrapped up in his description of the subject term of logic. When he tells us that the subject individuals of logic are well-defined meanings, he adds two other points.

The first point is that the subject individuals have to be taken in the second of what he calls ‘the two wujūds’ (Madkāl 15.3, 19f, 16.1, 34.7–9, 13, Maqūlat 4.15f). This is an ontological notion and we must explore it in a moment. But first, please be clear that there are not two different classes of meanings, those in first wujūd and those in second wujūd. The meanings in these two wujūds are the same meanings but with a different ontological status. This is very clear for example in the discussion at Madkāl 34.5–16 (as at Madkāl 34.8–10 ‘The propria and accidents which belong to the māhiyya can be attached to it in each of the two wujūds’). Marmura ([36] p. 46) translates wujūd in this context as ‘[kind] of existence’. 

22
Ibn Sīnā gives his main explanation of second *wujūd* at Madkāl 15.1–7. He explains there that a whatness can be considered in three different ways. The first is on its own; the second and third are the first and second *wujūds*. In the first *wujūd* a whatness is considered as being true of (or ‘attaching to’) things in the world. In the second *wujūd* a whatness is considered in such a way that it can be a subject or a predicate, or predicated of all or some, etc. These are features that a whatness can have only as a part of a compound meaning. This is certainly what Ibn Sīnā has in mind here, since in the parallel passage of Mašrīqiyyūn he has ‘meanings in the context of their being subject to composition’ (*ma`ánt min ḥaytu hiya mawdū`atun īl-ta`līf*, Mašrīqiyyūn 10.15).

So what Ibn Sīnā is telling us with his references to second *wujūd* and being subject to composition is that in the truths of logic, meanings are described in terms of how they fit into compound meanings. A glance at the examples (13)–(16), (18) will confirm that this is absolutely correct for the examples of truths of logic that we have examined so far. The compound meanings are the meanings of propositions.

We turn to the second added point in Ibn Sīnā’s description of the subject individuals of logic. This second point is that the truths of logic are in aid of making available new information either by definition in terms of known meanings or by deduction from known meanings (Mašrīqiyyūn 10.15f, Madkāl 15.11f, Ilāhiyyāt 10.18 ‘in the context of how they bring about a progression from [already] known (ma`lūm) things to [previously] unknown (majhūl) things’ (*min jihati kayfiyyati mā yatawassalu bihā min ma`lūmin ilā majhūlin*). So not any true proposition about meanings as parts of compound meanings counts as a truth of logic. There is a further requirement that the proposition is a help towards the aim of gaining new information in either of the two mentioned ways.

In several places Ibn Sīnā adds remarks about the kinds of accident or feature that can be ‘attached to’ the subject individuals in a truth of logic. There is a list at Madkāl 15.5f:

- being a subject, being a predicate, being predicated of all or some, 
- being essential, being accidental, and some other things that you will learn about.

This list is given in an explanation of second *wujūd*, so it might be meant just as an explanation of things that can be said about a component of a proposition. But a later list at Madkāl 22.10–12 is specifically said to be
about what properties are ascribed to simple meanings in the context of the art of logic:

(22) whether one of these whatnesses is a predicate, or a subject, or a universal, or a particular, etc.

Further lists appear in Ta‘līqāt 502.4–505.12 (and I assume we can count at least this part of Ta‘līqāt as the authentic words of Ibn Sīnā himself):

being universal (kullī), being existential (juz‘ī), being singular (šakṣī), … being necessary (wājīb), being absolute (mutlaq), being possible (mumkin), …, being affirmative (mājīb), being negative (ṣālib), …, being contradictory (tunāqīdī), being a premise (muqaddama).

Note that the items in these lists are not themselves subject individuals of logic; they are ‘essential accidents’ (lawāzīm, singular lāzīm, Ta‘līqāt 503.3) or ‘features’ (ahwāl, singular ḫāl, Madkāl 15.16, Ta‘līqāt 507.4) of the individuals. Hence they are items that appear not as subject terms or subject individuals of truths of logic, but as ingredients of the predicates of truths of logic. Again a glance at the concrete examples in (13)–(16), (18) will confirm that it has to be this way round.

The passage in Ta‘līqāt comments on some of the items listed, that although they can be used in logic, they are established (tuḥbatu, e.g. Ta‘līqāt 504.11) in metaphysics or First Philosophy. Thus

(24) being a genus (jinsiyya), being a differentia (fasliyya) and being a species (nawṣiyya)

are used as accidents of things in logic, but are established in First Philosophy (Ta‘līqāt 506.6f). A few lines later we read that

(25) genus, differentia, species, proprium (kāss) and accident (‘araḍ)

as ‘features in the teaching of the existent as existent’ are studied not in logic but in theory of the universal, i.e. in First Philosophy (Ta‘līqāt 506.9–11). Exactly what is intended here I am not sure, but it seems clear that Ibn Sīnā is somehow limiting the use of these notions in logic.

4.4 The boundaries of logic

The previous two subsections give us enough facts about the truths of logic to allow a comparison with the passages in which Ibn Sīnā says that something is not the concern of the logician. These passages are overwhelmingly
We can note straight away that there are no category words of any kind in the lists (21)–(24), except for ‘accidental’ in (21). If ‘accidental’ is in (21) as one of the features of the subject individuals of logic, and not just part of the explanation of second *wujūd*, then we should note that it is contrasted there with ‘essential’ and not with ‘substantial’; this is not a category distinction. In (25) ‘accident’ appears; but this is with a list of predicables, not categories; and in any case it is not described here as playing any role in logic.

In the opening pages of *Maqūlāt* Ibn Sinā says forwards and backwards and sidewards that the categories are no use for logic. (Thus *Maqūlāt* 3.13–4.1 not all features of the components of compound expressions used in logic are themselves helpful for logic, since some are not relevant to reaching new concepts or information; *Maqūlāt* 5.1–9 the student of logic, as opposed to First Philosophy, never needs to learn the ten categories; see Gutas [11] 300–303.) In fact he seems to say too much here, suggesting that the notions of genus and species are useless for logic (*Maqūlāt* 5.7–9), in contrast to (24). It could be that at *Maqūlāt* 5.7–9 he is saying just that the contrast between genus and species is irrelevant to logic. Perhaps more likely, he is concentrating on the central part of logic that studies syllogisms; the notion of genus is not needed here, though it certainly is needed in the theory of definitions.

The other references in *Maqūlāt* point out specific issues that don’t concern the logician. These are most of them things that we would be unlikely to have thought of putting into laws of logic. One case worth noting is *Maqūlāt* 143.15, where Ibn Sinā says that it is no business of the logician to establish the theory of relations. Today we regard the theory of relations as an integral part of logic. It could be that Ibn Sinā’s notion of logic is less inclusive than ours, or alternatively that his concept of establishing the theory of relations is semantic and linguistic rather than logical. At any rate nothing like ‘relation’ (*’idāfa*) appears in the lists (21)–(24).

At *Maṣriqiyyān* 82.13 Ibn Sinā says that the truth of sentences ‘as a fact of nature and not of necessity’ is not a concern of logicians. Again we note that ‘true’ is not in the lists (21)–(24), though ‘necessary’ is. At *Išrāt* 94.16 Ibn Sinā says that a logician examines a proposition without being concerned with whether the proposition is true.

At *Būrān* 87.10–12 he says that the question whether *X* is possible in the case of matter *Y* is a question that can’t be dealt with in logic but has to be investigated in First Philosophy. Now ‘possible’ was one of the terms in the list (21)–(24), so Ibn Sinā does accept that a truth of logic can talk in
terms of whether a certain proposition is possible. His point here seems to be that truths of logic, even if they can use this notion, can’t stipulate what is possible in medicine or biology (two fields he has been discussing).

Although Ibn Sinā is adamant that categories play no role in the truths of logic, he is by no means so sure that they play no role in the practice of logic. For example we know, and (23) acknowledges this, that there are rules of logic about what is contradictory to what. But in several places Ibn Sinā indicates that when we have a proposition \( \phi \) and we want to find the contradictory negation of it, we should make sure that the contradictory negation carries the same ‘additions’ as \( \phi \), and he uses the categories as a check-list of what additions we might need to look for. Thus at ⁶Ibāra 43.6–44.9 he mentions potential, place, time, relation. A similar list at Mašriqiyyūn 48.6f mentions relation, time, place, quality, dimension, act, passion, potential, act. In the discussion of the subject term of logic at Mašriqiyyūn 10.15–19, Ibn Sinā remarks that while there is no requirement that the subject individuals of logic should be ‘substances or quantities or qualities or the like’, a logician may pay attention to these features when looking for expressions that are ‘suitable to form parts of an explanatory phrase or an inference’. Presumably features like these will play some role in ensuring that the meanings being used are well-defined.

In the light of the facts above, what can Ibn Sinā have meant when he said that the difference between permanent and necessary is not one of concern to the logician as logician? The most straightforward reading is that he means that the laws of logic never need to refer to this distinction. And in fact we note that the lists (21)–(24) don’t contain any temporal notion such as ‘permanent’, though they do contain alethic modal notions like ‘necessary’ and ‘possible’. Again the most straightforward reading of this fact is that Ibn Sinā thinks that the truths of logic can stipulate what holds in general for necessity and possibility, just from the meanings of these two words, but it is not any part of a logician’s task, as logician, to determine what laws hold for any specific category of modality. (In view of ‘essential’ and ‘accidental’ in (21), the ontological modalities might be an allowed exception.)

This account leaves several possibilities open. For example Ibn Sinā might still be able to point to some laws that hold for temporal modalities but not for the abstract alethic ones. We will find that in fact he doesn’t, but this is something we will have to discover from his texts. We turn now to his temporal logic.
(Forgive me an aside here. I have suggested elsewhere that a modern reader can probably make best sense of Ibn Sīnā’s notion of second *wujūd* by thinking of meanings in second *wujūd* as *occurrences* of meanings, by analogy with the difference between words and the occurrences of words in sentences. One shouldn’t lose sight of another aspect of all this. For Ibn Sīnā the fact that we can handle meanings as parts of compound meanings is a criterial divide between humans and all other beings in the lower world. Hence for him it is reasonable to think of this ability as providing a guarantee of our personal immortality. The fact that Ibn Sīnā invests so much religious significance in a fairly abstruse point in the foundations of logic is both shocking and incisive. I suspect Ibn Sīnā and Lukasiewicz would have found they had a lot in common here.)

5 Two-dimensional logic

5.1 Ibn Sīnā’s introduction of two-dimensional logic

At the close of section i.7 of the *Prior Analytics*, where Aristotle rounds off his survey of the assertoric syllogisms, the Paris manuscript of the Arabic translation has a rubric:

(26) It was an innovation of the Alexandrians to read only this far in the book; they refer to what follows it in the book as ‘the part that is not read’. This [part] is the discussion of syllogisms composed of premises that have modalities. ([33] pp. 210f.)

Whether or not Ibn Sīnā had this rubric in his text of the *Prior Analytics*, he certainly wasn’t discouraged from reading on. In fact this ‘part that is not read’, and perhaps even more the discussions of it by Theophrastus, Alexander of Aphrodisias and Themistius, had a profound effect on Ibn Sīnā’s understanding of logic.

The texts of Theophrastus, Alexander and Themistius that Ibn Sīnā refers to (for example at Najāt 39.10f) are now mostly lost—though the relatively recent publication of a Hebrew paraphrase of a relevant work of Themistius [42] gives hope that more of this material may yet turn up. But this is not so important for us, because our main concern is not how Ibn Sīnā treated his sources, but the conclusions that he came to himself after reading those sources.

Immediately after the rubric just quoted, the Arabic Aristotle proceeds:

(27) Because the *muṭlaq*, the *darūrī* and the *mumkin* premises differ from each other … ([33] 29b29)
This translates a passage in which the Greek Aristotle says that there is a difference between being something, necessarily being something and possibly being something. Here the Arabic ْdarّارّت means ‘necessary’ and the Arabic ْمٓعِلَق means ‘possible’ (with some nuances to be discussed below). The Arabic ْمٓعِلَق, normally translated ‘absolute’, means ‘not qualified’ or ‘not subject to any condition’, which is not an item in the Greek original. The Arabic translator has taken what in Aristotle’s Greek are three different things that a sentence might express, and has converted them into three kinds of sentence; in the process he has invented a new kind of sentence, the ‘absolute’ sentence. (See Lameer [35] 55–59 on how this innovation might have crept in as the translation passed through Syriac.)

Ibn Sīnā, reading the Arabic Aristotle, thought that in the part of the Prior Analytics ‘that is not read’, Aristotle was discussing the logical properties of sentences with one or other of three modes, ‘necessary’, ‘absolute’ and ْمٓعِلَق. He was aware that the Arabic Aristotle’s ْمٓعِلَق could mean either ‘possible’ (i.e. not necessarily not the case) or ‘contingent’ (i.e. not necessarily the case and not necessarily not the case). In cases of ambiguity like this, Ibn Sīnā distinguishes between a ‘broad’ (‘امّت) or more inclusive sense, and a ‘narrow’ (کّن) or ‘strict’ (حاّقّ) or less inclusive sense. So we find in Ibn Sīnā frequent references to ‘broad ْمٓعِلَق’ and ‘narrow (or strict) ْمٓعِلَق’, which are different though closely related modes. The modes ‘necessary’, ‘absolute’, ‘broad ْمٓعِلَق’ and ‘narrow ْمٓعِلَق’ together form the main modes that Ibn Sīnā finds studied in Aristotle; we will call them the alethic modes.

The Arabic Aristotle adds these modes to assertoric sentences. Thus we find sentences like ‘A is with necessity found in some B’ ([33] 34b23) which Ibn Sīnā would normally write as ‘Some B is an A, with necessity’. Adapting the notation (i)(B, A), we can abbreviate this to

\[(i\text{-nec})(B, A)\].

Similarly we have sentence forms \((a\text{-pos})(C, B)\), \((i\text{-con})(C, A)\), \((o\text{-abs})(A, D)\) with pos, con and abs for necessary, possible, contingent and absolute. Often Aristotle and Ibn Sīnā are unclear about whether they intend possible or contingent, so we will sometimes need to write such things as \((a\text{-mum})(B, A)\) with mum for ْمٓعِلَق.

The ambiguity between pos and con was pretty blatant, but Ibn Sīnā believed that he could find in Aristotle, Theophrastus, Alexander and Themistius discussions of ambiguities in nec and abs too. As we pass from Ibn Sīnā’s
...Muktaṣar through Najāt and Qiyās to Maṣriqīyyān, we can sense a steady progression. In Muktaṣar Ibn Sīnā is concerned to set out the views of these earlier logicians, and to give some of his own reactions. By the time we reach Maṣriqīyyān, his reactions have settled into a collection of new sentence forms that amount to a new form of logic, and he no longer mentions the earlier logicians. It will become clear below that the effects of this new form of logic were already well entrenched in Ibn Sīnā’s account of modal syllogisms in Muktaṣar. So probably the progression from Muktaṣar to Maṣriqīyyān marks an improvement in presentation rather than a change of content. The account in Iṣārat is if anything a step backwards from Maṣriqīyyān, since it is less clear about the range of new sentence forms. Probably this is the result of the extreme brevity of the discussions in Iṣārat.

For example Ibn Sīnā believed that Theophrastus and Themistius on the one side, and Alexander on the other side, disagreed about what is expressed by an absolute sentence. For Alexander, an absolute sentence always expresses that all or some of the things that are Bs are sometimes As and sometimes not As. The other two logicians thought that an absolute sentence could express that all or some of the things that are Bs are sometimes As, without ruling out that some of these things might always be As. From Ibn Sīnā’s discussions it is not at all clear (at least not to me) whether Ibn Sīnā thinks these earlier logicians are disagreeing about the meaning of the word translated as ‘absolute’, or whether they agree about the sense of the word but disagree about how one should interpret the sentences that fall under it; and if the latter, whether he thinks this is a disagreement about how these sentences are normally used, or a disagreement about how logicians should use them. Maybe he thinks these authors were themselves unclear about which of these they meant.

But at least by Qiyās and Maṣriqīyyān, Ibn Sīnā is clear in his own mind: the view he attributes to Alexander should be read as a description of a particular type of sentence, which he calls wujūdī. A wujūdī sentence is one which expresses something of the form

\[(29) \text{ Every (or some) } B \text{ is sometimes an } A \text{ and sometimes not an } A.\]

(Cf. Maṣriqīyyān 65.13f.) Ibn Sīnā also refers to sentences of this kind as ‘the kind after the broad absolute’, where a broad absolute sentence is one which expresses something of the form

\[(30) \text{ Every (or some) } B \text{ is sometimes an } A.\]
(E.g. Mašriqiyyūn 77.1–6, 79.1–3.) Ibn Sinā also refers to these wujūdī sentences as ‘narrow absolute’ (e.g. at Qiyāṣ 130.4, 162.8, Išārāt 145.1), in analogy with the distinction between broad and narrow mumkin. Ibn Sinā believes that both broad and narrow absolute sentences occur regularly in normal scientific discourse. (He also believes that there is a particular problem about how negative universal broad absolute sentences are expressed, at least in Arabic; but I say no more about this here.)

Ibn Sinā also believed that in Theophrastus he could find a speculation about three different ways in which a sentence ‘Every B is an A’ can be read as expressing a necessary truth. Here I skip over the historical evidence ([9] p. 187ff, [42], Ibn Sinā Qiyāṣ i.5, 41.5–13) and concentrate on what Ibn Sinā took from it. It seems that Ibn Sinā had in front of him a claim that ‘Every B is an A’ can be read as expressing a necessity in the following three ways:

(a) Unconditionally.

(31) (b) Under a condition that the subject is mawjūd.

(c) Under a condition that the predicate is mawjūd.

Here mawjūd could mean either ‘existing’ or ‘true’, and the subject could be either the subject term or the subject individual; so there is multiple ambiguity. Ibn Sinā picked out two readings of (b) that he found significant, namely

Every B is an A throughout the time while its [individual] essence is satisfied (i.e. while the individual exists).

and the second as

Every B is an A throughout the time during which it is a B (i.e. while the subject term is true of the individual).

Setting out the paraphrases (32) and (33) in Mašriqiyyūn, Ibn Sinā proposes for (32) the name ḏarūrt, i.e. ‘necessary’; for (33) he proposes the name lāzim, which could be read as ‘adherent’. Although both of these sentences express a kind of conditional necessity, Ibn Sinā also calls the adherent sentences the ‘adherent absolutes’ (Mašriqiyyūn 79.14f). They also appear with names that only make sense in context, like ‘this kind of absolute’ (Qiyāṣ 40.16, 128.14).
One can speculate about why Ibn Sīnā mentions essences in sentences like (32). But from his examples and comments it is clear that he just intends ‘throughout the time while the individual exists’. Often he drops the mention of essence and just says ‘while it continues to exist’, as at Qiyās 77.3 and 91.2 and at Mašriqiyyūn 71.14f.

5.2 Features of two-dimensional sentences

By the end of these reflections, Ibn Sīnā has managed to transform Aristotle’s alethic modal sentences, and some early reflections on how these sentences should be understood, into a whole raft of new sentence forms. These new forms have several things in common.

First, they contain no alethic modes, and no alethic modes are used in defining them.

This is implicitly denied by Thom [47] p. 74, who includes the word ‘necessarily’ in his definitions of both \((d)\) and \((\ell)\). This must be a misunderstanding between Thom and his informant, because there is no textual evidence to support it. We have seen at (4) above that Ibn Sīnā allows that a thing can be permanent without being necessary. Ibn Sīnā does describe \((d)\) sentences as ‘necessary’ (\(\text{\texttt{\textit{\text{\texttt{\text{]]}}}}\)}\), but this surely means that he counts permanence as a kind of necessity, not that necessity has to be read into the definition.

In this context it is perhaps unhelpful that a number of published works refer to Ibn Sīnā’s sentences (32) as ‘substantial’, apparently mistranslating Ibn Sīnā’s word \(\text{\texttt{\textit{\text{\texttt{\text{]]}}}}\)}\) ‘essence’ as ‘substance’. It’s hard to see how this came about. Al-Fārābī does say that jawhar (the normal Arabic word for ‘substance’) is sometimes used to mean essence (Hurūf [8] 63.9), and Ibn Sīnā confirms this at Hudūd Definition 15 ([20] p. 23) and at Qiyās 22.3. But if Ibn Sīnā ever goes the other way and uses \(\text{\texttt{\textit{\text{\texttt{\text{]]}}}}\)}\) to mean substance—and Goichon [10] records no cases where he does—it would need an extremely strong argument to show that Ibn Sīnā has this in mind when he uses the word \(\text{\texttt{\textit{\text{\texttt{\text{]]}}}}\)}\) in the sentences (32). Goichon [10] pp. 134, 136 describes the translation of \(\text{\texttt{\textit{\text{\texttt{\text{]]}}}}\)}\) by ‘substantia’ as an unfortunate and confusing error, and I can only agree.

Second, these new sentence forms all contain a reference to time. In fact nearly all of them contain, besides the usual Aristotelian quantifier which we can now call the object quantifier, a second quantification over times. Because of this double quantification I will refer to these new sentences as
two-dimensional sentences, borrowing this name from Oscar Mitchell who in the early 1880s independently began to develop Aristotle’s assertoric logic in a similar direction [38]. As Ibn Sinā must have observed from the outset, these two-dimensional sentences have logical relationships between them. And so we can refer to the logical study of these sentences as two-dimensional logic.

Third, these sentence forms, at least in Ibn Sinā’s mature account of them, come in four flavours like the four kinds of assertoric sentence: (a), (e), (i) and (o), and at least the main forms have contradictory negations that Ibn Sinā describes. For example the contradictory negation of

(34) Every B is an A for as long as it exists.

is the (o) sentence

(35) Some B is, at some time during its existence, not an A.

So the two-dimensional forms include existential time quantifications that are dual to the universal ones in (32) and (33). The form

(36) Every B is, at some time during its existence, an A.

is one we can recognise as the form that Ibn Sinā thought he found in Theophrastus and Themistius, which he called ‘broad absolute’. (Cf. (30) above and Maṣriqyyūn 68.3–5.)

I add a remark that will play only a marginal role in this paper, but it may help for orientation. Another development that we owe to Ibn Sinā is his extension of the classes of muttaṣil and munfaṣil propositional compound sentences to (a), (e), (i) and (o) forms. I believe this development took place within the framework of Ibn Sinā’s two-dimensional logic, broadly as follows. He reversed the relative scopes of the object and time quantifiers in two-dimensional sentences, and this gave him sentence forms that could be regarded as propositional compounds, generalising the propositional compound forms discussed by earlier Peripatetic logicians. In doing so he noticed that the muttaṣil sentences can be presented as exactly analogous to the assertoric sentences. The resulting propositional syllogisms obey exactly the same formalism as the assertoric ones: same moods, same justifications, but with time quantification in place of object quantification. Ibn Sinā presents this result in Qiyās vi.1, spelling out the syllogisms with almost exactly the same order and commentary that he had used for the assertoric syllogisms in Qiyās ii.4.

32
A fourth feature of these two-dimensional sentences is that their truth-conditions are completely clear and unambiguous, at least after one has navigated a path through Ibn Sīnā’s confusing explanations. This is partly the result of his removing the modal expressions ‘necessary’, ‘possible’ etc. from the sentences—the first feature above. But Ibn Sīnā takes a further step to guard against a possible ambiguity in the quantifiers. Some Peripatetic logicians had noted that a quantification over ‘all Bs’ can be over things that are actually Bs, or it can be over things that could possibly be Bs. Ibn Sīnā tells us frequently that he restricts these quantifications to things that are ‘actually’ (bīl fīʾl) Bs. (Thus Muktaṣar 40a10–44a10; the phrase bīl fīʾl occurs thirty-three times in this passage, always with reference to this point about the quantification. Also Mašrīqiyyūn 68.3, 6f; this last is with reference to a ‘necessary’ sentence, blocking the suggestion sometimes made, that Ibn Sīnā’s quantification over actual Bs might not apply to modalised propositions.)

This feature has also been denied by Thom. At [45] p. 362 Thom quotes Inati’s translation of Išārat ([29] 93.10–12, [17] p. 99), and comments

[Avicenna] takes the subject-term of an absolute or modal proposition to apply to whatever falls under the term, “be it so qualified in a mental assumption or in external existence, and be it so qualified always or not always, i.e., in just any manner”. This formulation self-consciously rejects the idea that the subject-term of an absolute or modal proposition applies just to what actually exists.

If Thom is right then Ibn Sīnā in Išārat has abandoned one of his most cherished positions in his earlier logical writings; we would certainly not be entitled to read his new position back into the logic of Najāt or Qiyās, as Thom goes on to do. Thom doesn’t say what features of the quoted passage he takes as evidence for his conclusion, but let me guess that they are any or all of the following three: (a) the reference to ‘mental assumption’ as opposed to ‘external existence’, (b) the phrase ‘in just any manner’, and (c) the absence of any qualifying phrase ‘in actuality’ (bīl fīʾl).

As to (a): Ibn Sīnā has forestalled this reading at Qiyās 21.6–10, where he spells out that for him, existence in thought counts as actual. He wants to be able to say that mathematical objects like the icosahedron are actual though they are not in the material world.

As to (b): the phrase ‘in just any manner’ (kayfa ittafaqa) is a stylistic variant of the more usual kayfa kāna ‘however it is’. It certainly doesn’t rule out a requirement for things to be actual; for example at Išārat [29] 143.10
Ibn Sinā writes ‘Every C is a B in act, in any way’ (bil fi‘l, kayfa kāna).

As to (c): in reading Ibn Sinā it is always dangerous to infer anything from the absence of a phrase in one passage when the phrase occurs in other parallel passages. This is particularly true of Išārāt, which was written in a telegraphic style. Even in Muktaṣar, where Ibn Sinā leaves us in no doubt about the requirement of actuality, he sometimes doesn’t mention this requirement. An example is at Muktaṣar 40a5, explaining ‘Every B is an A’, where incidentally he also says ‘however it is described, permanently or not permanently, we don’t know when’.

Although Thom’s particular piece of evidence doesn’t hold up, there are two other reasons why he is right to be cautious.

The first is that although Ibn Sinā consistently says that he intends the object quantification to be over actuals, he never says the same for the time quantification. In fact some of his examples suggest that he must be including times that never were or will be actual, for example at Qiyās 30.10 ‘imagine a time when there are no animals except humans’, or at Qiyās 134.11 ‘some time when nothing is coloured white or red’. I believe these passages occur only with wide time scope, which puts them outside the range of most of the passages discussed in this paper. I also have an impression that they are partly a hangover from earlier Peripatetic speculations about reducing propositional logic to predicate logic. But a complete account will need to say something about them.

The second reason for caution is that Ibn Sinā, when he discusses possibility, accepts i-conversion from ‘Some B can be an A’ to ‘Some A can be a B’. There are obvious counterexamples to this conversion if we require that the quantification in the second sentence is only over actual As. For example it seems entirely possible that there never was and never will be a purple cow, though some accident of biology could turn a cow purple. In this case some cow can be purple; but things that aren’t cows don’t have the potential to become cows, and it is not true that any actual purple thing ever was or will be a cow, so it is false that some actual purple thing can be a cow. The problem is not that Ibn Sinā disowns his statements about quantifying over actuals when he comes to discuss possibility—he doesn’t. Rather it is that the things that he says in different places seem not to be compatible.

This is not the kind of problem that has a quick fix. It need not trouble us until we come to consider in general how Ibn Sinā deals with statements of possibility. But we should come back to the problem when we have a
better broad perspective on what Ibn Sinā is trying to do in the alethic logic of possibility.

6 Formalities

6.1 Formalising two-dimensional logic

Most of the two-dimensional sentence forms that Ibn Sinā introduces are clearly enough described to allow formalisation in a two-sorted first-order language with an object sort and a time sort. We use lower case latin letters for the object variables and greek letters for the time variables. The relations all take the form \( Rx \tau \), meaning that the object \( x \) is an \( R \) at time \( \tau \). There is one distinguished relation \( Ex \tau \), which means that \( x \) exists (or as Ibn Sinā would prefer, the essence of \( x \) is satisfied) at time \( \tau \).

We can reach most of the relevant sentences by starting with the assertoric sentence forms as in (8) above and making some replacements as in the following example. We have the assertoric formal (a) sentence

\[(a)(B, A), \text{ i.e. } (\forall x (Bx \to Ax) \land \exists x Bx).\]

We also have a modality (d) as follows:

\[(d) (B, A) : \forall \tau (Ex \tau \to Ax \tau)\]

expressing that \( x \) is an \( A \) throughout the time while \( x \) exists. We combine these two ingredients by putting the modality in place of \( Ax \), and then replacing \( Bx \) by \( \exists \tau Bx \tau \). This gives the formal sentence

\[(a-d) (B, A) : (\forall x (\exists \tau Bx \tau \to \forall \tau (Ex \tau \to Ax \tau)) \land \exists x \exists \tau Bx \tau).\]

Since this sentence comes from combining \( (a)(B, A) \) with the modality \( (d) \), we call it

\[(a-d)(B, A).\]

The same recipe works if we start from the \((e)\), \((i)\) or \((o)\) forms, with a suitable twist on the augment of the \((o)\) form:

\[
\begin{align*}
(e-d)(B, A) & : \forall x (\exists \tau Bx \tau \to \forall \tau (Ex \tau \to \neg Ax \tau)) \\
(i-d)(B, A) & : \exists x (\exists \tau Bx \tau \land \forall \tau (Ex \tau \to Ax \tau)) \\
(o-d)(B, A) & : (\exists x (\exists \tau Bx \tau \land \forall \tau (Ex \tau \to \neg Ax \tau)) \lor \forall x \forall \tau \neg Bx \tau)
\end{align*}
\]
Note that in the negative cases \((e)\) and \((o)\) we replace \(\neg Ax\) by the modality with the negation immediately in front of \(A\).

Three other modalities behave the same way, namely the modalities \((\ell)\), \((m)\) and \((t)\):

\[
\begin{align*}
(\ell) & : \forall \tau (Bx \tau \rightarrow Ax \tau) \\
(m) & : \exists \tau (Bx \tau \land Ax \tau) \\
(t) & : \exists \tau (Ex \tau \land Ax \tau).
\end{align*}
\]

(The letters are taken from the descriptions in Qiyās and Mašriqiyyūn; see [15].) For example we have

\[
(44) \quad (o-m)(B, A) : (\exists x (\exists \tau Bx \tau \land \exists \tau (Bx \tau \land \neg Ax \tau)) \lor \forall x \forall \tau \neg Bx \tau)
\]

which says that some sometimes-\(B\) is, at some time while it is a \(B\), not an \(A\).

Ibn Sīnā’s general assumptions ([14]) allow us to add that nothing has a positive property at any time when the thing doesn’t exist; in a phrase, nonexistents have no positive properties. So for every relation \(R\),

\[
(45) \quad \forall x \forall \tau (Rx \tau \rightarrow Ex \tau).
\]

Also we quantify only over things that exist at some time:

\[
(46) \quad \forall x \exists \tau Ex \tau.
\]

We call the sentences (45) and (46) the theory of \(E\), and we adopt them as background assumptions (or meaning postulates) whenever we are dealing with two-dimensional logic. Under these assumptions, each of the sentences in the following list entails all the sentences after it:

\[
(47) \quad (g-d)(B, A), \quad (g-\ell)(B, A), \quad (g-m)(B, A), \quad (g-t)(B, A)
\]

where \(g\) is any of \(a, e, i, o\). So we count \(d\) as stronger than \(\ell\), which is stronger than \(m\), which is stronger than \(t\).

We call \(a, e, i\) and \(o\) the aristotelian forms, and we call \(d, \ell, m\) and \(t\) the core avicennan forms. The sentence forms \((g-h)(B, A)\), where \(g\) is an aristotelian form and \(h\) is a core avicennan form, and \(R\) and \(S\) are any two distinct relation symbols, will be called the core two-dimensional forms. Ibn Sīnā himself doesn’t distinguish them by a name, but they are the leading forms in his account in Qiyās i.3 and Mašriqiyyūn, and they allow us to build a sensible logical theory around them.
In fact Ibn Sīnā uses other forms besides these core two-dimensional forms. He often calls attention to the *wujūdī* sentences which express that something is sometimes an *A* and sometimes not an *A*. Formally these are most smoothly handled by applying the modality (*t*) to the following four fictitious assertoric forms:

\[
\begin{align*}
(\ddot{a})(B, A) & : \forall x(Bx \rightarrow (Ax \land \neg Ax)) \\
(\ddot{e})(B, A) & : \forall x(Bx \rightarrow (\neg Ax \land Ax)) \\
(\ddot{i})(B, A) & : \forall x(Bx \land (Ax \land \neg Ax)) \\
(\ddot{o})(B, A) & : \forall x(Bx \land (\neg Ax \land Ax))
\end{align*}
\]

The modality (*t*) is applied separately to both *Ax* and *\neg Ax*. So for example we have

\[
(\ddot{a}-t)(B, A) : \forall x(\exists \tau Bx \tau \rightarrow (\exists \tau (Ex \tau \land A x \tau) \lor \exists \tau (Ex \tau \land \neg Ax \tau))]
\]

We will call these forms the *double-dot forms*, and the process of passing from a form *g* to *\ddot{g}* will be called *double-dotting*. In *Mašriqiyûn* 80.14–20 Ibn Sīnā also discusses the corresponding forms with (*m*) in place of (*t*), but we will not need to consider these.

Passing from (\ddot{a}) to (\ddot{e}), or from (\ddot{i}) to (\ddot{o}), is called *reduction to the negative* (*rujūc* 'alā sâlibih, *Qiyās* iii.5 174.16), and the move in the opposite direction is *conversion to the affirmative* (*'aks* 'alā 'ijâbih, *Qiyās* iv.4 208.17). Since Ibn Sīnā allows the moves in both directions, it seems that he regards (\ddot{a}-t)(B, A) as logically equivalent to (\ddot{e}-t)(B, A), and likewise for the existential forms. Our formalisations reflect this. But it follows that Ibn Sīnā adds the augments in both affirmative and negative cases, or in neither. For simplicity we assume neither, though I suspect he plays it by ear. The only moods that it affects are *Darapti* and *Felapton*.

Ibn Sīnā also considers sentences got by fixing the time to a particular moment or interval α, for example

\[
(\alpha-\ddot{z})(B, A) : \forall x(Bx \alpha \rightarrow (Ex \alpha \rightarrow Ax \alpha))
\]

Here \(z\) abbreviates Ibn Sīnā’s name for these, *zamânî* or ‘temporal’. If α is the present then these forms correspond to the Latin *ut nunc* sentences.

If \(\phi\) is any one of these new sentence forms, then we can uniquely recover from \(\phi\) the assertoric sentence form that gave rise to it. We write \(\tau_{\alpha}\phi\) for this assertoric sentence form, and we call it the *assertoric projection* of \(\phi\).
Among these various forms, the ones that will chiefly concern us are those where the avicennan form is \(d\) or \(t\). The reason for this is that Ibn Sinā associates these two forms with the alethic modes of necessary, broad absolute and possible. Most of the other forms above he groups together as other forms of absolute. We will refer to the two-dimensional sentences with avicennan form \(d\) or \(t\) as the \(dt\) fragment.

Tying these various sentence forms to Ibn Sinā’s text is not always straightforward. In \(Qiyāṣ\) for example, the sentences spelt out in the introductory sections are mostly two-dimensional, but when Ibn Sinā comes to study rules of inference he switches mainly to alethic modal forms. However, he often sprinkles temporal words over these alethic forms, and he sometimes switches back to straightforwardly two-dimensional forms in order to discuss a particular point. So when he writes an alethic modal form, the reader has to ask whether it should be read straightforwardly as an alethic modal form, or whether it is really a disguise for a two-dimensional form. Unfortunately these could both be the case together; Ibn Sinā is not above writing things that are intended to be read in two different ways simultaneously. See [13] p. 374f for a case in point, from \(Qiyāṣ\) ix.6.

### 6.2 Metatheorems of two-dimensional logic

We assemble here some facts about the validity of inferences in two-dimensional logic. Mathematical proofs are given in [15]. Ibn Sinā himself will have verified as many as he cared to by the kind of \(istikrāj\) that we saw him applying to propositional logic in (11).

#### Contradictory negations

**Fact 6.1** To find the contradictory negation of a core two-dimensional sentence \((g-h)(B,A)\), where \(g\) is an aristotelian form and \(h\) is an avicennan form, apply the following swaps to \(g\) and \(h\):

\[
\begin{align*}
a & \leftrightarrow o \\
e & \leftrightarrow i \\
d & \leftrightarrow t \\
\ell & \leftrightarrow m.
\end{align*}
\]

#### Conversions and other one-premise inferences
Fact 6.2 The following entailments hold between pairs of two-dimensional sentences with a given subject relation symbol and a given predicate relation symbol.

\[(a-d) \Rightarrow (a-\ell) \Rightarrow (a-m) \Rightarrow (a-t)\]
\[(i-d) \Rightarrow (i-\ell) \Rightarrow (i-m) \Rightarrow (i-t)\]
\[(e-d) \Rightarrow (e-\ell) \Rightarrow (e-m) \Rightarrow (e-t)\]
\[(o-d) \Rightarrow (o-\ell) \Rightarrow (o-m) \Rightarrow (o-t)\]
\[(\ddot{a}-t) \Leftrightarrow (\ddot{e}-t) \Leftrightarrow (\ddot{i}-t) \Leftrightarrow (\ddot{o}-t)\]

(51)

Fact 6.3 The following, and their immediate consequences by Fact 6.2 above, are the only conversions that hold between core two-dimensional sentences:

(a-t)-conversion: \[(a-t)(B, A) \Rightarrow (i-t)(A, B)\]

(e-d)-conversion: \[(e-d)(B, A) \Leftrightarrow (e-d)(A, B)\]

(e-\ell)-conversion: \[(e-\ell)(B, A) \Leftrightarrow (e-\ell)(A, B)\]

(i-m)-conversion: \[(i-m)(B, A) \Leftrightarrow (i-m)(A, B)\]

(i-t)-conversion: \[(i-t)(B, A) \Leftrightarrow (i-t)(A, B)\]

(52)

Valid moods

We write a mood with premise-pair \((\phi, \psi)\) and conclusion \(\chi\) as \((\phi, \psi, \chi)\). This mood is optimal in a given figure, if it is valid, but if either we weaken a premise or we strengthen the conclusion, staying within that figure, then the resulting triple is not a valid mood. The assertoric projection of \((\phi, \psi, \chi)\) is the triple \((\pi_0\phi, \pi_0\psi, \pi_0\chi)\) of assertoric projections of the three sentences. Likewise the avicennan form of the triple \((\phi, \psi, \chi)\) is the triple \((h_1, h_2, h_3)\) where \(h_1\) is the avicennan form of \(\phi\), \(h_2\) is the avicennan form of \(\psi\) and \(h_3\) is the avicennan form of \(\chi\).

Fact 6.4 Suppose \((\phi(C, B), \psi(B, A))\) is a premise-pair and \(\chi(C, A)\) is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for \((\phi(C, B), \psi(B, A), \chi(C, A))\) to be optimal in first figure:
(a) The assertoric projection of \((\phi(C, B), \psi(B, A), \chi(C, A))\) is optimal in assertoric logic.

(b) The avicennan form of \((\phi, \psi, \chi)\) is one of the following five triples:

\[(t, t, t), (t, d, d), (d, \ell, d), (\ell, \ell, \ell), (m, \ell, m).\]

**Fact 6.5** Suppose \((\phi(C, B), \psi(A, B))\) is a premise-pair and \(\chi(C, A)\) is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for \((\phi(C, B), \psi(B, A), \chi(C, A))\) to be optimal in second figure:

(a) The assertoric projection of \((\phi(C, B), \psi(B, A), \chi(C, A))\) is optimal in assertoric logic.

(b) The avicennan form of \((\phi, \psi, \chi)\) is one of the following five triples:

\[(t, d, d), (d, t, d), (\ell, \ell, \ell), (m, \ell, m), (t, \ell, t).\]

**Fact 6.6** Suppose \((\phi(B, C), \psi(B, A))\) is a premise-pair and \(\chi(C, A)\) is a sentence, all in core two-dimensional logic. Then the conjunction of (a) and (b) below is a necessary and sufficient condition for \((\phi(C, B), \psi(B, A), \chi(C, A))\) to be optimal in third figure:

(a) The assertoric projection of \((\phi(C, B), \psi(B, A), \chi(C, A))\) is optimal in assertoric logic.

(b) The avicennan form of \((\phi, \psi, \chi)\) is one of the following five triples:

\[(t, t, t), (t, d, d), (d, t, m), (m, m, m), (m, \ell, m).\]

**Fact 6.7** If a mood in core two-dimensional logic is valid, then it remains valid if one or both of the premises are double-dotted, unless the mood is Darapti or Felapton and both premises are double-dotted. Double-dotting the second premise in first or third figure allows the conclusion to be double-dotted too.

As noted in Subsection 3.3 above, Ibn Sīnā lists those moods that are conclusion-optimal, i.e. they are valid, but they become invalid if the conclusion is strengthened. In each figure, the conclusion-optimal moods \((\phi, \psi, \chi)\) can be found from a list \(S\) of the optimal moods as follows. First, we check that the premise-pair \((\phi, \psi)\) is productive by checking that

\[(53) \text{ there is a triple } (\phi', \psi', \chi) \text{ in } S \text{ where } \phi \text{ is or entails } \phi' \text{ and } \psi \text{ is or entails } \psi'.\]
Then if the pair is productive, the strongest conclusion is the strongest sentence $\chi$ such that there is a triple $(\phi', \psi', \chi)$ as in (53).

A metaprinciple

**Fact 6.8 (Orthogonality)** For each triple $(\phi, \psi, \chi)$ of core two-dimensional sentences in one of the three figures, the necessary and sufficient conditions for this triple to be a valid conclusion-optimal mood consist of two conditions, one of which says that the assertoric projection is valid and conclusion-optimal in assertoric logic, and the other refers only to the avicennan form of the triple.

There is enough evidence that Ibn Sīnā was well aware of this principle, at least as a heuristic.

By the Orthogonality principle, every (valid, conclusion-optimal) two-dimensional mood has an assertoric projection that is an assertoric mood. We can name the two-dimensional mood by naming its assertoric projection and then listing the avicennan forms of its sentences. Thus for example $\text{Barbara}(t,d,d)$, which is a two-dimensional mood by Fact 6.4 above, is $\text{Barbara}$ with a $(t)$ first premise, a $(d)$ second premise and a $(d)$ conclusion.

Internal justifications

**Fact 6.9** There are four valid two-dimensional moods in the $dt$ fragment where the internal justification of their assertoric projection by conversion or ecthesis doesn’t lift to the two-dimensional case. They are as follows:

- In second figure,
  
  $\text{Cesare}(d, t, d), \text{Camstres}(t, d, d), \text{Festino}(d, t, d)$.

- In third figure,
  
  $\text{Disamis}(t, d, d)$.

Fact 6.9 implies that the internal justifications using ecthesis can all be lifted to the $dt$ fragment. There are four such cases:

(54) $\text{Baroco}(t, d, d), \text{Baroco}(d, t, d), \text{Bocardo}(t, t, t), \text{Bocardo}(t, d, d)$.

The next Fact assures us of the ectheses needed in these four cases.
Fact 6.10 We have the following ectheses:

\begin{itemize}
  \item[(1)] \((o-t)(C, B) \vdash (i-t)(C, D), (e-\ell)(B, D)\)
    where \(Dx \equiv (\exists \sigma (Ex \land Cx \sigma) \land \neg Bx)\)
  \item[(2)] \((o-d)(C, B) \vdash (i-d)(C, D), (e-d)(B, D)\)
    where \(Dx \equiv (Cx \land \forall \sigma (Ex \to \neg Bx))\)
  \item[(3)] \((o-t)(B, A) \vdash (a-t)(D, B), (e-t)(D, A)\)
    where \(Dx \equiv (Bx \land \exists \sigma (Ex \land \neg Ax))\)
  \item[(4)] \((o-d)(B, A) \vdash (a-t)(D, B), (e-d)(D, A)\)
    where \(Dx \equiv (Bx \land \exists \sigma (Ex \land \neg Ax))\)
\end{itemize}

Fact 6.10 deserves three remarks. First, these ectheses are quite hard to find and check; they are as good examples as you can find of inference rules that are not self-evident.

Second, note the \(\ell\) in (1). By Fact 6.4 this \(\ell\) is needed for Celarent\((d, \ell, d)\); so we see that even operating the \(dt\) fragment sometimes requires us to use \(\ell\) sentences. This is not the only example of this phenomenon. Ibn Sīnā himself cites another at Isārāt 145.5–11.

Third, Street [43] p. 152 doubts that Baroco\((abs, nec, nec)\) can be proved by ecthesis. If Baroco\((abs, nec, nec)\) is read as Baroco\((t, d, d)\) then (1) of Fact 6.10 shows how the proof goes. But in Street’s paper Baroco\((abs, nec, nec)\) is treated as an alethic mood in an unspecified modal system, and in that setting I am not sure that the question whether this mood is provable by ecthesis need have a determinate answer.

6.3 The dt reduction

All the moods in the \(dt\) fragment can be derived by a reduction to assertoric logic. The reduction proceeds by changing the terms so that they include the references to time. We can call this method incorporation, in the sense that the terms are expanded to incorporate extra material. This is a move that Ibn Sīnā recognises and refers to quite often as ‘making (a modality) a part of the predicate’ (ju‘ila juz’an min al-mahmūl), for example at Muktaṣar 44b7, Najāt 37.1f, Qiyās 42.4f, 86.4, 130.11, Isārāt 98.13f. He speaks less often of making a modality a part of the subject; but this may be because he includes a time reference in the subject by default, reading ‘Every \(B\)’ as ‘Everything that was, is or will be a \(B\) at some time’.

To apply incorporation to the \(dt\) fragment, we introduce for every term, say \(A\), two new terms ‘always \(A\)’ and ‘sometimes \(A\)’, in symbols \(A^+\) and
Formal definitions are

\[ A^+ x : \forall \tau (E \tau \rightarrow A \tau) \]
\[ A^- x : \exists \tau (E \tau \land A \tau). \]

With these new terms we can translate any sentence of the \( dt \) fragment into an assertoric sentence, as follows:

<table>
<thead>
<tr>
<th>( dt ) sentence</th>
<th>assertoric translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a-d)((B, A))</td>
<td>(a)((B^-, A^+))</td>
</tr>
<tr>
<td>(a-t)((B, A))</td>
<td>(a)((B^-, A^-))</td>
</tr>
<tr>
<td>(e-d)((B, A))</td>
<td>(e)((B^-, A^-))</td>
</tr>
<tr>
<td>(e-t)((B, A))</td>
<td>(e)((B^-, A^+))</td>
</tr>
<tr>
<td>(i-d)((B, A))</td>
<td>(i)((B^-, A^+))</td>
</tr>
<tr>
<td>(i-t)((B, A))</td>
<td>(i)((B^-, A^-))</td>
</tr>
<tr>
<td>(o-d)((B, A))</td>
<td>(o)((B^-, A^-))</td>
</tr>
<tr>
<td>(o-t)((B, A))</td>
<td>(o)((B^-, A^+))</td>
</tr>
</tbody>
</table>

Together with these translations, we write \( \text{Th}(\pm) \) (the theory of plus and minus) for the set of all sentences of the form

\[ \forall x (A^+ x \rightarrow A^- x). \]

These sentences are provable from the theory of \( E \). Note that this reduction to assertoric logic is quite different from the assertoric projection.

One can show:

**Fact 6.11**

(a) The valid moods in the \( dt \) fragment are exactly those whose translations are provable (by compound syllogisms) in assertoric logic if we allow \( \text{Th}(\pm) \) as added premises.

(b) The optimal valid moods are exactly those whose translations are valid syllogisms in assertoric logic.

(c) The conclusion-optimal valid moods are exactly those whose translations are provable in assertoric logic if we allow as added premises the sentences of \( \text{Th}(\pm) \) for the terms which are predicates in the premises.

Fact 6.11 has a consequence that might be important for understanding Ibn Sinā. By the Fact, the laws of the \( dt \) fragment will apply to any other logical system that translates down into assertoric logic in the same way. So we should look at the reduction and see what it presupposes. Each term
A comes in two forms, a strong one $A^+$ and a weak one $A^-$; the strong implies the weak. In every sentence of the logic being reduced, the subject term is in the weak form. That’s all. In particular the reduction doesn’t assume anything along the lines that $A^-$ is the De Morgan dual of $A^+$ (as for example that ‘sometimes’ means ‘not always not’).

So Ibn Sīnā would get exactly the same valid moods as in the $dt$ fragment if he replaced ‘sometimes’ by ‘throughout every Tuesday’ and ‘always’ by ‘throughout every Tuesday and Thursday’, and then read his quantifiers as ‘Everything (or something) that is a $B$ throughout every Tuesday’. Or coming closer to Ibn Sīnā’s metaphysical interests, he could read $A^-x$ as ‘$x$ is a consistent meaning that is compatible with $A$’ and $A^+x$ as ‘$x$ is a consistent meaning that is incompatible with not-$A$’, and again he would get the same laws as those of the $dt$ fragment.

(Temporary note: At present [15] has $A^+$ and $A^-$ the other way round. I had reckoned that the one with $E$ positive should be $A^+$. But I now think it’s more intuitive the other way round. Sorry; this will be repaired.)

7 Ibn Sīnā reports the $dt$ fragment

7.1 Ibn Sīnā lists the moods

So now we have two kinds of ‘necessary’ sentence and two kinds of ‘broad absolute’ sentence. One kind is the alethic sentences with modality either $nec$ or $abs$, except where Ibn Sīnā indicates that he means some other kind of absolute. We will refer to this class of alethic sentences as the $nec/abs$ fragment of alethic modal logic. The other kind is the two-dimensional sentences with avicennan form $(d)$ or $(t)$; these are the ones that Ibn Sīnā himself refers to as ‘necessary’ or ‘broad absolute’. What is the relationship between the alethic and the two-dimensional versions?

We are going to do an experiment. First we will list, as list A, all the conclusion-optimal moods in the $dt$ fragment. Then quite separately from this, we will list, as list B, all the moods in the alethic $nec/abs$ fragment that Ibn Sīnā himself accepts. Then we will compare the two lists.

List A. By the Orthogonality principle (Fact 6.8), the list A need only list the avicennan forms, since the assertoric forms that go with them are determined by assertoric logic.

We take each figure in turn. For each figure we consider the four pairs $(d, d)$, $(d, t)$, $(t, d)$ and $(t, t)$. For each such pair $(h_1, h_2)$ we can check from
the appropriate one of Facts 6.4–6.6 whether the pair is productive, by looking to see whether there is a listed triple \((k_1, k_2, k_3)\) with \(h_1 \geq k_1\) and \(h_2 \geq k_2\). If there is such a triple, we look for the strongest value of \(k_3\) among such triples, and we call it \(h_3\). Whenever \((h_1, h_2)\) is productive, we count the triple \((h_1, h_2, h_3)\) as validated and we put it into List A.

First Figure:

<table>
<thead>
<tr>
<th>premise-pair</th>
<th>productive</th>
<th>strongest conc</th>
<th>validated triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((d, d, d))</td>
</tr>
<tr>
<td>((d, t))</td>
<td>Yes</td>
<td>(t)</td>
<td>((d, t, t))</td>
</tr>
<tr>
<td>((t, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((t, d, d))</td>
</tr>
<tr>
<td>((t, t))</td>
<td>Yes</td>
<td>(t)</td>
<td>((t, t, t))</td>
</tr>
</tbody>
</table>

Second Figure:

<table>
<thead>
<tr>
<th>premise-pair</th>
<th>productive</th>
<th>strongest conc</th>
<th>validated triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((d, d, d))</td>
</tr>
<tr>
<td>((d, t))</td>
<td>Yes</td>
<td>(d)</td>
<td>((d, d, d))</td>
</tr>
<tr>
<td>((t, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((d, d, d))</td>
</tr>
<tr>
<td>((t, t))</td>
<td>No</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Third Figure:

<table>
<thead>
<tr>
<th>premise-pair</th>
<th>productive</th>
<th>strongest conc</th>
<th>validated triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((d, d, d))</td>
</tr>
<tr>
<td>((d, t))</td>
<td>Yes</td>
<td>(m)</td>
<td>((d, t, m))</td>
</tr>
<tr>
<td>((t, d))</td>
<td>Yes</td>
<td>(d)</td>
<td>((t, d, d))</td>
</tr>
<tr>
<td>((t, t))</td>
<td>Yes</td>
<td>(t)</td>
<td>((t, t, t))</td>
</tr>
</tbody>
</table>

The \(m\) in Third Figure looks like a misprint. But we can check it with any third figure mood, say Datisi:

(61) Some sometime-\(B\) is a \(C\) throughout its existence.
    Every sometime-\(B\) is sometimes an \(A\).

What is the strongest core two-dimensional conclusion we can get in this figure, i.e. with subject \(C\) and predicate \(A\)? Answer:

(62) Some sometime-\(C\) is an \(A\) sometime while it’s a \(C\).
The ideal conclusion would be that some sometime-\( A \) is always a \( C \); but to get into third figure we need to convert this, and by Fact 6.11 the best conversion available is \((i-m)\)-conversion. It will be interesting to see what Ibn Sinā does with this case.

**List B.** Appendix B of [15] will give full references to the relevant passages of Muktaṣar, Najāt, Qiyāṣ and Iṣārāt. I checked Ibn Sinā’s text myself and then compared with Street’s list on page 160 of his paper [43]. Since our lists agreed in every detail, Street’s published list will serve here. Street puts the major premise before the minor, in the Latin style, so we need to reverse these two. He writes \( L \) for \( nec \) and \( X \) for \( abs \). Translating across into our present notation, we reach:

List B:

<table>
<thead>
<tr>
<th>First figure</th>
<th>((abs, abs, abs), (nec, abs, abs), (abs, nec, nec), (nec, nec, nec))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second figure</td>
<td>((nec, nec, nec), (nec, abs, nec), (abs, nec, nec))</td>
</tr>
<tr>
<td>Third figure</td>
<td>((abs, abs, abs), (nec, nec, nec), (abs, nec, nec), (nec, abs, abs))</td>
</tr>
</tbody>
</table>

**Results.** Under the mapping \( nec \mapsto d \) and \( abs \mapsto t \), the lists are identical except for the third figure case where List A has \((d, t, m)\). This discrepancy is completely accounted for if we suppose that Ibn Sinā is working within the \( dt \) fragment, so that he is looking not for strongest conclusions but for strongest \( dt \) conclusions. There is a reason to expect him to do this, namely the genetic hypothesis. By that hypothesis one should expect that the modality of the strongest conclusion is a modality of one of the premises. The triple \((d, t, m)\) is the only counterexample to that expectation.

With that proviso, the result of our experiment is that the two lists are a hundred per cent identical.

Readers of Street’s [43] will see that his listing of valid moods includes two other items; we should check that they don’t disturb the pattern. One is that he describes two of the first figure moods as ‘imperfect’. This refers not to their validity but to the justification that Ibn Sinā gives for them; we will return to this below.

The other is that Street includes two further moods with a sentence form that he labels \( A \); this form is what on his page 136 he describes as ‘perpetual \( (al-dā’ima) \)’. This form is a figment. Ibn Sinā has no such form; he does label some sentences as \( dā’im \), but these are the same sentences that he calls ‘necessary’, and very often he uses both labels together. The class of ‘perpetual’
sentences as a separate class was introduced a century and a half later by Rāzī (e.g. *Mulakkas* 184.2), as a conscious departure from Ibn Sīnā’s logic. Readers with no Arabic can confirm the point from Street’s own translations. On his page 146 the $A$ sentence is in a proof which he says is ‘not given in Avicenna’. On the next page he has an $A$ sentence in a proof said to be from *Najāt*; his translation at 2.2.2 on page 159 has no mention of perpetuity, and in fact the passage comes from a place where Ibn Sīnā is reporting Aristotle’s assertoric syllogisms (*Najāt* 63.9–11).

**Review** The first point to make is that these results are highly significant. The two lists were compiled from completely different data sets. List A was calculated from the semantics of a class of sentences described by Ibn Sīnā in the early parts of *Qiyās* and *Mašriqiyyān*. List B records Ibn Sīnā’s verdicts on alethic modal moods in other parts of Ibn Sīnā’s texts. Compare for example with tables of moods accepted by Latin scholastic logicians (such as Buridan, cf. [5] pp. 41–44). In those cases known to me, the Latin logicians present their material proof-theoretically, and the main thing that one can check is that they have followed their own proof rules correctly. This is not our situation, because the information in List A makes no appeal of any kind to Ibn Sīnā’s proof procedures. In this respect our results are more like, say, finding the right gear ratios in the Antikithera mechanism—though admittedly less startling than that case.

Nor are there any symmetries or obvious patterns in List A that could have led Ibn Sīnā to the information in List B by a happy accident.

Prima facie the results give strong support to the view that Ibn Sīnā, when he lists valid moods in the *nec/abs* fragment of alethic modal logic, is in fact reporting what is true in the *dt* fragment. But in view of the results of Subsection 6.3 above, we need to phrase this carefully. Ibn Sīnā is clearly working from some source that gives exactly the same valid moods as in the *dt* fragment. But are there other possible sources with this property?

The answer is certainly Yes. For example we would have the same List B in front of us if Ibn Sīnā was using modal predicate logic and reading $A^+x$ as $\Box Ax$ and $A^-x$ as $\Diamond Ax$. For the moment this particular suggestion is idle. Ibn Sīnā has already told us what sentences he is working with, namely the two-dimensional ones; and his insistence that he is quantifying only over actuals is hard to reconcile with the idea that his subject terms all take the form $\Diamond A$. But it’s best not to close this door before seeing more evidence.

Our second slice of evidence will consist of the internal justifications that Ibn Sīnā offers for *nec/abs* moods in second and third figures. Do
these agree with what is reported in Facts 6.9 and 6.10 about what methods of internal justification are available for the \( dt \) fragment? Do they throw any other light on the kind of sentences that he thinks he is dealing with?

We should note two other conclusions that can be drawn from the precise agreement of Lists \( A \) and \( B \). One is the mundane but reassuring point that Ibn Sīnā is indeed considering only conclusion-optimal moods, as we have been supposing. This is reassuring because he never says explicitly that this is what he is doing.

The other conclusion is that Ibn Sīnā was capable of sustained and accurate work in formal logic, including work in areas that had not been considered before. (This conclusion will have to lapse if it turns out that Ibn Sīnā knew a work in which Galen had already described the \( dt \) fragment, but I don’t suppose anybody expects this.) The results create a presumption that Ibn Sīnā’s other claims in formal logic should also be taken seriously.

7.2 Ibn Sīnā checks the internal justifications

We review the justifications that Ibn Sīnā gives for second- and third-figure syllogisms in the \( nec/abs \) fragment. The passages in question are \( Muktaṣar \) 54a1–55a14, \( Najāt \) 67.1–68.9, \( Qiyaṣ \) 130.4–159.16 and sections of \( Isārāt \) 147.10–153.2.

Some of the material in these sections is irrelevant to our purpose. There are sections that report and discuss what is in Aristotle and his commentators. There are sections that simply list what moods Ibn Sīnā accepts; we took these into account in the previous section. \( Qiyaṣ \) iii.1 and iii.3 contain long digressions on sentences with wide time scope; for the present I am not counting these as part of the \( nec/abs \) fragment. In \( Isārāt \) the things that we are looking for are mixed up with some material on other modalities.

When these irrelevances are removed, virtually all of what remains falls into three groups:

(i) Discussion of proofs of \( Baroco \) and \( Bocardo \) by ecthesis or contraposition. This occupies \( Muktaṣar \) 54a17–54b7 (Baroco) and 55a10–13 (Bocardo); \( Najāt \) 69.9–12 (Bocardo); \( Qiyaṣ \) 159.6 (Bocardo); \( Isārāt \) 152.10–153.2 (Bocardo).

(ii) Discussion of the proof of \( Disamis(\text{abs, nec, nec}) \). This occupies \( Muktaṣar \) 55a14; \( Najāt \) 69.12f; \( Qiyaṣ \) 158.3; \( Isārāt \) 152.1–4.

(iii) Discussion of the proofs of \( Cesare, Camestres \) and \( Festino \) where one
premise is nec and the other is abs. This occupies Muktaşar 54a8–16; Najât 67.8–68.9; Qiyâs 130.10–132.14.

In some cases wujûdî sentences are mentioned too.

We note at once that by Fact 6.9 and (54) these are exactly the places where the justifications in the assertoric case don’t carry over straightforwardly to the dt fragment. Anybody who wants to claim that Ibn Sînà is doing something other than reporting the situation with the dt fragment will need to show that these three topics are also an appropriate choice of topics for Ibn Sînà to discuss in relation to that something other. For example if Ibn Sînà is following not dt but its reduction to assertoric logic, then none of these three topics will need special discussion, because the assertoric proofs are already adequate. If he is following some version of modal predicate logic, then we need to be shown what kinds of rule he is using, and how these rules produce the same problems as the adaptations of the assertoric rules to the dt fragment.

Proofs of Baroco and Bocardo by ecthesis

Ibn Sînà is already using ecthesis for these cases in assertoric logic. The ecthetic proofs adapt to the cases listed at (54). What is not routine is to find ectheses that work in the dt case, as in Fact 6.10. In fact Ibn Sînà never spells out the ectheses for all four cases. He outlines the proofs for Baroco(nec,ab,nec) at Muktaşar 54a17–54b3 using alethic modal language, and for Bocardo(abs, nec, nec) at Muktaşar 55a10–12, again in alethic modal language. The alethic language is not well set up for specifying the ecthetic term; for example at Muktaşar 54b1f he says that D is what is a C and not an A; but he needs ‘what is a C and necessarily not an A’. For the Bocardo case he doesn’t even attempt to give a full description of D. The proof of Bocardo(abs, nec, nec) at Najât 69.8–13 likewise gives an inadequate explanation of D. The treatment of Bocardo(abs, nec, nec) at Qiyâs 159.6f doesn’t even attempt a description of the proof. At Isârât 153.1f there is an incomplete description of D, followed by an instruction to the reader to complete the argument. In none of these texts does Ibn Sînà attempt a description of D for Baroco(abs, nec, nec), the case where $\ell$ is needed.

I suspect that the reason why Ibn Sînà is not more forthcoming about these ecthetic terms is that he didn’t know how to be more precise. Our descriptions of them use two variables, but variables in this style were not part of Ibn Sînà’s tool-kit. He was after all the first logician to work with a logic where every sentence has two quantifications. He could reasonably reckon that a description like ‘what is a C and not an A’, so far as it goes,
is self-evidently in the right area to make the proof work, and he had no formal apparatus for taking the matter any further. But see also what he does in the proof of Disamis below.

**Proof of Disamis(abs,nec,nec)**

Ibn Sīnā mentions this case in all four sources as something needing special treatment. The statement in Muktaṣar is very brief and barely says more than that ecthesis will give us what we want. The account in Iṣārāt 152.1–5 is fuller, essentially as follows:

\[
\begin{align*}
(a-t)(B, C) & & (i-d)(B, A) & \text{(ecthesis)} \\
(a-t)(D, B) & & (a-d)(D, A) & \\
(a-t)(D, C) & & (i-d)(C, A) & \text{(conversion)} \\
(i-t)(C, D) & & & \text{(Darii}(t,d,d))
\end{align*}
\]

The proof is given with no modalities. As in the cases discussed just above, Ibn Sīnā specifies \( D \) with the inadequate description ‘some \( B \) that is an \( A' \), but he adds at once that this should be adjusted so as to prove a conclusion with the same modality as the second premise. (In fact the definition

\[
D \equiv (Bx \land \forall \sigma (Ex \rightarrow Ax\sigma))
\]

works here.)

Ibn Sīnā gives this same proof by ecthesis at Qiyāṣ 118.7–9 in his treatment of assertorics and absolutes; it is needed there to cope with the case where the second premise is \( wujūdī \). He refers back to this proof at Qiyāṣ 226.16 for Disamis(mum,nec,nec). Aristotle had already mentioned that there is a proof of Disamis by ecthesis, but we don’t know what he used it for.

**Cesare, Celarent and Festino**

This is the most interesting case. Street [43] p. 148 comments that Ibn Sīnā finds that these moods have a necessary conclusion ‘without however giving the proofs’. To my eye this is not correct; in Qiyāṣ Ibn Sīnā gives two
proofs for this result. But both proofs are odd, and one can see how they might be missed.

The first proof that Ibn Sinā proposes for these moods is by incorporation, Qiyās 130.11–131.3. The proof that he points to is as follows for Festino(nec, abs, nec):

\[
\begin{align*}
(i\text{-nec})(C, B) & \quad (e\text{-abs})(A^-, B^+) \\
(i)(C^-, B^+) & \quad (e)(A^-, B^+) \\
(o)(C^-, A^-) & \quad (e)(B^+, A^-) \\
(o\text{-nec})(C, A) & 
\end{align*}
\]

Yes, that works. So do the other two. Ibn Sinā’s description concentrates on the process of incorporating; some of his text here is incoherent and looks like rough notes to be sorted out later. Possibly Ibn Sinā had it in mind to put a better account into his Appendices (Qiyās 139.1), which as far as we know were never written.

Although this proof is watertight and completely solves the problem of these second-figure moods, there may be reasons why Ibn Sinā would be reluctant to rest with this use of incorporation. There are several places where he indicates that he dislikes justifying a logic by reduction to another logic. We quoted one at (11) above. Also the method is not in Aristotle—though one might cite Prior Analytics i.35, 48a29–39 to show that Aristotle was aware of the possibility. And third, Ibn Sinā has no formal procedure to regulate the kinds of paraphrase that can be used. Over eight hundred years later, Frege was to condemn this lacuna as one of the major faults of the old Aristotelian logic.

The second proof is described in all of Muktaṣar, Najāt and Qiyās. It has a different character from all the proofs that we have considered so far in this paper. A modern logician would describe it as a ‘proof by handwaving’. It doesn’t describe what you need to write on the page to reach the required conclusion. Instead it proposes a way in which the student can look at the data, with a hope that this will convince the student. (If the student isn’t convinced, he should try other books [sic] where Ibn Sinā goes into more detail, Najāt 68.3f.) As Ibn Sinā expresses it in Muktaṣar 54a13, if every \( C \) is a \( B \) with necessity, and no \( A \) is a \( B \), then there is an ‘essential distance’
(bawn dā'tī) between the natures of $C$ and $A$. There is no more content in this argument than there was in the incorporation argument, but the reference to essence and nature is a cue to the reader that we might here be formulating a principle on the basis of First Philosophy. Let me call this principle the Essential Distance principle and leave it there for the moment. (If I said any more, it would be that the principle looks a very plausible origin for Suhrawardi’s Illuminationist Second Figure Principle [44] 23.6ff.)

A further point to mention is that a large part of the discussion in this part of Qiyās is explicitly in the temporal language of two-dimensional logic. For example this applies to the whole of the discussion at Qiyās 131f where Ibn Sīnā introduces the problem about adapting the assertoric justification to Cesare($d,t,d$).

**Failed moods**

We should note what Ibn Sīnā says about the failed moods when all the sentences are absolute. For example at Muktasar 51b15 he says that the ecthetic proof in second figure doesn’t work because the proof ‘reduces to proof through the same figure’. The only place where he used ecthesis in second figure in the assertoric case was to prove Baroco. We can verify from DIAG ABOVE that if we try to copy this proof for Baroco($t,t,t$), then the ecthesis rule works only in the form

\[(67) \quad (o-t)(C, B) \vdash (i-t)(C, D) \text{ (or } (i - t)(D, C)), (e-t)(D, B)\]

so that the next step would be to maka a deduction from

\[(68) \quad (e-t)(D, B), (a-t)(A, B)\]

which is again in second figure. He also remarks that the proof by contraposition fails because no contradiction is found. For Baroco($t,t,t$) the argument by contraposition would draw a conclusion from the second premise $(a-t)(A, B)$ and the contradictory negation of the conclusion, i.e. $(a-d)(C, A)$. But the optimal conclusion from this premise-pair in two-dimensional logic is $(a-t)(C, B)$, which doesn’t contradict the first premise $(o-t)(C, B)$. So his brief remark in Muktasar is an exact description of the failure of two methods for proving Baroco($t,t,t$). (There is no corresponding remark in the treatment at Najāt 60.10–61.4, or at Qiyās 116.7–12.)
7.3 Conclusions so far

In this section we have assembled evidence for the following conclusion:

**Conclusion One.** In his reports of the valid syllogisms of the nec/abs fragment of alethic modal logic, and his justifications of the second- and third-figure syllogisms in this fragment, Ibn Sinā is taking his information from the corresponding facts about the dt fragment of two-dimensional logic.

It may be helpful to review that evidence in its proper context.

By far our most complete account of Ibn Sinā’s logic is his *Qiyās*. In that book, after two introductory sections on the nature of logic, Ibn Sinā begins his account of logic itself in *Qiyās* i.3 by introducing his two-dimensional sentences. From that point onwards there is a continuous development, at least up to the end of *Qiyās* iv where he concludes his treatment of modal logic. (I believe that the continuous development actually runs up to the end of *Qiyās*, including the propositional logic, but this is a battle for another day.) So it makes no sense to try to understand his treatment of modal logic without studying the material that leads into it, particularly since Ibn Sinā himself blends the alethic modal logic and the two-dimensional logic into each other.

This view, that the two-dimensional logic is the basis for what follows it in Ibn Sinā’s logic, is reinforced by *Mašriqiyān*, written a few years later than *Qiyās*. What we have of *Mašriqiyān* is clearly incomplete, but it roughly matches *Qiyās* i, and its whole treatment of propositions is dominated by the two-dimensional sentences, which are treated very systematically. The role of the two-dimensional sentences is not so clear in the earlier *Muktaṣar* and *Najāt*, but they are certainly present, and *Muktaṣar* in particular gives us a picture of how Ibn Sinā sifts out the temporal logic as he forms his own understanding of the alethic modal logic in the Peripatetic tradition.

So we checked the logical properties of the two-dimensional sentences, regarded as an extension of the system of assertoric logic as Ibn Sinā himself presented it. We found that what Ibn Sinā says about the validity of moods, and the internal justification of second- and third-figure moods, in the alethic modal logic of necessary and broad absolute, is an exact match for what is in fact true about the two-dimensional logic of (d) and (t) sentences, which are those two-dimensional sentences that Ibn Sinā himself describes as respectively ‘necessary’ and ‘broad absolute’.
It may yet turn out that what Ibn Sīnā says about this part of alethic modal logic is an equally close match to some other system of logic that we have not considered here. But that would be a very remarkable fact, given how close the match is in both results and methods, and given the fact that the two-dimensional logic is at present the only visible candidate. The fact that some of Ibn Sīnā’s discussion of moods that he introduces in alethic form is conducted using two-dimensional sentences wholesale is also a piece of relevant evidence.

In fact the match is so good that it provides strong evidence of Ibn Sīnā’s accuracy and originality in logic, putting his professional skills at a level at least comparable to that of any logician known to us between Aristotle himself and the nineteenth century. (This is of course not to say that he never got anything wrong.)

**Conclusion Two.** Ibn Sīnā in his treatment of alethic modal logic works with two classes of sentence, though they are not always clearly distinguished. One is alethic modal sentences in the style of the Arabic Aristotle, and the other is Ibn Sīnā’s own two-dimensional sentences.

This second conclusion seems to me inescapable from a reading of the texts. In a perceptive paper on Ibn Sīnā’s modal logic, Asad Q. Ahmed [2] observes that Ibn Sīnā discusses alethic modal sentences in the Aristotle style, and contrasts these sentences with what he calls Ibn Sīnā’s ‘peculiar manner of reading’ some of these sentences (p. 21). This peculiar manner turns out to be what we have been calling the \((a\cdot d)\) and \((e\cdot d)\) sentences. In a footnote on the same page, Ahmed refers to Ibn Sīnā’s ‘several different ways of looking at a proposition’. These two-dimensional sentences are surely not just ways of reading alethic sentences; they are sentences in their own right. This should be clear from a study of *Qiyās* 21–23, where Ibn Sīnā provides a number of scientific statements (some taken from biology, geography, physics and astronomy) as illustrations of the two-dimensional forms. From this passage the presumption should be that Ibn Sīnā selects the two-dimensional forms because they illustrate logically significant features found in normal scientific discourse. If these ‘peculiar manners of reading’ are recognised as sentences in their own right, then Ahmed’s account falls into line with Conclusion Two.

Ahmed says on his opening page (p. 3) that Ibn Sīnā is ‘trying to find an interpretation of the theory [of modal syllogisms] amenable to Aристо-
tle’s conclusions’. This is said before any evidence is presented, and it may be one of the assumptions with which Ahmed has approached the text. Certainly people have made such an assumption. But our evidence so far has to count against this assumption. In every case where Ibn Sinā notes a difference between his own views and those of Aristotle, his own views coincide with what is true in the dt fragment. There is never any contest; the dt fragment wins and Aristotle loses every single time. I will put this down as a third conclusion:

**Conclusion Three.** Ibn Sinā’s account of the alethic modal logic of necessary and broad absolute is not an attempt to interpret or accommodate Aristotle.

Ahmed also remarks (p. 22): ‘As there are different manners of construing a premise, the same syllogism will sometimes yield one conclusion, sometimes another’. This is a very interesting remark, because it points to the dog that didn’t bark in the night. We have seen no single case where Ibn Sinā presents a syllogism in necessary and broad absolute, and finds that its correctly deduced conclusion is different from the conclusion of the corresponding dt syllogism. That suggests a conjecture, which for the moment is only temporary:

**Conjecture.** For Ibn Sinā, the logical truths of the nec/abs fragment are identical to those of the dt fragment under the mapping nec → d and abs → t.

Our next section will address this conjecture.

Meanwhile let me mention some pieces of evidence which I take as crucial for understanding Ibn Sinā’s logic, and which seem to have been widely neglected in the existing literature.

(1) The first is Ibn Sinā’s own statements about the nature of logic and his intentions in his various writings on logic. His own statements of his intentions in the Šifā’ and Mašriqīyyūn in his preface to the Šifā’ are partic-
ularly helpful. To quote Gutas’ translation:

I also wrote another book . . . , in which I presented philosophy as it is naturally [perceived] and as required by an unbiased view which neither takes into account [in this book] the views of colleagues in the discipline, nor takes precautions here against creating schisms among them as is done elsewhere; this is my book on Eastern philosophy. But as for the present book, it is more elaborate and more accommodating to my Peripatetic colleagues. Whoever wants the truth [stated] without indirec-

tion, he should seek the former book; whoever wants the truth [stated] in a way which is somewhat conciliatory to colleagues, elaborates a lot, and alludes to things which, had they been per-
ceived, there would have been no need for the other book, then he should read the present book. (Maḏkal 10.11–17, trans. Gutas [11] p. 44f)

Of course it would be naive to take all Ibn Sīnā’s statements about himself at face value. But here he is describing his intentions, not boasting of his achievements, so there is less likelihood of distortion. In any case a reading of Qiyās and Mašriqīyyūn will confirm what he says about their relationship. (I have to state this as bald fact. At the time of writing I don’t have any evidence that anybody else since A.-M. Goichon has actually advanced any further into Mašriqīyyūn than the prologue.) Ibn Sīnā’s description of the bias in Šīfā’ will be helpful to us below.

(2) The second is the reconstruction of the logical content of Ibn Sīnā’s two-dimensional logic, and of his logical proof methods. There are two in-

gredients to this. One is the calculation of the logical properties of Ibn Sīnā’s two-dimensional sentences, as reported in Subsection 6.2 above. There is no reason why any of the information in that subsection should not have been available to Ibn Sīnā, by hands-on calculation of the kind that he boasted of doing. The one proviso, as stated earlier, is that he seems to have had difficulties specifying the ethtetic formulas precisely, given the logical tools at his disposal.

The second ingredient is his formalism for giving formal proofs. This has to be partly conjectural since he never gives a full exposition of it. But he does give enough hints to allow us to put together a plausible picture; I plan to give a fuller account in a book [16] with Amrouche Moktefi. This ingredient is less crucial for the results of the present paper, but it does play a supporting role.
(3) Mainly as a result of (1) and (2), we bring to bear a wider range of Ibn Sinā’s writings than is usual. Our use of *Qiyās* book i and its companion *Mašriqiyān* is one example, our use of material from *Burhān* and *Ta’līqāt* on Ibn Sinā’s notion of logic as a science is another. It was also good to have access to *Muktaṣar*, which has undoubtedly helped to fill out the picture.

On the other hand we make no use at all of the views of later Arabic logicians as evidence for the views of Ibn Sinā. The reason for this is very simple: they are *not* evidence for the views of Ibn Sinā. The one possible exception to this is Bahmanyār, who was Ibn Sinā’s student; we will discuss his input in BELOW.

I would add: the relationship between the logic of Ibn Sinā and that of his Arabic-speaking successors is an important question both for the history of logic and for understanding medieval Arabic culture. To study this relationship we need to have an account of Ibn Sinā’s views which is not contaminated with views of those successors.
References


Arabic terms

akbar, 7
aks, 8
caks, 2
al¯ a '¯ ıj¯ abih, 37
al-falsafa al-¯ ala, 15
c¯ amm, 28
carađ, 24
as̱ gar, 7
awsat, 7
bawn ḏ aṯ ı, 52
bayyin bi-nafsih, 18
bil fi̱ l, 33
burhān, 16
da'ım, 47
darb, 7
daruri, 28, 30
ḏ āt, 31
fušliyya, 24
gayr mu'ayyan, 21
ha'dd, 6, 9
hāl, 24
haqiqi, 28
cibra, 14
'iḍāfa, 25
iṭtiřād, 21
cilm, 15
intiḥān, 17
istikrāj, 17, 38
jawhar, 31
jinsiyya, 24
juz' min al-mahmul, 42
juzʾ, 5, 24
kabīr, 7
kalf, 8
kāmil, 8
kamm, kammīyya, 5
kāss, 24, 28
kayfa ittafaqa, 34
kayfa kāna, 34
kayfa, kayfiyya, 6
kubrā, 7
kulür, 5, 15, 24
lāzim, 24, 30
mabda’, 15
māhiyya, 19, 22
mahmūl, 6
majhūl, 23
ma'ālum, 23
ma'ānā, 19
ma'qūl, 19
mas’ala, 15
mašhūr, 6
maṭlūb, 7
mawdūʿa, 6
mawjud, 30
mu'ayyan, 21
muhmal, 5
mūjīb, 6, 24
mumkin, 24, 28
munfaṣil, 17, 32
muntij, 7
muqaddama, 7, 24
muṭlaq, 24, 28
muttaṣil, 32
nāqīd, 6
nati̱ ja, 7
nawṣiyya, 24
qānnūn, 15
qarına, 7
qiyas, 7
raft al-kalām, 8
raja'a, 8
ra's, 7
rujū' al-sālibih, 37
ṣağır, 7
šakl, 7
šakšt, 5, 24
sālib, 6, 24
šarā'īt al-'intaj, 13
šinā'ī, 15
šinā'ī naẓariyya, 15
ṣugrā, 7
ṣūra, 20
tajriba, 17
ta'līf, 23
ṭaraq, 7
tunāqiṣṭu, 24
tuṭbatu, 24
ulhiqa, 8
wājib, 24
wuḍā'a, 8
wuṣūd, 22
wuṣūd, 30, 37, 49, 50
yatba'ī, 14
yufrāḍu, 8
yukmālu, 8
zāmānī, 37
English terms

absolute, 24, 28, 29
  broad, 44
  broad ~, 30, 32
  narrow ~, 30
absurdity, 8
accident, 23, 24
accidental, 25, 26
accidental v. essential, 16
actual, 33, 34
affirmative, 24
alethic modal, 26, 28, 31, 38, 49
alethic modal logic, 53
all, 6, 23
aristelian form, 38
aristotelian form, 36
art, 15
assertoric, 6
  ~ logic, 6
  ~ projection, 39
  ~ sentence form, 6
assertoric projection, 37, 43
astronomy, 54
attachment, 8
augment, 37
  existential ~, 11
  universal ~, 11
avicennan form, 36, 38, 39
  core ~, 36
biology, 26, 34, 54
broad, 28

category
  Aristotelian ~, 25, 26
  modal ~, 3, 26
composition of meanings, 23
cclusion, 7
  strongest ~, 11, 14, 22

  conclusion-optimal, 11
condition
  ~ of productivity, 13
condition of productivity, 18, 19
contingent, 28
contradictory negation, 6, 24, 26, 32, 38
contraposition, 8, 12, 52
conversion, 8, 21, 34, 41
core two-dimensional form, 36

De Morgan dual, 44
deduction, 23
definition, 23, 25
demonstration, 16
denyng the statement, 8
differentia, 24
dominance, 14
double-dot form, 37
double-dotting, 37
dt fragment, 38, 42, 46, 49, 53

cethesis, 8, 12, 21, 41, 42, 49, 50, 56
empiricism, 17
essential, 26
essential distance, 52
existential, 24
extreme, 7
feature
  ~ of meaning, 23
figure, 7
First Philosophy, 15, 16, 18, 24–26, 52
following
  rules of ~, 13
follows, 14
form
  ~ of argument, 20
proof
  by handwaving, 52
proof theory, 56
proprium, 24

quality, 6
quantifier, 34
  object ∼, 32
quantity, 5
quiddity, 19

reduce, 8
reduction, 42, 51
relation, 25
rule, 15
rule of following, 18, 19

science, 15, 22
second kind of existence, 22, 27
self-evident, 9, 42
sentence
  (a), (e), (i), (o), 6
  affirmative ∼, 6
  assertoric ∼ form, 6
  concrete ∼, 6
  existential ∼, 5
  formal ∼, 6
  munfasıl ∼, 32
  muttasıl ∼, 32
  negative ∼, 6
  particular ∼, 5
  singular ∼, 5
  two-dimensional ∼, 32, 35
  universal ∼, 5
  unquantified ∼, 5
  ut nunc ∼, 37
singular, 24
some, 6, 23
species, 24, 25
stipulate, 8
strict, 28

subject, 6, 23
  ∼ individual, 7
  ∼ term, 7
subject term
  ∼ of a science, 20
  ∼ of logic, 19, 22
substance, 31
substantial, 25
syllogism, 7
  Barbara, 13, 18
  Baroco, 12, 13, 41, 49, 52
  Bocardo, 13, 21, 41, 49
  Camestres, 41, 49
  Celarent, 12, 50
  Cesare, 41, 49, 50, 52
  conclusion-optimal, 11
  Darapti, 14, 37
  Disamis, 41, 49, 50
  Felapton, 14, 37
  Ferio, 12, 13
  Festino, 41, 49–51

term, 6
terms
  method of ∼, 9
that v. why, 16
theorem, 15
time, 4, 32, 35, 52
time quantifier, 34
truth, 26
truths of logic, 15, 18, 22, 23, 25,
  26, 44, 55
  existential ∼, 16
  internally justified ∼, 18
  negative ∼, 17
  self-evident ∼, 18, 20, 21
two-dimensional, 27, 32
two-dimensional form, 36
  core ∼, 36
<table>
<thead>
<tr>
<th>Term</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>universal</td>
<td>24</td>
</tr>
<tr>
<td>valid</td>
<td>11</td>
</tr>
<tr>
<td>variable</td>
<td>19, 50</td>
</tr>
<tr>
<td>object</td>
<td>~, 35</td>
</tr>
<tr>
<td>time</td>
<td>~, 35</td>
</tr>
<tr>
<td>whatness</td>
<td>19, 22</td>
</tr>
</tbody>
</table>
Cited passages of Ibn Sīnā

Appendices, 51
Aqsām al-ṭulūm
  112.15–17, 15
Burhān
  87.10–12, 26
  87.13, 16
  118.18, 16
  155.1–7, 15
  166.1f, 15
  179.12f, 16
  188.8, 16
  188.8f, 15
  220.8, 15
Dānešnāmeh, 10
  67.5–80.2, 11
  78.4f, 21
Hayawān
  159.7, 14
Hudūd
  Def 15, 31
Ibāra
  43.6–44.9, 26
  79.11–80.12, 11
Ilāhiyyāt
  5.7f, 15
  10.17–11.2, 19
Iṣārat, 10, 29, 34
  i.4.2
    89f, 4, 16, 31
  i.4.5
    93.10.12, 33
  i.4.7
    98.13f, 42
  i.5.5
    94.16, 26
Jadal
  21.11, 15
Madkal
  10.11–17, 56
  10.16, 9
  15.1–7, 23
  15.3, 22
  15.4–7, 19
  15.16, 24
  15.19f, 22
  15.5f, 23
  16.1, 22
  22.10–12, 24
  34.5–16, 22
  34.7–9, 22
  34.13, 22
Maqūlāt
  3.13–4.1, 25
  4.15f, 22
  5.1–8.15, 25
  5.1–9, 15, 25
  5.7–9, 25
  29.11, 25
  38.3–5, 25
  62.11, 25
  87.2, 25

68
106.3, 25
118.15, 25
145.3, 25
152.13, 25

Mas’ā’il
103.12–14, 17, 51

Mašriqiyūn, 1, 10, 36, 53, 56
10.15, 19, 23
10.15–19, 26
48.6f, 26
65.13f, 30
68.3, 33
68.3–5, 32
68.6f, 33
71.14f, 31
77.1–6, 30
79.1–3, 30
79.14f, 30
80.14–20, 37
82.13, 26

Muktaşar, 9, 29, 53
40a5, 34
40a10–44a10, 33
44b7, 42
49b09–53a06, 11
51b15, 52
51b3f, 21
54a1–55a14, 48
54a8–16, 49
54a13, 52
54a17–54b3, 49
54a17–54b7, 49
54b1f, 49
55a10–12, 49
55a10–13, 49
55a14, 49

Najāt, 9, 29, 53
8.8, 15
37.1f, 42

57.1–64.3, 11
60.10–61.4, 52
61.11f, 21
63.9–11, 47
67.8–68.9, 49
68.3f, 52
69.8–13, 49
69.9–12, 49
69.12f, 49
135.12–136.3, 20

Qiyās, 9, 29, 53
i.1
4.4, 15
i.2
10.11f, 15
11.11–12.2, 16
11.17, 18
13.6f, 15
i.3, 1, 36
21–23, 54
21.6–10, 33
22.3, 31
i.4
30.10, 34
i.5
40.16, 30
41.4–13, 30
42.4f, 42
48.14–17, 5
i.7
71.1, 20

Najāt, 9, 29, 53
ii.1
77.3, 31
86.4, 42
ii.2
91.2, 31
ii.4, 32
108.10, 18
108.12–119.8, 11
109.11–13, 5  
116.7–12, 52  
116.10f, 21  
118.07–09, 50  
iii.1  
128.14, 30  
130.4, 30  
130.10–132.14, 49  
130.11, 42  
130.11–131.3, 51  
130.11f, 43  
131f, 52  
134.11, 34  
139.1, 51  
iii.1–3  
130.4–159.16, 48  
iii.3  
158.3, 49  
159.6, 49  
159.6f, 49  
iii.4  
162.8, 30  
iii.5  
174.6, 37  
iv.1  
118.2f, 19  
iv.4  
208.17, 37  
iv.6  
226.16, 50  
vi.1, 32  
vi.4  
296.1–304.4, 11  
viii.3, 12  
ix.6, 38  

expired  

Tā‘līqat  
502.4–505.12, 24  
503.3, 24  
504.11, 24  
506.9–11, 24  
507.4, 24  

cUyun al-hikma  
50.2f, 14
People

Ahmed, A., 9, 54
Alexander of Aphrodisias, 19, 27, 29
Alexandrian logicians, 27
Arabic logicians
later ~, 57
Aristotle, 2, 5, 6, 11, 13, 20, 22, 28, 48, 50, 51, 54, 55
Arabic ~, 5, 6, 8, 9, 13, 28, 54
Bahmanyār, 57
Bertolacci, A., 16
Buridan, Jean, 47
Bybee, J., 3
Chatti, S., 2
al-Fārābī, 31
Fleischman, S., 3
Frege, G., 51
Galen, 48
Gentzen, G., 21
Goichon, A.-M., 31, 56
Gutas, D., 9, 10, 17, 25, 56
 Hunayn b. ʿIšāq, 5
Inati, S., 4, 10, 33
Irāq b. Ḥunayn, 5
Jabre, F., 5
Kalbarczyk, A., 9
Lameer, J., 5, 28
Latin logicians, 6, 7, 37, 46, 47
Lukasiewicz, 27
Marmura, M., 22
McGinnis, J., 17
Mitchell, O., 32
Moktefi, A., 56
Movahed, Z., 2
Peripatetic logicians, 7, 9, 32, 34, 53, 56
Philoponus, John, 13
al-Rāzī, Fakhr-al-Dīn, 2, 47
Street, T., 21, 42, 46, 47, 51
Suhrawardī, 52
Themistius, 27, 29, 32
Theodorus, 5
Theophrastus, 27, 29, 32
Thom, P., 21, 31, 33
Venema, Y., 3
Von Wright, G., 3
Rules and principles

Conditions of productivity, 13
Essential Distance principle, 52
Genetic hypothesis, 14, 46
Nonexistents have no positive properties, 36
Orthogonality principle, 41, 44
Rules of ethesis, 21
Rules of following, 13
Second figure principle (Suhrwardi), 52