# Ibn Sīnā's new vision of logic

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http://wilfridhodges.co.uk/arabic63.pdf

I nominate the passage iii.2, 140.8–143.14 of Ibn Sīnā's *Qiyās* (from his *Šifā'*) as the most outstanding achievement of logic between Aristotle and Leibniz, for its depth, originality and precision.

The whole of the rest of this lecture will be spun out of this short passage.

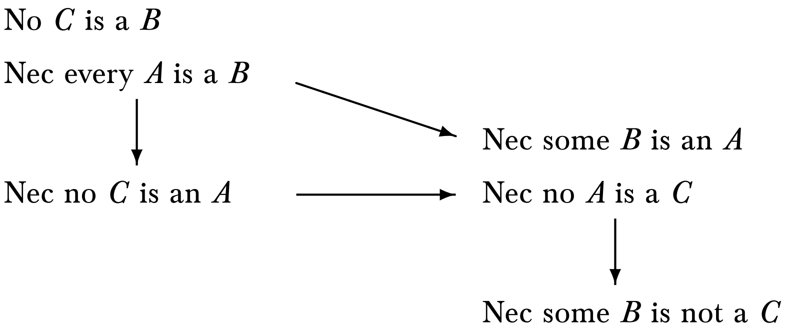
Aristotle, *Prior Analytics* i.10, 30b25–31, considers the 2nd figure inference

> *No C is a B.*
> *With necessity, every A is a B.*
> *Therefore with necessity, no C is an A.*

(Aristotle has *A*, *B* transposed. We follow Ibn Sīnā's lettering.)

Aristotle argues that this inference is invalid.
Normally he would show this by finding *A*, *B* and *C* that make the premises true and the conclusion false.
But this time he does something different.

He argues:
Assume the premises, and suppose the inference is valid.
Then we can extend the inference as follows.



*But we can choose B and C so that every B is a C with possibility, but no C is in fact a B.*
*So if the inference was valid, we could deduce a contradiction from true premises.*

This is Aristotle's argument, and it's both neat and ingenious. (Robin Smith on a parallel argument of Aristotle: 'sophisticated and flawless'.)

But Ibn Sīnā will use this same argument,
not to demonstrate a failure of inference,
but to debug Aristotle's procedure and uncover a fatal mistake (which in the West was apparently first noticed in 1996, by Paul Thom).

Early in *Qiyās* book i, Ibn Sīnā lays out the temporal sentences that will be the basis of all his new logical systems,
both alethic modal (i.e. with 'necessarily' and 'possibly') and hypothetical.
Here we will be concerned only with the temporal and the alethic modal sentences.

These temporal sentences contain two quantifications,
one over individuals and one over 'times' or 'situations'.
Following Oscar Mitchell in the 1880s,
we call these sentences 'two-dimensional' (2D).

Al-Rāzī later commented that 'The logical literature has found itself stumbling around as a result of using the expression "necessary" sometimes for what is inevitable, and sometimes for what is permanent'.

In fact Ibn Sīnā does use 'necessary' in both these ways,
but he also distinguishes.
He has a translation from alethic modal to temporal,
and in key places he *justifies* an alethic modal inference by translating it to temporal. (Never the other way round, unlike later logicians such as Ḥillī.)

The translation for affirmative sentences:

Necessarily every/some $A$ is a $B$:

(1) Every/some sometime-$A$ is a $B$ throughout its existence.

Possibly every/some $A$ is a $B$:

(2) Every/some sometime-$A$ is a $B$ at least once during its existence.

Similarly for negative sentences:

Necessarily every/some $A$ is not a $B$:

(1) Every/some sometime-$A$ is not a $B$, throughout its existence.

Possibly every/some $A$ is not a $B$:

(2) Every/some sometime-$A$ is not a $B$, at least once during its existence.

*Qiyās* section iii.2 deals with syllogisms where one premise has 'necessarily' and the other premise is absolute (i.e. non-modal). In such passages Ibn Sīnā reads the 'necessarily' as (1) and the 'absolute' as (2).
He is *translating* Aristotle's alethic 'necessarily' to (1).

But (confusingly, as Rāzī says) Ibn Sīnā himself in *Qiyās* and the later work *Mašriqiyyūn* uses 'necessary' and 'absolute' as *names* for the two kinds of 2D sentence (1) and (2).

In short, his own calculations in *Qiyās* iii are 2D temporal, not alethic.

These procedures allow Ibn Sīnā to read Aristotle's argument as if it was within 2D logic.
Within this logic there is no room for disagreement about what inferences hold.
For example the inference that Aristotle is proving invalid is clearly valid:

> *Every sometime-C is at least once in its existence not an A.*
> *Every sometime-B is an A throughout its existence.*
> *Therefore every sometime-C is throughout its existence not a B.*

(NB For Ibn Sīnā a thing can't be a $C$ when it doesn't exist.)

But also Aristotle's added steps work (with a slight adjustment) in the 2D setting:

*Every sometime-A is always a B.*
*Therefore some sometime-B is sometimes an A.*

*Every sometime-C is never an A.*
*Therefore every sometime-A is never a C.*

The first of these conclusions is weaker than Aristotle's.
But it's enough for Aristotle's last inference:

*Some sometime-$B$ is sometimes an $A$.*
*Every sometime-$A$ is never a $C$.*
*Therefore some sometime-$B$ is never a $C$.*

Then as before, Aristotle can say:
We could have chosen $B$ and $C$ so that every sometime-$C$ is sometimes not a $B$, but every sometime-$B$ is sometimes a $C$.

We seem to have proved a contradiction out of thin air. Help!

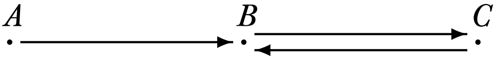
Ibn Sīnā's solution (and Paul Thom's):
We can choose $B$ and $C$ that way, but not if we also choose $A$ so that every sometime-$A$ is throughout its existence a $B$.
The reason is that this choice of $B$ and $C$ entails that nothing is a $B$ throughout its existence.

So we have a minimal inconsistent set:

*Every sometime-C is sometimes not a B.*
*Every sometime-B is sometimes a C.*
*Every (or some) sometime-A is always a B.*

Ibn Sīnā notes that 'some' works in place of 'every' in the third proposition.

We draw the 'graph' of a minimal inconsistent set by drawing an arrow from $A$ to $B$ for a sentence with subject $A$ and predicate $B$. The minimal inconsistent set above has graph




**Theorem**. In categorical syllogisms every minimal inconsistent set has a circular graph.

This was probably known to Aristotle, at least subconsciously. He wrongly assumed it applied to all logic.

The inference

$$\phi_1, \ldots, \phi_n \ \vdash \ \theta.$$

is valid without redundancy if and only if the set

$$\phi_1, \ldots, \phi_n, \text{not-}\theta$$

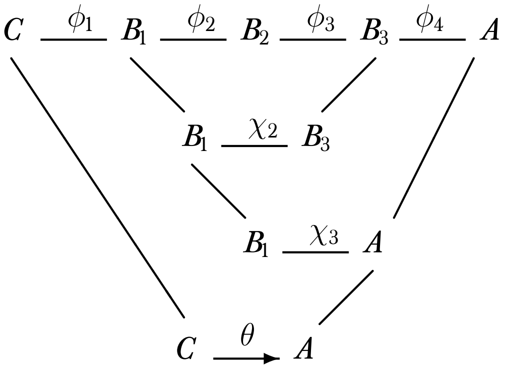
is minimal inconsistent. So in categorical syllogisms, every inference has the form

$$C \ \overset{\phi_1}{\overline{\qquad}} \ B_1 \ \overset{\phi_2}{\overline{\qquad}} \ B_2 \quad \ldots \quad B_{n-1} \overset{\phi_n}{\overline{\qquad}} \ A$$

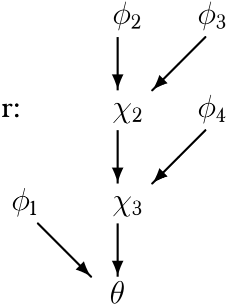
$$\vdash \quad C \ \xrightarrow{\ \theta\ } \ A$$

The lines for $\phi_1, \ldots, \phi_n$ are arrows that might go either way.

We prove this inference by applying syllogisms to pairs of formulas, as in:




or:




'Demonstrations . . . don't go in a circle. They advance in a straight line by addition of terms.' (Aristotle *De Anima* i.3, 407a27–29)

Western philosophers at least up to Descartes assumed that all proofs must have this straight-line form.

Ibn Sīnā, having proved otherwise in *Qiyās* iii.2 (around 1024), looked around for other examples.

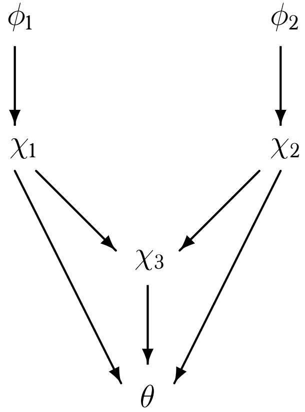
Within 2D logic he discovered one other kind of noncircular minimal inconsistent set of three propositions (in his later *Išārāt* i.7):

*Every A is a B throughout its existence.*
*Every B is a C throughout the time while it's a B.*
*Every B is, throughout its existence, not a C.*

Note the 'while it's a $B$'. This is the best possible within 2D logic, and there are no other examples (except that we can swap 'a $C$' and 'not a $C$'.)

The two sentences with subject $B$ and predicate $C$ combine as: 'Every $B$ is a $C$ throughout the time while it's a $B$, but not throughout its existence'.
Al-Rāzī in his *Mulakhkhaṣ* introduces this form under the name 'conventional necessary'.
He presumably extracted it from *Išārāt* i.7.

In his *Dāneshnāmeh* Ibn Sīnā pointed out that the first Proposition of Euclid's *Elements* also has a non-straight-line form:




Note that two premises here have to be used twice—which never occurs in categorical syllogisms.

## The translation from alethic to temporal

Leave aside for a moment the question of what Ibn Sīnā thought he was doing with this translation.

In fact the translation is formally identical with the modern procedure of justifying an inference 'semantically', i.e. by translating it to an inference between statements about Kripke structures.

This is shown in detail in Hodges and Johnston, 'Medieval modalities and modern methods: Avicenna and Buridan', *IfCoLog Journal of Logics and their Applications* 4 (4) (2017) 1029–1073. (Issue in memory of Grisha Mints.)

This formal correlation came to light vividly in 2015, when Spencer Johnston in his doctoral thesis gave a Kripke semantics for the divided modal logic of Buridan (early 14th century France).
Unexpectedly, Johnston's propositions about Kripke structures were exact translations of Ibn Sīnā's 2D propositions.

Caution: Ibn Sīnā has no language for saying 'This is a semantics for that'.
So we should be cautious about assuming he has concepts for thinking it either.

Ibn Sīnā doesn't have enough set theory for a set-theoretic description of Kripke structures.

So instead of justifying his 2D inferences set-theoretically, he adapts Aristotle's proof theory for categorical syllogisms.

In every case where a 2D inference holds but can't be proved by Aristotle's method for the corresponding categorical syllogism, Ibn Sīnā indicates an alternative proof, and these alternative proofs all work correctly.

For example the 2D version of the inference that Aristotle took to be invalid:

> *Every sometime-$C$ is sometime in its existence not a $B$.*
> *Every sometime-$A$ is a $B$ throughout its existence.*
> *Therefore every sometime-$C$ is throughout its existence not an $A$.*

can't be proved by Aristotle's method, which was to convert 'No $C$ is a $B$' to 'No $B$ is a $C$' and use *Celarent*.

Ibn Sīnā suggests instead changing the meanings of the letters so as to include the 'sometime' or 'always'.

Thus:

*No sometime-C is an always-B.*
*Every sometime-A is an always-B.*
*Therefore no sometime-C is a sometime-A.*

This reduces the inference to categorical *Camestres*, which is straightforward to prove.

This method is often known today as *Morleyisation*, though in the West it was invented by Thoralf Skolem in 1920.

## A conclusion

Imagine two scholars working on Ibn Sīnā's logic. The first reads Ibn Sīnā's own explanations and tries to paraphrase them in today's language. The second concentrates not on Ibn Sīnā's explanations but on his *calculations*. To the first scholar, Ibn Sīnā will very likely seem to be a more careful version of al-Fārābī. To the second, Ibn Sīnā is one of the most radical and perceptive logicians in the history of logic.

In my view, the second scholar is right.
Ibn Sīnā did many things that he himself was unable to explain.
His explanations come nowhere near the insights contained in his methods.

One scholar who has done as much as anybody to illustrate Ibn Sīnā's logical perception is Zia Movahed,
for which I warmly thank him.