Identifying Ibn Sīnā's hypothetical sentence forms

Wilfrid Hodges
Herons Brook, Sticklepath,
Okehampton, Devon EX20 2PY, England
wilfrid.hodges@btinternet.com

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1 Hypothetical sentence forms

In his logical writings Ibn Sīnā introduces some sentence forms which he calls *sharṭī* (literally 'conditional'), and he presents argument forms that use these sentence forms. We refer to this part of his logical work as his 'hypothetical logic', so that Ibn Sīnā's 'hypothetical sentences' will be the *sharṭī* sentences studied in this logic. (Some writers say 'propositional' in place of 'hypothetical'.)

Ibn Sīnā says in his *Mašriqiyyūn* 61.7–17 that logicians use the word *šarṭī* as if it meant compound sentences with subclauses which don't make an affirmation or a denial when the whole sentence is uttered. This description includes standard conditional sentences of the form 'If p then q', but it includes many other sentences too. Ibn Sīnā distinguishes the hypothetical sentences from the predicative (*ḥamlī*) sentences which typically have forms like 'Every A is a B' or 'Some C is not a D' as in Aristotle's categorical syllogistic. (Predicative sentences also include modalised versions of the categorical sentences, but this will mostly be irrelevant below.)

Comparison with his predecessor al-Fārābī makes it clear that Ibn Sīnā took over some features of his hypothetical logic from earlier Peripatetic logicians. For example he took over a division of all hypothetical sentences into two mutually exclusive kinds, called in Arabic *muttaṣil* and *munfaṣil*. I won't attempt a translation of these two terms until we have retrieved a clear picture of the meanings that Ibn Sīnā had in mind for the sentences involved. In al-Fārābī the main *muttaṣil* sentences are 'If … then' sentences, and the main *munfaṣil* sentences are 'Either … or' sentences.

Besides these inherited features, Ibn Sīnā's hypothetical logic contains other features that seem to be Ibn Sīnā's own invention. One of these is that both the *muttaṣil* and the *munfaṣil* sentences are classified as universal (*kullī*) or existential (*juz'ī*), and as affirmative (*mūjib*) or negative (*sālib*), just like the sentences of categorical logic. It will be convenient to use a later Scholastic shorthand, writing a for 'universal affirmative', e for 'universal negative', i for 'existential affirmative' and o for 'existential negative'. In fact we can conveniently list the eight hypothetical forms as

(1) $(a, mt),\ (e, mt),\ (i, mt),\ (o, mt),\ (a, mn),\ (e, mn),\ (i, mn),\ (o, mn).$

with *mt* for *muttaṣil* and *mn* for *munfaṣil*. Besides universal and existential, the categorical sentences include some described as 'singular' (*shakhsī*) and some described as 'unquantified' (*muhmal*). Ibn Sīnā recognises counterparts of the singular and unquantified sentences in his hypothetical logic, but in his discussions of formal proofs they are practically invisible compared with the universal and existential sentences.

So what are Ibn Sīnā's eight main types of *muttaṣil* and *munfaṣil* sentences? We know well enough how Ibn Sīnā wrote and spoke them in Arabic; they are sentences of the following eight forms, with their symbolic shorthand on the left:

(2)

$(a, mt)(p, q)$ *kullamā kāna* p *fa* q.

$(e, mt)(p, q)$ *laysa albatta iḏā* p q.

$(i, mt)(p, q)$ *qad yakūnu iḏā kāna* p *fa* q.

$(o, mt)(p, q)$ *laysa kullamā* p q.

$(a, mn)(p, q)$ *dā'iman immā an yakūna* p *aw* q.

$(e, mn)(p, q)$ *laysa albatta immā* p *wa-immā* q.

$(i, mn)(p, q)$ *qad yakūnu immā an yakūna* p *aw* q.

$(o, mn)(p, q)$ *laysa dā'iman immā* p *wa-immā* q.

There are some variants that seem to be purely stylistic. Thus Ibn Sīnā sometimes puts *fa* before q in the *muttaṣil* sentences and sometimes he leaves it out. In the *munfaṣil* sentences he sometimes writes 'or' as *aw* and sometimes as *wa-immā*. The verb *kāna* or *yakūnu* ('is the case that') is sometimes present and sometimes absent. I will generally ignore these two variations. At *Qiyās* 316.15 Ibn Sīnā writes *qad lā yakūnu* 'sometimes not'; I read this as meaning the same as *laysa dā'iman* 'not always'.

Potentially more serious is that sometimes Ibn Sīnā writes *laysa immā*, leaving out the *albatta* ('ever' or 'altogether') or *dā'iman* ('permanently') that distinguish (e, mn) from (o, mn). In the crucial section *Qiyās* vi.2 this happens at 308.4, 311.17, 312.4, 312.5, 314.6, 316.3, 316.13 and 316.16. In all these cases except 316.16 the context shows that a universal sentence is meant rather than an existential one, so that *laysa immā* should be read as *laysa albatta immā*, giving (e, mn). (For this kind of editing it's helpful to keep an eye on the overall structure of the section, cf. Subsection 11.2 below.) On 316.16 see the note in Subsection 11.3.

The letters p and q in (2) stand for sentences and are called respectively the 'antecedent' (*muqaddam*) and the 'consequent' (*tālī*); the same names apply to any sentence put in place of p or q. We call the antecedent and the consequent the 'clauses' of the sentence. The use of these letters for sentences is ours and not Ibn Sīnā's; like Aristotle he almost never puts single letters for sentences. (*Qiyās* 544.18f is a rare counterexample.) Instead he gives what seem to be abbreviated categorical sentences. Thus a typical example of (a, mt) at *Mukhtaṣar* 125.14 appears literally as

(3) *kullamā kāna a b fa-ba*^{c}*ḍu j d*

which in our notation is

(4) $(a, mt)(A\ B$, Some $C\ D)$.

He says at *Qiyās* 296.2 that '$A\ B$' in expressions of this kind can stand for any predicative proposition; but does he mean *permissively* that it doesn't have to be literally '$A\ B$', or does he mean *restrictively* that it must be predicative? An instance that supports the permissive reading is at *Qiyās* 301.13, where he specifies a particular situation—or perhaps a class of situations—for an ecthetic argument. He specifies it by a sentence 'A is B'. But the context gives not the slightest reason to believe that the situation is describable by a single predicative sentence, let alone a categorical one, so at least here the 'A is B' seems to be a pure propositional variable consisting of two letters rather than one. But Ibn Sīnā sometimes uses the forms of the clauses to affect the meaning of the sentence form; for example we will see that in *Qiyās* vi.2 he lets the fact that the clauses in an 'Either ... or' sentence are affirmative be an indication that the disjunction is exclusive.

2 The meanings of the sentence forms

Of course we want to know what Ibn Sīnā's eight Arabic expressions (2) *mean*.

Here we run into a standard problem with Peripatetic logic, that sentences are classified by natural language forms, rather than by logical formulas as in modern logic. One and the same natural language sentence can be used for several different logical forms, as Ibn Sīnā himself emphasises in the case of 'absolute' (*muṭlaq*) predicative sentences (for example at *Qiyās* 43.16 'As you know, there are many kinds [of absolute proposition]', cf. *Qiyās* 135.11f). Sometimes a natural language form has a default reading, and extra text can be added if one wants to use the form in a different way from the default.

There is a second problem that is more specific to Ibn Sīnā's forms (2). This is that some of these forms are not standard usage in medieval Arabic. In fact they are not found in Ibn Sīnā himself except within his own logical examples. So any sense that we try to attribute to the eight forms by our knowledge of Arabic in general is going to be partly guesswork. We will see this in detail when we consider Rescher's guesses in Section 3 below.

The fact that some of Ibn Sīnā's sentence forms have no clear standard meaning suggests that he uses them as technical terminology. It also suggests that he didn't know straightforward non-technical ways of expressing the meanings of these forms. Ibn Sīnā does indicate in several places that hypothetical logic contains some modes of thought that don't come to us naturally: 'We will mention some of these [hypothetical syllogisms], but we will avoid those that are not close to our natural [ways of thinking]' (*Išārāt* 157.3, /432d/); cf. *Najāt* 84.12 '[these forms of argument are] very remote from our natures'. Given these remarks, one might wonder why he went to such trouble in *Qiyās* to develop a branch of logic that he regarded as largely 'unnatural'. One possible answer is that he recognised the formal beauty and the innovative nature of the structures that he was building.

Fortunately we need not rely on bare knowledge of Arabic to pin down the meanings of the eight forms. We have two other resources within Ibn Sīnā's text. The first resource is the large number of detailed logical calculations that Ibn Sīnā reports, using these forms. These are found mainly in *Qiyās* vi.1,2 and vii.1–3, a little over fifty pages of the book *Qiyās* in his encyclopedic *Shifā'*.

The main purpose of this paper is to show that this resource provides precise and robust answers to most of the main questions about the meanings of Ibn Sīnā's hypothetical sentence forms. Along the way we will note some reservations that have to be made in our use of these calculations. (See also Hodges and Johnston [16] §4 for some general considerations about using historical logicians' calculations as evidence for the meanings of their formulas.)

The second resource is the informal explanations that Ibn Sīnā gives in several of his books. (Subsection 11.1 below lists the main passages in which Ibn Sīnā discusses hypothetical logic. In *Qiyās*, book v is devoted to informal preliminaries for hypothetical logic.)

I will not make any use of the views of later Arabic logicians on hypothetical logic. Until we understand what Ibn Sīnā himself meant by his sentence forms, we are not in a position to judge whether or how these later writers can be used to explain his usages.

3 Rescher's formulas and translations

In a pioneering paper of 1963 Nicholas Rescher [31] examined Ibn Sīnā's hypothetical sentence forms. He proposed two kinds of interpretation of the forms, one by giving first-order formulas and one by giving English translations. This was a brave move, since in 1963 he had little information to go on. He used the brief discussions in *Dānešnāmeh* [21] and *Išārāt* [23], but beyond these he had to rely on his own intuition of what Ibn Sīnā's Arabic meant.

Rescher's formula interpretations are as follows (in a slightly differentt logical notation):

$$\begin{array}{ll} (a, mt)(p, q) & \forall\tau\ (p(\tau) \to q(\tau)) \\ & \text{or } \forall\tau\ \neg(p(\tau) \wedge \neg q(\tau)) \\ (e, mt)(p, q) & \forall\tau\ \neg(p(\tau) \wedge q(\tau)) \\ (i, mt)(p, q) & \exists\tau\ (p(\tau) \wedge q(\tau)) \\ (o, mt)(p, q) & \exists\tau\ (p(\tau) \wedge \neg q(\tau)) \\ (a, mn)(p, q) & \forall\tau\ (p(\tau) \vee q(\tau)) \\ (e, mn)(p, q) & \forall\tau\ \neg(p(\tau) \vee q(\tau)) \\ (i, mn)(p, q) & \exists\tau\ (p(\tau) \vee q(\tau)) \\ (o, mn)(p, q) & \exists\tau\ \neg(p(\tau) \vee q(\tau)) \end{array} \tag{5}$$

I will call these formulas 'Rescher's formulas'. (Rescher says that the variable τ ranges over 'times' or 'cases'. The two formulas for $(a, mt)(p, q)$ are logically equivalent.) His English translations are as follows:

(6)

$(a, mt)(p, q)$	Always: when p, then q.
$(e, mt)(p, q)$	Never: when p, then q.
$(i, mt)(p, q)$	Sometimes: when p, then q.
$(o, mt)(p, q)$	Not always: when p, then q.
$(a, mn)(p, q)$	Always: either p or q.
$(e, mn)(p, q)$	Never: either p or q.
$(i, mn)(p, q)$	Sometimes: either p or q.
$(o, mn)(p, q)$	Not always: either p or q.

I will call these 'Rescher's translations'.

Four of Rescher's *formulas* look very plausible renderings of Ibn Sīnā's Arabic. For $(a, mt)(p, q)$, the Arabic reads

(7) Whenever p, q.

Rescher's formula for $(a, mt)(p, q)$ expresses

(8) In every time or case in which p holds, q holds.

This is a good fit. There is a question exactly what the variable τ ranges over, but this may be something that should be left to the particular material that the logic is being applied to. There is also a question whether the sentence (7) should be taken to imply that there is a time or situation in which p. The question arises because Ibn Sīnā tells us in ^{c}*Ibāra* 79.11–80.12 that the categorical sentence 'Every A is a B' counts as false if there are no As (cf. [11]). If he makes the corresponding claim for $(a, mt)(p, q)$ then Rescher's formula should be amended to

(9) $$(\forall \tau \ (p(\tau) \rightarrow q(\tau)) \wedge \exists \tau p(\tau)).$$

The added clause $\exists \tau p(\tau)$ is called the 'existential augment', and the condition that it expresses is called the 'existential import'. Both Hodges [11] and Chatti [4] note that the reasons Ibn Sīnā gives for assuming existential import become less convincing the further we move from categorical sentences.

Similarly the Arabic for $(a, mn)(p, q)$ translates literally as

(10) Always either p holds or q holds.

and this fits Rescher's formula. We can ask the same question as above about the range of the quantifier. Instead of the question of existential import, we have a question whether the 'either ... or' should be read as inclusive or exclusive—we will come back to this point.

If the meanings of $(a, mt)(p, q)$ and $(a, mn)(p, q)$ are fixed, then the meanings of $(o, mt)(p, q)$ and $(o, mn)(p, q)$ should be fixed too, since the Arabic for (o, mt) is a straightforward sentence negation of that for $(a, mt)(p, q)$, and likewise for $(a, mn)(p, q)$ and $(o, mn)(p, q)$. Rescher's formulas for the sentences $(o, mt)(p, q)$ and $(o, mn)(p, q)$ are logically equivalent to the negations of his formulas for $(a, mt)(p, q)$ and $(a, mn)(p, q)$. So half the Rescher formulas, namely those for the a and o forms, are at first sight correct or nearly correct. We will need to study his formulas for the e and i forms more closely.

When we turn to Rescher's *translations*, the translation for $(a, mt)(p, q)$ looks straightforward provided we agree that Rescher's phrase 'Always: when' means 'Whenever'. But why the colon? Examining that colon more closely should set two alarm bells ringing.

The first alarm bell is that Rescher's translations for the *muttaṣil* sentences make it appear that the parts of the the four sentences after the quantifier are all identical, namely 'when p, then q'. Now in $(a, mt)(p, q)$ this phrase can't be symmetrical between p and q, since then 'Whenever p, q' would mean the same as 'Whenever q, p', which it doesn't. (Whenever it rains the flowers are watered; but sometimes the flowers are watered in some other way.) But then if 'when p, then q' means the same in $(i, mt)(p, q)$ as it did in $(a, mt)(p, q)$, then $(i, mt)(p, q)$ should say something different from $(i, mt)(q, p)$. This is surely wrong; $(i, mt)(p, q)$ is the hypothetical equivalent of 'Some A is a B', which is logically equivalent to 'Some B is an A'. In fact if we look at Rescher's formula for $(i, mt)(p, q)$ we see that it is logically equivalent to $(i, mt)(q, p)$.

Maróth implicitly calls attention to this anomaly when he explains the form $(i, mt)(p, q)$ by writing

(11) p [and] q *is equivalent to* Sometimes $(p \supset q)$.

([28] p. 111.) No logical system since the birth of mankind has recognised any such equivalence!

The second alarm bell should be triggered by the parts of Rescher's translations that lie to the left of the colons, namely the quantifiers 'always', 'sometimes' etc. One basic fact of Aristotelian logic is that quantifiers in categorical sentences are always relativised to the subject term: for example 'Every A is a B' ascribes being a B to 'every A'. By the same token, the quantifiers 'always' and 'sometimes' should be restricted to the class of times when p, at least in the *muttaṣil* sentences. So for example the sentence that Rescher writes as 'Always: when p, then q' needs to be parsed as 'At (every time when p), q'. His colons make the break in the wrong place.

Removing the colons in Rescher's translations for *muttaṣil* sentences is an essential first step. But it doesn't help for the more troublesome *munfaṣil* sentences, since with them it is not clear what if anything the quantifiers are relativised to.

In sum, it is not safe to assume that Rescher's translations are a reliable guide to Ibn Sīnā's intentions. In Section 10 below I will reassess them in the light of the results of this paper, and suggest some replacements.

4 Ibn Sīnā's informal explanations

Clearly any conclusions that we reach about the meanings of the sentence forms (2) should take proper account of Ibn Sīnā's own informal explanations. But there are several reasons why an uncritical reading of these explanations on their own is not going to give us conclusive evidence of Ibn Sīnā's intentions.

One reason is that informal discussions are inherently less precise than formalised ones, particularly if they dip into ontology or other kinds of deep philosophy.

A second reason is that Ibn Sīnā's informal explanations of hypothetical sentences (except perhaps those in *Mašriqiyyūn*) are meant to be read by fellow Peripatetics, so they concentrate on issues already raised within the Peripatetic tradition. For example Ibn Sīnā is aware that a major issue within that tradition is how the negative forms are related to the affirmative ones; in fact in *Qiyās* he devotes a whole section to this question (*Qiyās* v.5). Maróth [28] pp. 115–120 compares Ibn Sīnā's comments on this issue with earlier Peripatetic treatments of negation, and concludes on his p. 120: 'Thus Ibn Sīnā's words are a criticism of the view of the Peripatetic authors who served as his source' (my translation).

One result of this slant towards earlier Peripatetic issues is that Ibn Sīnā's informal explanations tend to concentrate on universal rather than existential sentences. Another is that in discussing *muttaṣil* sentences, they tend to rest on examples using *in* 'if' and *iḏā* 'when', rather than the *kullamā* 'whenever' that Ibn Sīnā prefers in his own forms (2).

Thirdly, not all of Ibn Sīnā's informal explanations can safely be assumed to be about the *meanings* of the forms. Let me mention two examples. One is that Ibn Sīnā often distinguishes 'absolute' hypothetical sentences from ones that carry information about 'correspondence' (*ittifāq*) or 'entailment' (*luzūm*). These are two notions that refer to the kind of reason given for taking the sentences to be true in a proof. In the case of correspondence the reason is that a clause of the sentence is known to be true 'on its own'; in the case of entailment the two clauses are known to be connected in some way. In some places Ibn Sīnā introduces new sentence forms that add words about entailment to the formulations in (2). An example is a variant of (a, mt) at *Qiyās* 366.7:

(12) *kullamā kāna* p *fa laysa yalzamu anna* q.
('... it doesn't follow that q.')

Since these variant sentence forms are not in (2), I ignore them below. See [13] for a fuller study.

For a second example of an explanation which might seem to be about meanings but is not, consider what Ibn Sīnā says about the element of 'doubt' in conditional statements. One might claim, as some earlier writers had done, that a statement of the form 'If p then q' indicates a doubt about the truth of p. Ibn Sīnā notes this at *Qiyās* 233.10. He responds at *Qiyās* 236.19f that we doubt q if we are aiming to derive it from p, and we doubt p if we are aiming to refute it from not-q. Thus the 'doubt' that Ibn Sīnā recognises is not a part of the meaning of the sentence; it belongs to the vocabulary of scientific methodology.

5 Negation and metathesis

We need to take on board some of Ibn Sīnā's informal comments on negation—not least because the more advanced of Ibn Sīnā's logical calculations will undermine these same comments.

One kind of negation recognised by Aristotelian logic is contradictory negation (*naqīḍ*): a contradictory negation of a sentence ϕ is a sentence equivalent to the negation of ϕ. We write $\overline{p}$ for the contradictory negation of p, and we note that $\overline{\overline{p}}$ is logically equivalent to p.

In categorical logic an o sentence is the contradictory negation of the a sentence with the same terms, and vice versa; likewise an i sentence is the contradictory negation of the e sentence with the same terms, and vice versa. We would expect the same to hold for the corresponding *muttaṣil* and *munfaṣil* sentences, for example that $(a, mt)(p, q)$ and $(o, mt)(p, q)$ are contradictory negations of each other. Ibn Sīnā confirms this many times over.

For example at *Qiyās* 366.15-367.1 he says explicitly that the second of the following *muttaṣil* sentences is the contradictory negation of the first:

(13) (a, mt)(Every A is a B, Not every C is a D).
(o, mt)(Every A is a B, Not every C is a D.)

At *Qiyās* 380.6–8 he gives a contradictory pair of *munfaṣil* sentences:

(14) (e, mn)(No every A is a B, Every C is a D.)
(i, mn)(Not every A is a B, Every C is a D.)

In both cases we could put p for the first clause and q for the second, but Ibn Sīnā likes to give fully explicit examples even if it multiplies the labour by sixteen. Besides these explicit statements, there are many cases where he appeals to reductio ad absurdum, and each of these cases produces an example of a contradictory pair. For example there are reductio proofs with *muttaṣil* sentences at *Qiyās* 300.17, 301.5, 301.10, 301.13, 302.14, 303.1, 303.5, 303.8, 303.11 and 304.3. The proof at *Qiyās* 300.17 confirms that (e, mt) and (i, mt) are a contradictory pair; and so on.

So we can take these contradictory negations for *muttaṣil* and *munfaṣil* sentences as solidly establshed. Hence determining the meanings of the universal hypothetical sentences will determine those of the existential sentences too.

In several of his discussions of categorical logic (for example *ᶜIbāra* [18] 78.8–79.10) Ibn Sīnā describes a phenomenon called 'metathesis' (Arabic *ᶜudūl*), which Arabic logic inherits from earlier Peripatetic work. Metathesis occurs when a sentence contains a negation, but the negation is part of the matter of the sentence rather than its form, so that it is invisible to logical rules. Take for example the syllogism

(15) Some horse is not a human.
Every Greek is a human.
Therefore some horse is not a Greek.

Here the 'not' in the first premise is part of the sentence form, so that one can apply a logical rule that recognises 'human' in the second premise as being the same term as 'human' (without the 'not') in the first premise. But as Ibn Sīnā observes at *ᶜIbāra* 92.3, one can also take 'not a human' as a subject term. An example might be

(16) Something not a human is a horse.

Here there is no syllogistic rule that could be applied so as to separate 'not' from 'human'; the subject term has to be taken as an indivisible unit. So the 'not' in (16) is metathetic.

We should make two comments at once. The first is that the distinction between metathetic and non-metathetic negation applies only to sentences for which a formalisation is given or implied. For example the English sentence that appears as first premise in (15) could have been formalised so as to treat 'not a human' as a term with metathetic negation, giving the whole sentence the affirmative form (a). That would block the syllogism, but it would still be a tenable formalisation. Ibn Sīnā himself in ^{c}*Ibāra* 78.13–79.10 suggests some linguistic points that can be used as clues to whether a negation should be read as metathetic or not. But a language is a living entity and nobody has to be bound by Ibn Sīnā's suggestions. So we as readers of Ibn Sīnā may need to make calculated decisions about whether he means a particular negation in an Arabic sentence to be metathetic or not.

The second comment is that the distinction between metathetic and non-metathetic has to take into account the available logical rules. The negation in (16) is metathetic because there is no rule of categorical logic that could take it otherwise. But if we added a new logical rule that took into account a 'not' attached to the subject term, it would be open to us to treat the negation in (16) as not metathetic. We will see below that this is exactly what happens in Ibn Sīnā's more advanced hypothetical calculations, where he mixes *muttaṣil* and *munfaṣil* sentences together in the same syllogism. As a result we will meet plethoras of negated subject clauses. In some earlier notes in the internet I described these negated subject clauses as 'metathetic' by analogy with (16). But this was wrong; a better view is that these subject negations are precisely not metathetic when the new rules can reach them. So below I will describe negations as 'ametathetic' when they negate terms or clauses but are part of the sentence form.

Although Ibn Sīnā doesn't himself use the word 'metathetic' in hypothetical logic, the concept is there. Clauses are distinguished as affirmative or negative, and the negativeness of a negative clause is in effect metathetic when it is not available to the logical rules, for example those logical rules that distinguish whole sentences as being affirmative or negative. As Ibn Sīnā puts it in an informal explanation in the early work *Mukhtaṣar* 78.14–

17:

(17) The affirmation affirms the connecting (*ittiṣāl*) or the separating (*infiṣāl*), and the denial denies the connecting or the separating, regardless of how the antecedent and the consequent are. Likewise if you say 'If no A is a B then no C is a D', this is affirmative even if it is made from two negative clauses.

Thus an affirmative *muttaṣil* sentence affirms a 'connecting' of p and q, and an affirmative *munfaṣil* sentence affirms a 'separating' of these two clauses; the corresponding negative sentences deny the connecting or separating. The affirmativeness or negativeness of the whole sentence is independent of whether its clauses are affirmative or negative. So if the logical rules are the analogues of those of categorical logic, then the negativeness of a negative clause is metathetic.

Here Ibn Sīnā is telling us that the negative forms (e, mt) and (e, mn) come from the affirmative forms by introducing a denial. Presumably a denial takes the form of a negation, but where should the negation be put? Let us pursue this question for (a, mt), assuming that Rescher's formula for (a, mt) is correct. (We ignore the existential augment for the present.) Rescher's formula is

$$\forall\tau\ (p(\tau) \to q(\tau)) \tag{18}$$

Since (e, mt) is universal, the negation can't be put at the front where it would turn $\forall\tau$ into $\exists\tau$. There are just three other places where the negation might be added:

$$\begin{array}{ll} (i) & \forall\tau\ \neg(p(\tau) \to q(\tau)) \\ (ii) & \forall\tau\ (\neg p(\tau) \to q(\tau)) \\ (iii) & \forall\tau\ (p(\tau) \to \neg q(\tau)) \end{array} \tag{19}$$

In principle we should be able to choose between (i), (ii) and (iii) on the basis of how Ibn Sīnā uses (e, mt) sentences in formal inferences.

For example at *Mukhtaṣar* 125.18f Ibn Sīnā claims that the following inference is valid:

(20)
(e, mt)(A is B, No C is a D.)
Every D is an H.
So (e, mt)(A is B, No C is an H.)

This rules out (ii). (I leave the detailed check to the reader. A guide is that $D \subseteq H$ justifies replacing D by H in a sentence ϕ just when D occurs only positively in ϕ. The relevant logical theory is in [26] and will be explained more fully in [15].)

Ibn Sīnā gives other examples of syllogisms with (e, mt) sentences at *Mukhtaṣar* 125.19f, 125.20f, 125.22f, 126.9f, 126.10f, 126.11f, 126.13f; but all of these give just the same logical information about (e, mt) as the example above. However, patience is rewarded at *Mukhtaṣar* 130.11f, where we read

(21)
Every C is a B.
(e, mt)(Every C is a D (or some C is a D), H is a Z).
It yields: (e, mt)(Every B is a D, H is a Z)

This result eliminates the reading (i), so we conclude that (iii) is the correct reading of (e, mt). Happily (iii) is logically equivalent to the formula that Rescher himself gave for (e, mt).

So the formal inferences in *Mukhtaṣar* confirm Rescher's formula for (e, mt). By implication they also confirm his formula for (i, mt), since we know that it is the contradictory negation of (e, mt). This confirmation of these two formulas is weak because the elimination of (i) rests only on the entailment (21), and any single example might contain errors. But we will see below that further inferences in *Qiyās* provide overwhelming evidence in support of Rescher's formulas for (e, mt) and (i, mt).

Could we have reached the same conclusion from Ibn Sīnā's explanation that the negative form (e, mt) 'denies the connecting' that is affirmed by (a, mt)? Hardly. It would be natural to assume that a negation denies what comes immediately after it. So according to Rescher's formula, the negation in (e, mt) denies the second clause, not the connecting between the two clauses. Ibn Sīnā's informal explanation is thoroughly misleading at this point. The moral is that any attempt to find the meanings of Ibn Sīnā's hypothetical sentence forms by means of his informal explanations alone, disregarding his formal inferences, is likely to be worthless.

6 PL1 in *Qiyās* viii.1,2

Ibn Sīnā assembles in *Qiyās* viii.1,2 a group of inference rules for hypothetical logic that he inherits from al-Fārābī. The section *Qiyās* viii.1 is devoted to sentence forms that he calls *muttaṣil*, and the section *Qiyās* viii.2 to forms that he calls *munfaṣil*. These sentence forms are as follows, together with the number of times that they occur in the two sections. First the *muttaṣil* forms:

	form	occurrences
1.	If (*in*) p then q	9
2.	If (*in*) p then possibly q	2
3.	When (*iḏā*) p then q	1
4.	If (*in*) $\overline{\overline{p}}$ then q	1
5.	If (*in*) $\overline{p}$ then $\overline{q}$	2

Next the *munfaṣil* forms:

	form	occurrences
6.	Either (*immā an*) p or (*aw*) q	4
7.	Either (*immā an*) p or (*immā*) q	8
8.	It is not the case that (*lā yakūnu*) both p and q	2
9.	A is either B or not C	1
10.	A is either not B or it is C	2
11.	A is either not B or not C	3
12.	A is not (*lā yakūnu*) either B or C	2
13.	A is not (*laysa immā*) either B or C	8
14.	A is not (*laysa albatta*) either B or C	1

The letters p and q stand for affirmative categorical sentences. In 9–14 it is assumed that p and q have the same subject term but different predicate terms. From 9–11 we see that at least in the affirmative *muttaṣil* form, the antecedent and the consequent can each be either affirmative or negative independent of each other.

There are forty-six occurrences listed above. Only one of them is also in the list (2), namely number 14 which occurs at *Qiyās* viii.2, 403.8f and has the form (e, mn). From the context at 403.8f it seems that Ibn Sīnā reads this form as saying 'A is not B and not C', which agrees with Rescher's formula for (e, mn). But we will see below that it disagrees with all the other evidence that we have for the meaning of (e, mn). This one sentence form is the only one in *Qiyās* viii.1,2 which shows any sign of the temporal quantification that appears in all of Ibn Sīnā's forms (2). Conversely almost none of the forms listed above appear in *Qiyās* vi, vii where Ibn Sīnā demonstrates syllogisms using his forms (2).

In short, the hypothetical logic of *Qiyās* viii.1,2 has completely different sentence forms from those of the hypothetical logic or logics of *Qiyās* vi, vii. This is one reason why we separate off the logic of *Qiyās* viii.1,2 by labelling it Propositional Logic 1, or PL1 for short. At *Qiyās* 356.15–17 Ibn Sīnā himself attacks a form of hypothetical logic that he found in a book, precisely because it fails to understand how being affirmative, negative, universal, existential or unquantified apply to hypothetical sentences. PL1 is a plausible candidate for being that logic.

Since the sentence forms in PL1 in general have no temporal quantifiers, the question of existential import doesn't arise directly. But it might appear in some other guise, for example as a restriction that 'If p then q' implies that p is true or at least possible. However, at *Qiyās* 390.7–15 Ibn Sīnā discusses 'If ... then' sentences that are true 'because their consequent is known to be true'. Taken literally, this implies that the truth of the sentence doesn't require any property of p.

The logical rules of PL1 all fall into a simple pattern. We consider a particle or operator $\star$ that forms a sentence $p \star q$ from sentences p and q. The inference rules are as follows:

(22)

1. $p \star q, p$ *yields* q.
2. $p \star q, \overline{q}$ *yields* $\overline{p}$.
3. $p \star q, q$ *yields* p.
4. $p \star q, \overline{p}$ *yields* $\overline{q}$.
5. $p \star q, \overline{p}$ *yields* q.
6. $p \star q, \overline{q}$ *yields* p.
7. $p \star q, p$ *yields* $\overline{q}$.
8. $p \star q, q$ *yields* $\overline{p}$.

Rules of this type are called *istithnā'ī*, which has been variously translated, for example as 'exceptive' or 'duplicative'.

Ibn Sīnā tells us which hypothetical sentence forms, when they are read as $p \star q$, obey which of these rules. This yields a classification of the sentence forms of PL1 into four types:

(23)

muttaṣil obeys rules 1 and 2.

complete (*tāmm*) *muttaṣil* obeys rules 1 to 4.

munfaṣil obeys rules 5 and 6.

strict (*ḥaqīqī*) *munfaṣil* obeys rules 5 to 8.

In modern terminology, a complete *muttaṣil* sentence is a biconditional. A *munfaṣil* sentence is a disjunction, and it is an exclusive disjunction if it is strict.

The inference rules (22) are not conducive to being used with quantified hypothetical sentences, because it is not clear how the quantification would apply to the second premise and the conclusion. Take for example modus ponens as Ibn Sīnā presents it at *Qiyās* 390.14:

(24) If p then q. p. We infer: q.

If 'if' is read as 'whenever', then this implies that p and q carry a temporal parameter, say as $p(\tau)$ and $q(\tau)$. This allows at least three valid adaptations of (24):

(25)
$$\begin{array}{lll} \forall\tau\ (p(\tau) \to q(\tau)). & \forall\tau\ p(\tau). & \text{We infer: } \forall\tau\ q(\tau). \\ \forall\tau\ (p(\tau) \to q(\tau)). & p(\alpha). & \text{We infer: } q(\alpha). \\ \forall\tau\ (p(\tau) \to q(\tau)). & \exists\tau\ p(\tau). & \text{We infer: } \exists\tau\ q(\tau). \end{array}$$

(and of course some invalid adaptations). As far as I can see, Ibn Sīnā in *Qiyās* viii.1,2 never addresses this complication. Presumably it was not addressed in his sources either.

7 PL2 in *Qiyās* vi.1, 295–304

After his preliminary explanations in *Qiyās* v, Ibn Sīnā begins to expound the proof theory of hypothetical logic in *Qiyās* vi.1. This section is completely devoted to syllogisms whose premises and conclusions are *muttaṣil* sentences. It freely uses all the forms (a, mt), (e, mt), $(i.mt)$ and (o, mt), and overwhelmingly these forms are given as the first four forms in (2). So the logic in this section is Ibn Sīnā's own, unlike the Farabian logic of PL1. Anticipating that this logic is significantly different from PL1, we call it PL2.

The section contains seventy-two *muttaṣil* sentences or sentence forms spelled out in full. In all but four cases they are spelled out as in (2) above. Of these four exceptions, three are sentences with a label about entailment, as at (12) above. The fourth exception, at *Qiyās* 303.14, is (a, mt) with the typically PL1 particle *iḏā* 'when' in place of *kullamā* 'whenever'. The rarity of this PL1 form in this section is one of the indications that PL2 should be taken as a different logic from PL1.

Throughout this section, Ibn Sīnā makes remarks to the effect that this part of hypothetical logic is a copy of what he calls 'predicative' (*ḥamlī* logic. Thus syllogisms are grouped into three figures exactly as in the predicative syllogisms, taking subclauses as the counterpart of terms (*Qiyās* 295.7–10). The productivity conditions are exact copies of those for predicative syllogisms (*Qiyās* 296.1f, 299.10–12, 300.11–13, 302.7–9). Elsewhere Ibn Sīnā uses 'predicative' to cover both Aristotelian categorical logic and temporal and alethic modal logic. But in the present case it's clear that he means categorical logic. He repeats the productive moods with their conclusions, figure by figure, exactly as he does for the categorical syllogisms at *Mukhtaṣar* [17] 100.4–107.3, *Najāt* [20] 57.1–64.3, *Qiyās* [19] ii.4, 108.12–119.8 and *Dānešnāmeh* [21] 67.5–80.2, following *Prior Analytics* i.4–6 almost to the letter. The proofs that he gives for the second and third figure hypothetical syllogisms are exact copies of those that he gives for categorical syllogisms, and differ from Aristotle's only in that Ibn Sīnā always allows a proof of *Baroco* by ecthesis.

So these *muttaṣil* syllogisms form an exact formal copy of the categorical syllogisms. This correspondence has much in common with the correspondences described by Wallis [33] *Thesis Secunda* and Boole [3] Chapter xi, but one conspicuous difference is that Ibn Sīnā makes no claim that the categorical syllogism corresponding to a hypothetical syllogism is a paraphrase of the hypothetical syllogism. For him the hypothetical syllogisms need to be justified as they stand, rather than borrowing justification from the categorical syllogistic.

This formal correspondence creates a presumption that the hypothetical sentence forms can be expressed with first-order formulas that are copies of those that we know are correct for the categorical forms. Thus:

(26)
$$\begin{array}{ll} (a)(A,B) & (\forall x(Ax \to Bx) \wedge \exists x Ax) \\ (e)(A,B) & \forall x(Ax \to \neg Bx) \\ (i)(A,B) & \exists x(Ax \wedge Bx) \\ (o)(A,B) & (\exists x(Ax \wedge \neg Bx) \vee \forall x \neg Ax) \end{array}$$

Copying formally into hypothetical logic gives:

(27)
$$\begin{array}{ll} (a,mt)(p,q) & (\forall\tau\ (p(\tau) \to q(\tau)) \wedge \exists\tau p(\tau)) \\ (e,mt)(p,q) & \forall\tau\ (p(\tau) \to \neg q(\tau)) \\ (i,mt)(p,q) & \exists\tau\ (p(\tau) \wedge q(\tau)) \\ (o,mt)(p,q) & (\exists\tau\ (p(\tau) \wedge \neg q(\tau)) \vee \forall\tau \neg p(\tau)) \end{array}$$

which are logically equivalent to Rescher's guesses, except for the augments in (a, mt) and (o, mt).

It remains only to justify the existential augment in (a, mt); justification of it will justify the universal augment in (o, mt) too. Note what the existential augment says in the hypothetical case. It says that 'Whenever p, q' counts as false if there is no time or situation in which p; in other words, 'Whenever p, q' implies that it is possible that p, or that sometimes p.

The existential augment is needed in three places in this section. The first two are to justify (a, mt)-conversion in the proof of *Darapti* at *Qiyās* 302.12 in the proof of *Felapton* at *Qiyās* 303.1. The third is to justify the claim, in a proof by absurdity at *Qiyās* 302.12, that $(a, mt)(p, q)$ is incompatible with $(e, mt)(p, q)$. Besides these three places, there is a passage in this section where Ibn Sīnā departs from the listing of syllogisms and discusses questions arising from the notion of a universal *muttaṣil* sentence being 'true by agreement'. Unpicking the details reveals that for Ibn Sīnā an (e, mt) sentence has this kind of truth if its consequent is false, while an (a, mt) sentence has this kind of truth if its consequent is true *and its antecedent is possible*. Key statements are at *Qiyās* 297.15 for (a, mt) and 299.8 for (e, mt); see also 297.1f where an $(a, mt)(p, q)$ sentence with q true and p impossible is declared false. These statements are exactly in line with the position that (a, mt) has existential import and (e, mt) doesn't.

See Movahed [30] for another angle on these augments. Movahed argues that the existential import in (a, mt) is a mistake on Ibn Sīnā's part. We will see in the next section that in the context of his logic PL3, Ibn Sīnā is forced to abandon the automatic use of augments in (a, mt). But at least in PL2 he is using them with his eyes open; they are not an oversight.

In sum, the formulas (27) for *muttaṣil* sentences are confirmed as strongly as we have any right to expect, at least for PL2. The difference of the sentence forms, and the existential augments required in PL2, are two indicators of the difference between the logics PL1 and PL2.

8 PL3 in *Qiyās* vi.2, 305–318 and vii.2, 373–384

In the next section of *Qiyās*, section vi.2, Ibn Sīnā moves on from syllogisms composed of *muttaṣil* sentences. Now he considers syllogisms where one of the premises is *muttaṣil* and the other is *munfaṣil*. Just as with the *muttaṣil* sentences in *Qiyās* vi.1, he expands the class of *munfaṣil* sentences so that each of them has one of the forms (a, mn), (e, mn), (i, mn) and (o, mn). So the full range of sentence forms (2) is now in play.

Ibn Sīnā justifies his mixed *muttaṣil*/*munfaṣil* syllogisms by inferring a *muttaṣil* sentence from the *munfaṣil* premise, drawing a conclusion in *muttaṣil* logic, and then (as an optional extra) inferring a *munfaṣil* sentence from the *muttaṣil* conclusion. As a result of this approach, he gives us in *Qiyās* vi.2 a large number of inferences from *munfaṣil* sentences to *muttaṣil* ones, and several inferences in the opposite direction too. These provide us with plenty of information for inferring what he takes the *munfaṣil* forms to mean.

The section *Qiyās* vii.2 continues the comparisons made between *muttaṣil* and *munfaṣil* sentences. I use it with more caution than *Qiyās* vi.2, because in vii.2 more may be taken for granted. For example Ibn Sīnā uses sentences that could be read either as $(a, mt)(p, \overline{q})$ or as $(e, mt)(p, q)$. This is less likely to happen in vi.2 because the calculations are all tied to specific syllogisms.

The scheme that Ibn Sīnā uses to organise this section (see Subsection 11.2) gives rise in principle to 108 pairs of premises. Ibn Sīnā studies some of these premise-pairs in great detail, even giving several ways of stating the conclusion by using different sentence-forms. Other premise-pairs he leaves out altogether, or remarks briefly that the reader can reconstruct the facts from premise-pairs considered earlier. For some premise-pairs he gives the required proof but only sketchily. In order to extract the logical content, we need to begin with the cases that he describes most fully. The results for these can then be fed back to allow us to reconstruct the sketchier cases. In this way the evidence given by the inference rules is cumulative.

Besides the inferences quoted below, there is another source of evidence in *Qiyās* vi.2 for Ibn Sīnā's understanding of the hypothetical sentence forms. This is his proofs of nonproductivity. Following Aristotle's method, he gives natural language sentences that are instances of given forms, and which he considers to be true. This source turns out to be less useful than we might have hoped. The results gained below are enough to show that Ibn Sīnā seriously misunderstands Aristotle's method; several of the premise-pairs that he pronounces nonproductive are in fact productive, and this tends to discredit his examples. The point is discussed more fully in [14].

8.1 Comparison of *muttaṣil* sentences

Besides the inferences between *muttaṣil* and *munfaṣil*, *Qiyās* vi.2 also offers a number of inferences between *muttaṣil* forms. Since these don't entirely agree with those of PL2, we need to check them out. Here is a list of them. We use p for affirmative categorical sentences and q for negative categorical sentences.

Table 1: *Qiyās* vi.2, *muttaṣil* $\to$ *muttaṣil*				
1.	$(a,mt)(p,q)$	$\to$	$(e,mt)(p,\overline{q})$	306.5f, 318.6
2†.	$(a,mt)(p,q)$	$\to$	$(e,mt)(\overline{p},q)$	312.14
3.	$(a,mt)(p,\overline{q})$	$\to$	$(e,mt)(p,q)$	308.3, 317.16f
4.	$(i,mt)(p,q)$	$\to$	$(o,mt)(q,\overline{p})$	317.9
5.	$(i,mt)(\overline{p},\overline{q})$	$\to$	$(o,mt)(\overline{p},q)$	310.14, 310.18
6.	$(e,mt)(p,q)$	$\to$	$(a,mt)(p,\overline{q})$	306.3–6, 307.15–308.1, 310.16
7.	$(e,mt)(p,\overline{q})$	$\to$	$(a,mt)(p,q)$	310.10
8.	$(o,mt)(p,q)$	$\to$	$(i,mt)(\overline{q},p)$	310.13
9.	$(a,mt)(p,q)$	$\to$	$(i,mt)(q,p)$	311.13
10.	$(a,mt)(\overline{p},q)$	$\to$	$(i,mt)(q,\overline{p})$	312.8f

All of these inferences agree with what we found for PL2, except for inferences 2, 6, 7 and 8. Inference 2 is probably a copying error; see the comments on this passage in Subsection 11.3. Inferences 6 to 8 (five instances in all) are more significant: they all assume that there are no augments on (a,mt) or (o,mt) sentences. On the other hand inferences 9 and 10 assume that (a,mt) sentences do carry augments; they are backed by an occurrence of *Darapti* at 309.11, which makes the same assumption.

At 311.10 Ibn Sīnā says that the syllogism using inference 9 yields a conclusion *min jihatin mā*, i.e. when taken in a certain way. The obvious candidate for a 'way of taking it' is that we choose to give existential import to the (a,mt) sentence—though this would involve giving existential import to a *munfaṣil* sentence, which is a puzzling idea. The phrase is consistent with the possibility that Ibn Sīnā in the logic of *Qiyās* vi.2 drops the augments in general, but allows us to use them when the subject-matter justifies them. We will come back to the question later.

Qiyās vii.2 adds a few inferences:

Table 2: *Qiyās* vii.2, *muttaṣil* $\to$ *muttaṣil*				
11†.	$(i,mt)(\overline{p},\overline{q})$	$\to$	$(o,mt)(p,\overline{q})$	382.2
12.	$(o,mt)(p,\overline{q})$	$\to$	$(i,mt)(p,q)$	377.7
13.	$(o,mt)(p,\overline{q})$	$\to$	$(i,mt)(p,q)$	378.12
14.	$(o,mt)(\overline{p},q)$	$\to$	$(i,mt)(\overline{p},\overline{q})$	377.1

Here inferences 12–14 confirm that augments are no longer obligatory on (o,mt) sentences. Inference 11 makes little sense and is probably a copying error.

8.2 Pinning down (a, mn) and (o, mn)

Turning to the *munfaṣil* sentences, it will be convenient to settle the cases of (a, mn) and (o, mn) before we tackle (e, mn) and (i, mn), which are more challenging. For (a, mn) there are some natural expectations. If it expresses inclusive disjunction then, given that we no longer have existential import on (a, mt), we would expect that $(a, mn)(p, q)$ entails $(a, mt)(\overline{p}, q)$. (If at least one of p and q holds, but p doesn't, then q holds.) Writing $\sharp$ for contradictory negation, and putting together results already established, this yields the expectation

(28)
$$\begin{array}{ccccc} (a, mn)(p, q) & \Rightarrow & (a, mt)(\overline{p}, q) & \Leftrightarrow & (e, mt)(\overline{p}, \overline{q}) \\ \sharp & & \sharp & & \sharp \\ (o, mn)(p, q) & \Leftarrow & (o, mt)(\overline{p}, q) & \Leftrightarrow & (i, mt)(\overline{p}, \overline{q}) \end{array}$$

But if $(a, mn)(p, q)$ is an exclusive disjunction, then we expect also to deduce $(a, mt)(p, \overline{q})$. (If at most one of p and q holds, and p holds, then q doesn't.) This gives some further expected entailments:

(29)
$$\begin{array}{ccccc} (a, mn)(p, q) & \Rightarrow & (a, mt)(p, \overline{q}) & \Leftrightarrow & (e, mt)(p, q) \\ \sharp & & \sharp & & \sharp \\ (o, mn)(p, q) & \Leftarrow & (o, mt)(p, \overline{q}) & \Leftrightarrow & (i, mt)(p, q) \end{array}$$

We will test these expectations against the evidence of *Qiyās* vi.2 and vii.2. Before we do that, note the arrangement of syllogisms in *Qiyās* vi.2, as described in Subsection 11.2 below. The syllogisms are arranged in four blocks, which we name A, B, C and D. Within each block, Ibn Sīnā considers first the cases where the *munfaṣil* premise is read as strict, and then those in which it is not read as strict. In the strict case the two clauses of the *munfaṣil* premise are always taken to be affirmative, unlike the non-strict cases. It would seem that Ibn Sīnā is taking the fact that the two clauses in a *munfaṣil* sentence are affirmative as evidence that the sentence is strict. In *Qiyās* vi.2 he never quite says this. But he confirms it at *Qiyās* vii.2, 378.8f, saying that an entailment from $(a, mn)(p, q)$ to $(a, mt)(p, \overline{q})$ holds because the clauses of the *munfaṣil* sentence are affirmative, and wouldn't hold if either of the clauses was negative. (*Qiyās* vii.2, 376.7 refers to 'the affirmative strict *munfaṣil* proposition whose clauses are both affirmative'.)

For this reason, the tables below in this subsection add a superscript ^{S} to those (a, mn) sentences which are premises in the parts of *Qiyās* vi.2 where (a, mn) premises are taken to be strict, and to those (a, mn) sentences in *Qiyās* vii.2 whose clauses are both affirmative. These cases are the ones where we would expect to find some inferences agreeing with (29) above.

We consider first the inferences from a *munfaṣil* form to a *muttaṣil* form. These commonly appear near the beginning of a proof, so as to convert all the premises to *muttaṣil* forms.

Table 4: *Qiyās* vii.2, (a, mn) and (o, mn)

1.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(\bar{p}, q)$	306.4–7, 313.10, 314.4f, 317.4, 317.8
2.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(\bar{q}, p)$	309.11, 310.2
3.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(p, \bar{q})$	305.9–11, 309.10–13, 310.13f
4.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(q, \bar{p})$	313.12
5.	$(a, mn)^S(p, q)$	$\rightarrow$	$(e, mt)(p, q)$	316.10
6.	$(a, mn)(p, \bar{q})$	$\rightarrow$	$(a, mt)(\bar{p}, \bar{q})$	308.1, 315.11, 316.2
7.	$(a, mn)(p, \bar{q})$	$\rightarrow$	$(a, mt)(q, p)$	311.12
8.	$(a, mn)(\bar{p}, q)$	$\rightarrow$	$(a, mt)(p, q)$	314.12, 315.4, 318.5
9.	$(a, mn)(\bar{p}, q)$	$\rightarrow$	$(a, mt)(\bar{q}, \bar{p})$	312.10
10†.	$(a, mn)(\bar{p}, q)$	$\rightarrow$	$(a, mt)(p, \bar{q})$	317.15f
11.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(\bar{p}, q)$	376.7
12.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mt)(\bar{q}, p)$	376.7
13.	$(a, mn)^S(p, q)$	$\rightarrow$	$(e, mt)(p, q)$	377.5
14.	$(a, mn)^S(p, q)$	$\rightarrow$	$(a, mn)(q, p)$	377.14
15.	$(a, mn)(\bar{p}, \bar{q})$	$\rightarrow$	$(a, mt)(p, \bar{q})$	378.9
16.	$(a, mn)(\bar{p}, \bar{q})$	$\rightarrow$	$(a, mt)(q, \bar{p})$	378.9
17.	$(a, mn)(\bar{p}, \bar{q})$	$\rightarrow$	$(a, mt)(p, \bar{q})$	381.14
18.	$(a, mn)(p, q)$	$\rightarrow$	$(a, mt)(\bar{p}, q)$	383.11
19†.	$(o, mt)(p, \bar{q})$	$\rightarrow$	$(o, mn)(p, q)$	384.4

Inferences 3, 4, 5 and 13 are all correct for an exclusive disjunction, as in (29); and all of them carry the superscript ^{S}. The remainder are all correct for an inclusive disjunction as in (28), except for 10 and 19 which are presumably errors. The agreement with (28) and (29) is impressive.

Next we consider the inferences from a *muttaṣil* form to a *munfaṣil* form. In *Qiyās* vi.2 these always appear, if at all, at the end of a proof where they are used to find different ways of expressing the conclusion. A reasonable prediction is that they will confirm (28) with the one-directional arrows strengthened to ⇔.

Table 5: *Qiyās* vi.2, (a, mn) and (o, mn)				
1.	$(a, mt)(\bar{p}, q)$	$\to$	$(a, mn)(p, q)$	313.9
2†.	$(i, mt)(p, q)$	$\to$	$(o, mn)(p, q)$	311.10, 317.10
3.	$(i, mt)(p, \bar{q})$	$\to$	$(o, mn)(\bar{p}, q)$	309.12
4†.	$(i, mt)(p, \bar{q})$	$\to$	$(o, mn)(p, \bar{q})$	312.9
5.	$(a, mt)(p, \bar{q})$	$\to$	$(a, mn)(\bar{p}, \bar{q})$	379.4
6†.	$(i, mt)(p, q)$	$\to$	$(o, mn)(p, q)$	384.3
7.	$(i, mt)(\bar{p}, \bar{q})$	$\to$	$(o, mn)(p, q)$	306.12f

Half of these—those marked with †—disagree with our reasonable prediction, which is disappointing. But there is no sign of any alternative interpretation that would explain them, and a probable reason for the errors is that the inferences are all optional in the passages where they occur—they are used only after the conclusion has been found. In any event, given the strong confirmation that we had for (28), it is still sensible to judge that we have the following equivalents (here ignoring strict disjunctions and augments):

$$\begin{array}{ccccc} (a, mt)(p, q) & \Leftrightarrow & (e, mt)(p, \bar{q}) & \Leftrightarrow & (a, mn)(\bar{p}, q) \\ \sharp & & \sharp & & \sharp \\ (o, mt)(p, q) & \Leftrightarrow & (i, mt)(p, \bar{q}) & \Leftrightarrow & (o, mn)(\bar{p}, q) \end{array} \tag{30}$$

For example the equivalence of $(a, mt)(p, q)$ and $(a, mn)(\bar{p}, q)$ seems to be telling us that 'Always if p then q' is equivalent to 'Always either not-p or q', which is exactly as we would expect.

The equivalences between *muttaṣil* and *munfaṣil* have two major effects on the logic. The first is that negations can appear freely in either clause. For example replacing p by $\overline{p}$ gives the equivalence $(a, mn)(p, q) \Leftrightarrow (a, mt)(\overline{p}, q)$, telling us that disjunctions are expressible as hypothetical analogues of (a) sentences with negations on their subject terms. We have seen the effect in the large number of barred letters $\overline{p}$, $\overline{q}$ in the tables above.

Moreover the inference rules relating *muttaṣil* to *munfaṣil* often add or remove negations. The effect is that we now have logical rules that can see the difference between an affirmative clause and a negative one; so henceforth no negations of clauses are invisible to the logic. All clause negations become ametathetic, as we phrased it earlier. Strictly speaking this has the consequence that the bars over letters should now be counted as a part of the sentence forms; but for simplicity I overlook this point.

Allowing free use of negation gives us some new syllogisms, simply by substituting negated terms for some unnegated ones. But the new proof rules give us more. They allow proofs of three new syllogisms that are not substitution instances of the old Aristotelian ones, or of each other. Pseudo-Albertus in the 14th century noticed one of these new syllogisms; but Ibn Sīnā already knew and stated all three in *Qiyās* vi.2. Further details are in [15].

A further consequence of the new rules is that it is hard to find any good reason to maintain existential imports in general. Since all clauses are open to being negated, we have the equivalence $(a, mt)(p, q) \Leftrightarrow (a, mt)(\overline{q}, \overline{p})$. It seems hard to argue that the sentence $(a, mt)(\overline{q}, \overline{p})$ carries the implication that p is not empty. Likewise there is a shortage of plausible reasons for ascribing existential import to any kinds of disjunction.

In short, the present logic PL3 has three main features that distinguish it from PL2. First, *muttaṣil* and *munfaṣil* sentences are freely combined in arguments. Second, there are no obligatory existential imports, and hence no obligatory implied existential or universal augments. Third, negations can occur anywhere. The first of these features also distinguishes PL3 from PL1, as does the fact that PL3 uses time-quantified sentence forms.

8.3 Pinning down (e, mn) and (i, mn)

What inference should we expect for the form (e, mn)? Analogy with the categoricals, the two-dimensional sentences and the *muttaṣil* sentences suggests that the negation of $(a, mn)(p, q)$ should be—ignoring augments—either $(a, mn)(\bar{p}, q)$ or $(a, mn)(p, \bar{q})$. The symmetry of (a, mn) makes it hard to see why Ibn Sīnā should prefer either of these to the other. So we test them both against his inferences. If $(e, mn)(p, q)$ is $(a, mn)(\bar{p}, q)$, then we can use contradictory negations, together with equivalences already established, to get a complete set of equivalences between *muttaṣil* and *munfaṣil* forms, as follows (where $\sharp$ means contradictory negation):

OPTION α:

$$\begin{array}{ccccccc}
(a, mt)(p, q) & \Leftrightarrow & (e, mt)(p, \bar{q}) & \Leftrightarrow & (a, mn)(\bar{p}, q) & \Leftrightarrow & (e, mn)(p, q) \\
\sharp & & \sharp & & \sharp & & \sharp \\
(o, mt)(p, q) & \Leftrightarrow & (i, mt)(p, \bar{q}) & \Leftrightarrow & (o, mn)(\bar{p}, q) & \Leftrightarrow & (i, mn)(p, q)
\end{array}$$

On the other hand if $(e, mn)(p, q)$ is $(a, mn)(p, \bar{q})$ then we have

OPTION β:

$$\begin{array}{ccccccc}
(a,mt)(p,q) & \Leftrightarrow & (e,mt)(p,\bar{q}) & \Leftrightarrow & (a,mn)(\bar{p},q) & \Leftrightarrow & (e,mn)(\bar{p},\bar{q}) \\
\sharp & & \sharp & & \sharp & & \sharp \\
(o,mt)(p,q) & \Leftrightarrow & (i,mt)(p,\bar{q}) & \Leftrightarrow & (o,mn)(\bar{p},q) & \Leftrightarrow & (i,mn)(\bar{p},\bar{q})
\end{array}$$

Surprisingly it turns out that almost all the relevant inferences in *Qiyās* vi.2 and vii.2 support either Option α or Option β, and Option α is preferred by a ratio of about two to one.

Here are the inferences that agree with Option α and not with Option β:

Table 6: Option α				
1.	$(a,mt)(p,q)$	$\to$	$(e,mn)(p,q)$	306.4f, 310.10, 312.15, 314.10,382.5, 384.3
2.	$(a,mt)(p,\bar{q})$	$\to$	$(e,mn)(p,\bar{q})$	308.4, 311.17-312.1
3.	$(a,mt)(\overline{p},q)$	$\to$	$(e,mn)(\overline{p},q)$	315.8
4.	$(a,mt)(p,\overline{q})$	$\to$	$(e,mn)(p,\overline{q})$	381.15
5.	$(e,mt)(p,\overline{q})$	$\to$	$(e,mn)(p,q)$	310.9f, 384.4
6.	$(a,mn)(\overline{p},q)$	$\to$	$(e,mn)(p,q)$	384.2, 381.5
7.	$(a,mn)(\overline{p},\overline{q})$	$\to$	$(e,mn)(p,\overline{q})$	381.12
8.	$(i,mn)(p,q)$	$\to$	$(i,mt)(p,\bar{q})$	306.11f, 310.16f

The following inferences agree with Option β and not with Option α:

Table 7: Option β				
1.	$(e,mt)(p,q)$	$\to$	$(e,mn)(\overline{p},q)$	315.3, 316.12f, 382.15
2.	$(e,mt)(\overline{p},q)$	$\to$	$(e,mn)(p,q)$	314.6, 317.6
3.	$(e,mt)(\overline{p},\overline{q})$	$\to$	$(e,mn)(p,\overline{q})$	310.3f
4.	$(e,mn)(p,q)$	$\to$	$(a,mt)(\overline{p},\overline{q})$	383.12
5.	$(i,mn)(\overline{p},q)$	$\to$	$(i,mt)(p,q)$	380.7, 382.17
6.	$(i,mn)(p,\overline{q})$	$\to$	$(i,mt)(\overline{p},\overline{q})$	381.17 B

The following inferences agree with neither:

Table 8: Neither option				
1.	$(e,mt)(\bar{p},q)$	$\to$	$(e,mn)(\bar{p},q)$	316.4
2.	$(i,mt)(\overline{p},q)$	$\to$	$(o,mn)(\overline{p},q)$	316.16f
3.	$(a,mn)(p,q)$	$\to$	$(e,mn)(\overline{p},q)$	379.17, 380.1
4.	$(a,mn)(p,q)$	$\to$	$(e,mn)(\overline{q},p)$	380.3
5.	$(i,mn)(p,q)$	$\to$	$(o,mt)(\overline{p},q)$	381.3

Again a large majority of the inferences are correct for either Option α or Option β.

I know no proofs where Ibn Sīnā uses both Option α and Option β, but there are plenty of places where a proof using one option is followed at once by a proof using the other.

How can we explain Ibn Sīnā's apparent willingness to use two different interpretations for the same sentence form? The first part of the answer is that Ibn Sīnā already does this, in line with his predecessors, by allowing disjunctions to be taken sometimes as exclusive and sometimes as inclusive. But leaving open the choice between Option α and Option β seems somehow more brazen, particularly since there is no conceivable way that anybody could deduce either of the two options just by meditating on the Arabic texts that Ibn Sīnā uses for (e, mn) and (i, mn).

The following viewpoint may help. Suppose we take strict (exclusive) (a, mn) as our paradigm form of disjunction. Then (e, mn) should deny, at all times, what (a, mn) affirms at all times. Now strict $(a, mn)(p, q)$ affirms, for all times, that p and q have different truth values. So $(e, mn)(p, q)$ can be taken to say that at all times p and q have the same truth value. This is the conjunction of two statements:

(31) $$\forall\tau\ (p(\tau) \to q(\tau)), \quad \forall\tau\ (q(\tau) \to p(\tau))$$

which are exact equivalents of $(e, mn)(p, q)$ under Option α and Option β respectively.

So by assuming a premise $(e, mn)(p, q)$ and then choosing one of the options, a reasoner is choosing to follow one of the implications of the premise. There is never any harm in doing this if that implication is all that is needed to carry through the argument. And for logicians working in natural language, either of the two forms (31) is much easier to handle than their conjunction.

Harder to justify is using one of the options when a premise of the form $(i, mn)(p, q)$ has been assumed, because in this case we are choosing one of the two disjuncts of a disjunction, and hence we are claiming more than the premise stated. Even this can be defended by saying: Yes we are claiming more than we assumed, but the result is that the proofs become much easier to state and follow, and there is no harm if in the particular case under consideration we happen to know that the claimed disjunct is true. And if we do happen to know that, we can interpret the premise that way at the outset, so there is no threat to the formality of the logical argument.

There are places both in Ibn Sīnā and in other medieval Arabic writers which can be read as following the line just stated. One is a discussion of negation in al-Sīrāfī, analysed recently in [7]. I hope to discuss the issue more fully elsewhere. It bears on questions about how formal logic is used in practice.

9 Three phases of propositional logic

We gather up the results so far.

PL1 is the hypothetical logic of *Qiyās* viii.1,2. The sentence forms carry no time quantification. (The one counterexample at *Qiyās* 403.8f is read in a way incompatible with its use in either PL2 or PL3.) There are four main sentence forms:

(32)

muttaṣil	$(p \to q)$
complete *muttaṣil*	$(p \leftrightarrow q)$
munfaṣil	$(p \vee q)$
strict *munfaṣil*	$(p \leftrightarrow \neg q)$

The letters p, q stand for any categorical sentences, either affirmative or negative.

PL2 is the hypothetical logic of *Qiyās* vi.1. There are four sentence forms, all of them temporally quantified:

(33)

$\boldsymbol{(a, mt)(p, q)}$	$(\forall\tau\ (p(\tau) \to q(\tau)) \wedge \exists\tau p(\tau))$
$\boldsymbol{(e, mt)(p, q)}$	$\forall\tau\ (p(\tau) \to \neg q(\tau))$
$\boldsymbol{(i, mt)(p, q)}$	$\exists\tau\ (p(\tau) \wedge q(\tau))$
$\boldsymbol{(o, mt)(p, q)}$	$(\exists\tau\ (p(\tau) \wedge \neg q(\tau)) \vee \forall\tau\neg p(\tau))$

The letters p, q stand for any categorical sentences, and possibly other sentences too. The inference rules are the analogues of those of categorical logic.

PL3 is the hypothetical logic of *Qiyās* vi.2 and vii.2. There are ten main sentence forms, all of them temporally quantified:

$$
(34) \qquad \begin{array}{ll}
\boldsymbol{(a,mt)(p,q)} & \forall\tau\ (p(\tau) \to q(\tau)) \\
\boldsymbol{(e,mt)(p,q)} & \forall\tau\ (p(\tau) \to \neg q(\tau)) \\
\boldsymbol{(i,mt)(p,q)} & \exists\tau\ (p(\tau) \wedge q(\tau)) \\
\boldsymbol{(o,mt)(p,q)} & \exists\tau\ (p(\tau) \wedge \neg q(\tau)) \\
\boldsymbol{(a,mn)(p,q)} & \forall\tau\ (p(\tau) \vee q(\tau)) \\
\boldsymbol{(e,mn)_\alpha(p,q)} & \forall\tau\ (p(\tau) \to q(\tau)) \\
\boldsymbol{(i,mn)_\alpha(p,q)} & \exists\tau\ (p(\tau) \wedge \neg q(\tau)) \\
\boldsymbol{(e,mn)_\beta(p,q)} & \forall\tau\ (q(\tau) \to p(\tau)) \\
\boldsymbol{(i,mn)_\beta(p,q)} & \exists\tau\ (q(\tau) \wedge \neg p(\tau)) \\
\boldsymbol{(o,mt)(p,q)} & \exists\tau\ (\neg p(\tau) \wedge \neg q(\tau))
\end{array}
$$

The letters p, q stand for any categorical sentences, and possibly other sentences too; $\overline{p}$ is the contradictory negation of p. The inference rules for the *muttaṣil* sentences are the analogues of those of categorical logic, except for (a)-conversion and the syllogisms *Darapti* and *Felapton*. Further inference rules allow any one of the sentences below to be replaced by any other sentence in the same horizontal line:

$$
\begin{array}{lllllllll}
(a,mt)(p,q) & \Leftrightarrow & (e,mt)(p,\bar{q}) & \Leftrightarrow & (a,mn)(\bar{p},q) & \Leftrightarrow & (e,mn)_\alpha(p,q) & \Leftrightarrow & (e,mn)_\beta(\bar{p},\bar{q}) \\
(o,mt)(p,q) & \Leftrightarrow & (i,mt)(p,\bar{q}) & \Leftrightarrow & (o,mn)(\bar{p},q) & \Leftrightarrow & (i,mn)_\alpha(p,q) & \Leftrightarrow & (i,mn)_\beta(\bar{p},\bar{q}) \\
(a,mn)(p,q) & \Leftrightarrow & (a,mn)(q,p) & & & & & & \\
(o,mn)(p,q) & \Leftrightarrow & (o,mn)(q,p) & & & & & &
\end{array}
$$

Optionally a sentence of the form $(a,mn)(p,q)$ with affirmative clauses can be designated as 'strict', in which case it also allows the inference rule

$$(a,mn)(p,q) \ \Rightarrow\ (a,mt)(p,\overline{q}).$$

Also a sentence of the form $(a,mt)(p,q)$ can be designated as having existential import, in which case (a)-conversion can be applied to it, and it can be used as a premise in the analogue of *Darapti* or *Felapton*.

In spite of its relative complication, PL3 should be celebrated as one of the jewels in the crown of medieval logic. It contains for the first time a sound and complete proof calculus for boolean algebra, though severely limited in the sentence types that it can handle. To extend it to full first-order boolean algebra would involve marrying PL3 to the techniques of Ibn Sīnā's treatment of reductio ad absurdum [12], something that Ibn Sīnā never attempted. In fact the next advance in boolean algebra after *Qiyās* vi.2 was made by Leibniz, who did combine within a single system all the proof techniques needed for first-order boolean algebra [27].

The preceding two sections give us some leverage for distinguishing different stages within Ibn Sīnā's hypothetical logic. To speak of 'stages' suggests movement in time, and it looks to me very likely that the stages do represent different moments in Ibn Sīnā's exploration of hypothetical logic. For example the early *Mukhtaṣar* seems to represent a stage when Ibn Sīnā still regarded hypothetical logic as concentrated in the universal sentences (as in PL1), though that work does already mention the time quantification. But all three logics appear in *Qiyās*, so it's clear that Ibn Sīnā himself felt that each of them made sense on its own.

One can examine also how Ibn Sīnā's treatment of hypothetical logic in other places fits in with PL1–3. For example *Qiyās* vii.1 uses the *muttaṣil* sentence forms of (2), so it is not PL1. Like PL2 it restricts to *muttaṣil* and ignores *munfaṣil* sentences; but it is not PL2 since it uses the equivalences of PL3 between affirmative and negative forms (e.g. *Qiyās* 366.1f), and hence rejects the augments. So it belongs with PL3.

Relating Ibn Sīnā's discussion of reductio ad absurdum (*Qiyās* viii.3, cf. [12]) to PL1–3 is more complicated. Ibn Sīnā needs unquantified conditionals, which belong in PL1. On the other hand he also needs a rule saying that if q entails r then 'If p then q' entails 'If p then r', and his proof of this rule consists of the section *Qiyās* vi.4 which belongs with PL2. On the other hand again, one of his devices is to allow 'If every C is a B then every C is a B' to be assumed as an axiom; which is illegitimate if existential import is assumed, since in general there is no guarantee that there is any time or situation in which 'Every C is a B' is true. At *Qiyās* viii.3, 410.13 he writes the axiom as 'If not not every C is a B then every C is a B'; is this a device for adding a negation to the axiom so as to avoid the existential import? This all has the air of an untidy corner that he never got around to clearing up.

10 Revisiting the Rescher translations

Unfortunately Rescher's translations for (e, mn) and (i, mn) sentences are useless, not least because they fail to distingish between Option α and the very different Option β.

Also Rescher's translations for (e, mt) and (i, mt) are obscure and potentially misleading. Since this may be less clear than the problems with

(e, mn) and (i, mn), let me indicate two published works whose authors rely on the Rescher translations for (e, mt) and (i, mt) and seem to have been misled by them.

The first work is Nabil Shehaby's commented translation [32] of *Qiyās* v.1–viii.2. In this work Shehaby relies throughout on Rescher's translations, and shows no awareness of any problem with them. At least it can be said that Rescher's translations allow one to reconstruct Ibn Sīnā's Arabic text faithfully, regardless of whether they convey its meaning. But even here a reservation is needed: there is evidence that Shehaby has used the Rescher translations to 'correct' Ibn Sīnā's text in places where no correction was needed. At *Qiyās* 330.10 ([32] p. 130) Shehaby correctly calculates that a negation of the second clause of an (e, mt) sentence is needed, so he adds it to the Cairo text [19], not realising that Ibn Sīnā has already expressed it with *laysa albatta*.

We turn to a second example. In [5] (repeated in [24] p. xxxiiif) Khaled El-Rouayheb ascribes to Ibn Sīnā a claim which he describes as 'Avicenna's principle'. To state it he uses Rescher's translations but with 'if' for 'when':

(35) Avicenna ... held the following two conditionals to be logically equivalent:
(1) Always (*Kullamā*): If Every A is B then Every J is D.
(2) Never (*Laysa al-batta*): If Every A is B then Not Every J is D.

The reader will recognise this equivalence as

(36) $$(a, mt)(p, q) \;\Leftrightarrow\; (e, mt)(p, \overline{q}),$$

which belongs in PL3 since it uses temporally quantified sentences and doesn't assume existential import on (a, mt). In symbols it says:

(37) $$\forall\tau\ (p(\tau) \to q(\tau)) \;\Leftrightarrow\; \forall\tau\ (p(\tau) \to \neg\neg q(\tau))$$

which is true by cancellation of double negation, and would be recognised as valid by every classical logician. But El-Rouayheb regards this claim of equivalence as something peculiar to Ibn Sīnā. So he must be reading one or both of the translations wrongly.

El-Rouayheb goes on to explain that Khūnajī attacked 'Avicenna's claim' by showing that an impossible antecedent can imply both a consequent and the contradictory negation of that consequent. So it looks as if El-Rouayheb is ascribing to Ibn Sīnā a claim along the lines

(38) $$(p \to q) \to \neg(p \to \neg q).$$

This is confirmed by El-Rouayheb's statement that Ibn Sīnā's principle is discussed by Chris Martin [29] in connection with Boethius and Abelard. If we look up Martin's paper we find that the principle which he discusses is exactly (38). This principle differs from Ibn Sīnā's equivalence by having no temporal quantifiers. Leaving that aside, it seems clear that El-Rouayheb has read $(p \to q)$ as represented by $(a, mt)(p, q)$ and $\neg(p \to \neg q)$ as represented by $(e, mt)(p, \overline{q})$. In other words El-Rouayheb has opted for the reading of (e, mt) that we numbered (i) in (19) above and found reason to reject.

El-Rouayheb also includes a paraphrase of what he takes to be Ibn Sīnā's proof of (1) $\Rightarrow$ (2) at *Qiyās* 367.11–368.1. It goes by showing that (1) is incompatible with the contradictory negation of $(e, mt)(p, \overline{q})$, namely $(i, mt)(p, \overline{q})$. He states this contradictory negation

(39) Once (*Qad yakūn*): If Every A is B then Not Every J is D.

(This again follows Rescher's translations with colon, but again putting 'if' for Rescher's 'when'.) He makes Ibn Sīnā 'assume the antecedent', i.e. assume that every A is B. But in fact the statement translated as (39) states that at some time every A is B and not every J is D—so that 'Every A is B' is true at that time and doesn't need to be assumed. (Nor does Ibn Sīnā say he assumes it; *mawḍū^{c}atun* at *Qiyās* 367.15 means 'stated as antecedent', not 'assumed'.) The Rescher style of translation has been used to present (i, mt) as an 'If ... then' form, when in fact it is a quantified conjunction. (None of these comments detract from El-Rouayheb's discussion of Khūnajī, which is the main thrust of his paper.)

So Rescher's translations are not harmless. Finding acceptable replacements is not easy, but let me attempt it.

As remarked in Section 3 above, Rescher's translations of the *muttaṣil* sentences fail to relativise the quantifiers correctly. For example the translation of $(a, mt)(p, q)$ should express:

(40) Every time or situation τ such that $p(\tau)$ holds is a time or situation such that $q(\tau)$ holds.

The ambiguity between times and situations would be better represented by using 'Always', and of course we want to avoid the variable τ and the logician's jargon 'such that'. There is a way of meeting these requirements, namely to write

(41) $\boldsymbol{(a, mt)(p, q)}$ It is always the case when p that q.

The sentence forms (e, mt), (i, mt) and (o, mt) then fall out as:

(42)

$\boldsymbol{(e, mt)(p, q)}$	It is never the case when p that q.
$\boldsymbol{(i, mt)(p, q)}$	It is sometimes the case when p that q.
$\boldsymbol{(o, mt)(p, q)}$	It is not always the case when p that q.

For (a, mn) and (o, mn) we have natural translations:

(43)

$\boldsymbol{(a, mn)(p, q)}$	It is always the case that either p or q.
$\boldsymbol{(i, mt)(p, q)}$	It is not always the case that either p or q.

For the (e, mn) and (i, mn) cases I would suggest copying the strategy that we used for (e, mt) and (i, mt), as follows.

(44)

$\boldsymbol{((e, mn)_\alpha(p, q)}$	At every time other than when q, p..
$\boldsymbol{((i, mn)_\alpha(p, q)}$	At some time other than when q, p..
$\boldsymbol{(e, mn)_\beta(p, q)}$	It is not always the case when p that q.
$\boldsymbol{(i, mn)_\beta(p, q)}$	At some time other than when p, q.

These translations allow a reasonable amount of uniformity when standard translations are needed in a formal setting. For more loose-limbed work one can often simplify them, for example writing $(a, mn)(p, q)$ as 'Always either p or q'.

The following remark is for readers who are puzzled at the role of 'when' in the translations of all four *muttaṣil* sentences. In English the word 'when', followed by a sentence p, designates the class of times or situations in which p is true. In our translations, this class relativises the temporal quantifier to the left of it. The use of 'when' as a near equivalent of 'if' seems to be a derivative usage, short for 'at the time when'. Since Ibn Sīnā's own formulations for (e, mt) and (i, mt) in (2) play a similar trick with *iḏā*, I presume the semantics of Arabic gives the word a similar role to English 'when' in designating classes of situations. (For a deeper analysis from the point of view of formal semantics, see Kratzer [25] Chapter 4 or von Fintel [6], both on a now widely accepted theory of adverbial quantification, and [8] for application to medieval Arabic debates.)

We have not yet suggested any translations for *muttaṣil* and *munfaṣil*. Rescher [31] used 'conjunctive' and 'disjunctive' respectively. These are too specific: on the *muttaṣil* side, only (i, mt) and (o, mt) are conjunctions and on the *munfaṣil* side only (a, mn) is a disjunction. Shehaby [32] goes to the opposite extreme and translates as 'connective' and 'separative', which are good literal translations and do connect with the explanations that Ibn

Sīnā gives for *muttaṣil* and *munfaṣil* in PL1. But they don't bear any visible relationship to Ibn Sīnā's temporally quantified forms.

The paper [12] suggested two intermediate translations, that relate to the literal senses of *ittiṣāl* and *infiṣāl* and do suggest some of the logical content of the sentence forms. For *muttaṣil* it suggested 'meet-like', where 'meet' goes with *ittiṣāl* in the sense of joining, and with (i, mt) and (o, mt) in the sense of being built around a conjunction. For *munfaṣil* it suggested 'difference-like', where 'difference' is one of the senses of *faṣl*, and strict (a, mn) expresses logical difference. The suffix '-like' corresponds to the Arabic *-ī* and warns us that the names fit the logical content only approximately. I don't think the results of the present paper diminish the case for these two translations.

11 Appendix: Tables

11.1 The relevant texts of Ibn Sīnā

The main passages in which Ibn Sīnā discusses hypothetical logic are as follows. The works are listed in what is widely taken to be their chronological order, from *Mukhtaṣar* sometime before 1014 to *Išārāt* around 1030 (see Gutas [9] e.g. p. 165 for the dating).

Mukhtaṣar **= Middle Summary**

46.18–47.9 hypothetical sentences; 77.1–81.11 classification of hypothetical sentences; 121.8–136.17 hypothetical syllogisms

Najāt **= Deliverance (or Salvation)**

19.14–20.11 hypothetical sentences; 79.6–83.3 classification of hypothetical sentences; 83.4–92.5 hypothetical syllogisms

ᶜIbāra **= De Interpretatione, from The Cure**

32.15–17, 33.12–34.5, 37.9–11 hypothetical sentences.

Qiyās **= Syllogism, from The Cure**

v 231.1–292.16 classification of hypothetical sentences; vi 295.1–viii 411.5 hypothetical syllogisms

Dānešnāmeh **= Book of Wisdom**

French 83–86 hypothetical sentences; 98f hypothetical syllogisms

Mašriqiyyūn **= Easterners**

60.21–63.7 hypothetical sentences

Išārāt **= Pointers**

72.4–13 hypothetical sentences; 73.6–15 and 76.16–7.5 and 80.5–82.4 and 83.3–12 classification of hypothetical sentences; 139.9–144.8 and 157.1–159.2 hypothetical syllogisms

The following table shows how often the different hypothetical forms (2) occur in the main source documents.

(45)

	Mukhtaṣar	*Najāt*	*Qiyās*	*Mašriqiyyūn*	*Išārāt*
(a, mt)	49	2	321	1	3
(e, mt)	44	1	162	1	1
(i, mt)	4	1	75	0	1
(o, mt)	0	2	56	1	1
(a, mn)	0	1	54	1	1
(e, mn)	2	1	50	1	1
(i, mn)	0	1	25	0	1
(o, mn)	0	1	23	0	1

11.2 Summary of proof-theoretic material on hypotheticals in *Qiyās*

Qiyās vi.1, 295–304

After setting up a correspondence between *muttaṣil* sentences and categorical sentences, Ibn Sīnā lists all the valid syllogisms with *muttaṣil* sentences in the same order as he gives elsewhere for the valid categorical syllogisms, and with the same proofs. (*Mukhtaṣar* [17] 101.18–107.3, *Najāt* [20] 57.1–64.3, *Qiyās* [19] ii.4, 108.12–119.8 and *Dānešnāmeh* [21] 65.8–80.2. The account in *Išārāt* is sketchier and mixed with modal material.) He also gives the same productivity conditions as he gives elsewhere for categorical syllogisms. (*Mukhtaṣar* 100.1–3, 100.5–7, 101.19f, 104.6f, *Najāt* 53.11–13, 57.4, 58.8–10, 61.7f, *Qiyās* 108.8f, 109.8f, 114.2f, 116.14f, *Dānešnāmeh* 65.5, 66.6f, 69.10f, 74.6–75.1.)

Qiyās vi.2, 305–318

Qiyās vi.2 is devoted to syllogisms where one premise is *muttaṣil* and the other premise is *munfaṣil*. The section splits into four blocks A, B, C, D as follows:

A. The first premise is *muttaṣil*, the second is *munfaṣil* and the clauses are arranged as in first figure.

B. The first premise is *muttaṣil*, the second is *munfaṣil* and the clauses are arranged as in third figure.

C. The first premise is *munfaṣil*, the second is *muttaṣil* and the clauses are arranged as in first figure.

D. The first premise is *munfaṣil*, the second is *muttaṣil* and the clauses are arranged as in second figure.

If X is any one of A, B, C, D then X is divided into three subblocks as follows:

X1. The *munfaṣil* premise is read as strict when it is (a, mn), and the middle clause is affirmative.

X2. The *munfaṣil* premise is not read as strict, and the middle clause is affirmative.

X3. The *munfaṣil* premise is not read as strict, and the middle clause is negative.

Each subblock is subdivided into nine paragraphs numbered ij ($1 \leqslant i, j \leqslant 3$) and listed lexicographically by this numbering, as follows:

$i = 1$: Both premises are affirmative.

$i = 2$: Just the first premise is negative.

$i = 3$: Just the second premise is negative.

$j = 1$: Both premises are universal.

$j = 2$: Just the first premise is existential.

$j = 3$: Just the second premise is existential.

Some subblocks or paragraphs are missing.

***Qiyās* vii.1, 361–372**

Ibn Sīnā lists the equivalences and contradictory negations between all *muttaṣil* sentences, and proves one case in detail. The account is sixteen times longer than it needs to be, because Ibn Sīnā considers for each *muttaṣil* form the results of putting any one of the four categorical sentences in place of p and likewise in place of q (using different term letters for the two categorical sentences), though the rules that he illustrates can all be stated in terms of just p and q.

***Qiyās* vii.2, 373–384**

The section begins by doing for the *munfaṣil* sentences what the previous section did for the *muttaṣil* sentences. THEN WHAT?

***Qiyās* vii.3, 385f**

This very short section has an air of incompleteness. Ibn Sīnā derives (e, mt)-conversion from (i, mt)-conversion. He also observes that if an (e, mt) sentence is true by *ittifāq*, its converse need not be true by *ittifāq*.

11.3 Notes on textual readings in *Qiyās*

Qiyās vi.2, 312.14f. Here Ibn Sīnā states three conclusions for the same syllogism, namely $(e, mt)(\overline{p}, q)$, $(a, mt)(p, q)$, $(e, mn)(p, q)$ in that order. By the readings we found for PL2 and the *muttaṣil* part of PL3, the second sentence

$(a, mt)(p, q)$ is a correct conclusion, but the first one, $(e, mt)(\overline{p}, q)$ is not; the third, $(e, mn)(p, q)$, is correct under Option α. So it seems likely that the first sentence, $(e, mt)(p, \overline{q})$, is corrupted.

Qiyās vi.2, 316.16f, the sentence beginning *laysa immā*. The context requires a sentence saying that at some time we have $\overline{r}$ and p. Emending to *laysa albatta immā* gives a sentence beginning 'At all times ...', clearly wrong. Emending to *laysa dā'iman immā* gives, by the results of Subsection 8.2, the sentence 'There is a time when r and $\overline{p}$', which is the wrong way round. Most likely Ibn Sīnā intended the second but put the negation in the wrong place. Since the sentence is clearly wrong as it stands, I ignore it in the calculations of the paper.

Qiyās vii.2, 381.17–382.2. The text states two entailments: $(i, mn)(p, \overline{q}) \Leftrightarrow (i, mt)(\overline{p}, \overline{q}) \Leftrightarrow (o, mt)(p, \overline{q})$. The first of these is in agreement with Option β. The second is out of line with our results for *muttaṣil*, but it can be corrected by replacing $(i, mt)(\overline{p}, \overline{q})$ by $(i, mt)(p, q)$. This correction makes the first entailment correct for Option α. There is nothing in the manuscripts to support the text with this correction, but it seems likely to be what Ibn Sīnā intended.

Qiyās vii.2, 384.1–5. Here Ibn Sīnā gives rules for translating from negative *munfaṣil* sentences to other kinds of sentence. For (e, mn) sentences his rules agree exactly with the top line of the definition of Option α in Subsection 8.3. For (i, mn) sentences the translation to (o, mn) is clearly wrong. It seems most likely that Ibn Sīnā only intended the rules to be for (e, mn) sentences, so I have not included the rules that result from applying them to (o, mn).

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