How and why Rāzī created Post-Avicennan Logic

Wilfrid Hodges

St Andrews, May 2019

http://wilfridhodges.co.uk/arabic??.pdf

Avicenna c. 980–1037
Fakhr al-Dīn al-Rāzī 1149–1209

A gap of about 170 years, the same as the interval between Boole's logic and the present.

Rāzī takes on board the work of some people in the interim: Sawī, Sharaf al-Dīn Masʿūdī, Jīlī, Barakāt.
Tehran scholars have been doing good work making texts of some of these authors available.

Formal logic is based on a set of *sentence forms*.

Aristotle had categorical sentence forms 'Every *B* is an *A*' etc. He built a modal logic either by adding 'Necessarily', 'Contingently' etc. to the categorical forms:

'Necessarily every *B* is an *A*'

or by adding modal valuations of the categorical forms:

'Every *B* is an *A*' is a necessary truth.

Much of what he said is ambiguous between these two approaches.

This same ambiguity is sometimes found in Avicenna, though his clear tendency is to think in terms of new sentence forms.

In particular he investigates arguments involving new predicate forms that refer to time, e.g.:

Every *B* is an *A* at all times when it is a *B*.

Ibn Sīnā lists his new predicate forms, particularly in *Mašriqiyyūn*.
But the list has vague edges; several of the forms in *Mašriqiyyūn* play a minor role, or are not related at all to inference rules.

Rāzī (in the modal logic of his *Sharḥ al-Išārat* and *Mulakhkhaṣ*) is working with new predicate forms.
There is no ambiguity about this.

Also in these works he clearly lists and defines the predicate forms that he will study in inferences.

The list in *Mulakhkhaṣ* has thirteen predicate forms, including several that Ibn Sīnā didn't study.

The rest of this talk will expand on the claim that:

> *The post-Avicennan modal logic of Ṭūsī, Ḥillī, Abharī, Katibī etc. is a development of Rāzī's logic, not of Avicenna's.*

Since many people here will not have read these authors, part of my job is to tell them what they will find if they do read them.

I also need to explain what Avicenna was doing with his new sentence forms, since Rāzī and the later authors represent a reaction to Avicenna's innovations.

Avicenna inherited several kinds of sentence that appeared in logic not long after Aristotle, though several ambiguities make their status as sentence forms unclear.
Thus Theophrastus's threesome:

> Every B is an A at all times.
> Every B is an A while the subject exists.
> Every B is an A while it is an A.

Also from Theophrastus and Alexander:

> Every B is sometimes an A.
> Every B is sometimes an A and sometimes not an A.

Avicenna from his earliest surviving logical work rejigs the threesome as six forms:

1. Every sometimes-B is an A at all times.
2. Every sometimes-B is an A at all times when it exists.
3. Every sometimes-B is an A at all times when it is a B.
4. Every sometimes-B is an A at all times when it is an A.
5. Every sometimes-B is at certain specified times necessarily an A.
6. Every sometimes-B is at some unspecifiable times necessarily an A.

This list appears with minor variations in *Awsaṭ*, *Najat*, *Qiyas*, *Burhān* and *Išārāt*.

The overall aim of Avicenna's change to the list is clear:

> *DISAMBIGUATION. The sentence forms should have clearly distinguished meanings, at least enough to distinguish their logical properties.*

Avicenna himself achieved it with the six sentence forms.
But in two other areas he failed to carry out a complete correction of ambiguities in Aristotle.
These two failures gave an opening for Rāzī to make repairs, which went a long way further than needed just for disambiguation.

The areas are (1) possibility/contingency and (2) meanings of 'necessary'.

(1) Possibility/contingency

Aristotle used the same words for 'possible' and for 'contingent'. Avicenna disambiguated by distinguishing between 'broad possible' as in

> possibly an *A*.

and 'narrow possible' as in

> both possibly an *A* and possibly not an *A*.

But having done that, he frequently left out the word 'broad' or 'narrow', and in several places it is unclear which he meant. This might have been a misguided piece of loyalty to Aristotle.

Avicenna was more careful with the parallel distinction between what he called 'broad absolute' as in

> sometimes an *A*

and what he called either 'narrow absolute' or *wujūdī* (safest left untranslated):

> both sometimes an *A* and sometimes not an *A*.

Even in this case it might be ambiguous whether he intended 'sometimes during its existence'.

(2) Meanings of *ḍarūrī* ('necessary')

Avicenna several times defines 'necessary', used with no qualification, to mean permanent in the sense

> (an *A*) throughout its existence.

But he also uses 'necessary' for alethic necessary, i.e. unavoidable (*lā budda*), as in

> not possibly not an *A*.

In the version of his list of the six sentence forms in *Burhān* he adds alethic necessary at the head of the list as a further form. He is clear that these kinds of 'necessary' are not synonymous.

In Avicenna's presentations of modal logic he follows Aristotle's classification of the premises as necessary, possible/contingent, or neither.

So to make sense of this logic it is essential to clarify when Avicenna is using 'necessary' to mean permanent and when to mean alethic necessary.
(We will come back to this.)

We turn to Rāzī. He clearly accepts the aim of disambiguating. So to deal with possible versus contingent, he introduces a distinction between 'simple' and 'compound' predicates (in *Mulakhkhaṣ* with hints in *Sharḥ*).

> A 'compound' predicate is a logical conjunction of two predicates, one of them affirmative and one negative.

To avoid overuse of 'compound', I will call the compound predicates 'Janus' predicates.

Rāzī's distinction was immediately accepted and led to compound predicates being treated as a distinct subdivision within logic.

For example Kātibī's *Shamsiyya* lists thirteen predicates 'which it is customary to investigate' (para. 49). He lists six simple and seven compound. The simple:

(absolute necessary ?†) 'necessarily an A as long as it exists'
(absolute permanent) 'an A as long as it exists'
(broad conditioned †) 'necessarily an A at specific times T'
(broad conventional) 'an A at all times when it is a B'
(broad absolute) 'sometimes an A'
(broad possible) 'possibly an A'

† means not logically investigated by Avicenna.

The compound or Janus:

(narrow conditioned †) 'necessarily an A at specific times T,
and not necessarily an A whenever it exists'

(narrow conventional †) 'an A at all times when it is a B
and not an A whenever it exists'

(non-necessary *wujūdī* †) 'an A sometimes,
and not necessarily an A whenever it exists'

(non-permanent *wujūdī*) 'an A sometimes,
and not an A whenever it exists'

(*ut nunc* †) 'an A at specific time T
and not an A whenever it exists'

(scattered †) ‘an *A* at some unspecified time
and not an *A* whenever it exists’

(narrow possible) ‘possibly an *A* and possibly not an *A*’.

Observe that at least five of the compound predicates are not forms for which Avicenna provided any logical rules.
The remaining two are (I would argue) Avicenna’s narrow absolute (= his *wujūdī*) and his narrow possible.

Observe also that in several cases Kātibī takes a negative part that is not the negation of the affirmative part.
This asymmetrical usage is new with Rāzī, though Avicenna did mention a form of this kind.

The introduction of these compound forms has at least two major logical consequences.
First, it greatly increases the number of forms used within the system.
Second, though Avicenna made sure that he had a complete set of proof rules to cover all inferences using the non-Janus sentence forms (with one specific exception below), his treatment of the Janus cases is not adequately regulated.

Neither Rāzī nor his successors (as far as I know) made any attempt to give a system of inference rules covering the Janus cases.
In consequence a large part of their logic becomes an unsubstantiated catalogue.
I’m sure this contributed unfairly to an impression that later post-Avicennan logic rested on rote learning.

Also Rāzī made a point of tackling Avicenna’s failure to draw a firm line between permanent and alethic necessary:

> The logical literature has found itself stumbling around (*khabṭ*) as a result of using the expression ‘necessary’ sometimes for what is inevitable, and sometimes for what is permanent. (From *Mulakhkhaṣ*, but the same reference to *khabṭ* is in both *Sharḥ* and *Lubāb*.)

Rāzī’s solution was to use ‘necessary’ only for alethic necessary, ignoring the places where Avicenna defines ‘necessary’ as ‘permanent’.
This was a very welcome improvement in logical terminology, but it was disastrous for understanding Avicenna.

For an example of how, consider this from *Sharḥ*:

> The Shaykh (Avicenna) gave this a lengthy answer in his *Šifā’*, but was unsatisfied with it (the answer, I guess).

The ‘lengthy answer’, in *Qiyās* iii.2, is in fact a detailed refutation of a metatheoretic assumption that Aristotle apparently made in *Prior Analytics* i, 30b25–31.
It’s the most penetrating piece of argumentation that I know in any medieval logic.
Aristotle’s mistake was not noticed in the West until Paul Thom independently identified it in 1995.
Razi has missed the connection with Aristotle’s text and hence the whole drift of the argument.

The relevance of this is that Avicenna's argument in *Qiyas* depends on reading 'necessary' as 'throughout its existence'.
Even if Rāzī had grasped the point at issue,
he would have been unable to follow the argument if he read 'necessary' in his preferred way, as alethic necessary.

Rāzī's dismissive reference seems to have been the end of any study of the *Qiyas* passage in Post-Avicennan logic.
Nor am I aware of any later Arabic writer pointing out Avicenna's definition of 'necessary' as 'throughout its existence'.

We noted that Kātibī's list of sentence forms contained many that are not used as logical sentence forms by Avicenna.

These new sentence forms all have one thing that separates them from Avicenna's logical sentence forms:

> *They all use both alethic necessity (or possibility) and temporal notions.*

Also Kātibī's list is almost identical to a list that appears in Rāzī's *Mulakhkhaṣ*. (Rāzī doesn't separate off the simple from the complex. Also Rāzī adds the 'narrower possible', which has a temporal definition.)

Some obvious questions arise.

1. Why does Ibn Sīnā avoid these mixed alethic-temporal forms? (Of course it might be a coincidence, but is that likely?)

2. Why does Rāzī add these forms to those used by Ibn Sīnā?

3. Refining 2, what principle of selection does Rāzī use to decide which mixed alethic-temporal forms to use?
(With six simple forms there are thirty-six Janus forms.
Why pick Rāzī's seven?)

The answers below seem to me plausible, but they could be overruled by any better suggestions.

1. Why does Avicenna avoid using mixed alethic-temporal forms in logic?

The most obvious answer is that he doesn't know how to use them.
He regards logic as a theoretical science, based on principles that are known with certainty and have the power to explain the facts of logic.
What principles are there to perform this service for mixed forms?

He hints at this answer when he tells us in *Qiyas* that the relationship between necessity and permanence
'is not the business of the logician'.

To appreciate this answer, consider how Avicenna justifies logical claims using the sentence forms that he does accept.

Avicenna classifies syllogisms by the forms of their premises under the three heads of necessary, possible/contingent and absolute. ('Absolute' is how the Arabic translators described Aristotle's non-modal sentences.)
Using this classification to provide a schedule, Avicenna proceeds as follows.

☞ When both premises are absolute or necessary, Avicenna reads absolute as broad absolute and necessary as permanent.
He is able to adapt Aristotle's rules for justifying categorical inferences to this case.

☞ When the premises are possible or necessary, he reads necessary as alethic necessary, and (at least in several cases) he invokes a translation

$$\begin{array}{lcl} \text{alethic necessary} & \mapsto & \text{permanent,} \\ \text{possible} & \mapsto & \text{broad absolute} \end{array}$$

which reduces these cases to cases already handled. (This is obscured in the rearrangement of syllogisms in his late *Išārat*.)

The translation gives a sound justification of a semantic kind. Details are in Hodges-Johnston.

☞ What about the case where one premise is absolute and the other is possible?
Avicenna is committed to handling this case by his adoption of Aristotle's schedule.
But the methods of the other two cases give no basis for handling this case, so it is important to see what he does here.

This is the only place where he relates logical inferences to 'narrower possible' sentence forms.
The reason is clear: it matches a temporal 'broad absolute' to the temporally-defined 'narrower possible'.

Otherwise he proceeds empirically, testing individual premise-pairs and asking how we react to them.
This provides empirical answers but no general theory.

2. Why is Rāzī content to include mixed forms?

He tells us in his *Lubāb*:

> The logician needs to know the difference between the mode of necessity and the mode of permanence regardless of whether or not the one entails the other.

As Avicenna might have put it, necessity and permanence are among the relevant properties (*aḥwāl*) of the subject of logic. But as Avicenna might have pointed out, until we know the laws relating necessity to permanence, those laws will be problems of logic, not theorems of logic.

So how do Rāzī and his successors justify the claims that they make about inferences involving mixed sentence forms?

To a great extent they use the empirical method that Avicenna calls on for the possible + absolute case.

Some writers try to make the results more significant by summarising them in general 'conditions of entailment'.
Avicenna would surely have approved of the attempt.
But the resulting conditions of entailment are just records of empirical discoveries,
and they have little or no explanatory force.

Rāzī himself is one of those who go further and attempt rational proofs of the conditions.
But first, his arguments tend to use inadequately explained notions, and to rest rather heavily on hope.
And second, a number of passages indicate that he is not really attempting formal laws even when formal laws are available.
For just one example, in his proof that the two premises in second figure must have opposite qualities, he remarks

> Sometimes the two premises clearly have the same quality but nevertheless are productive because of the presence of a difference in the facts themselves.

3. How does Rāzī select which mixed sentence forms to work with?

Here I think I know the answer.
The sentence forms that are in the lists of Rāzī and Kātibī, but are not among those which Avicenna gives logical properties for, are all of them mentioned by Avicenna for some other reason.

The most striking example is the 'narrow conventional', quite an odd sentence form to choose. In fact it fell out of Avicenna's refutation of Aristotle's false metaprinciple in the passage in *Qiyas* iii.2 discussed above, and Avicenna has carried it over to *Išarat*. Rāzī has missed the interesting technical point about it that Avicenna revealed.

To sum up:

Major differences between Avicenna's logic and logic from Rāzī onwards are the result of a reasonable, though bookish, attempt to plug gaps in Avicenna's theory where he had mentioned some sentence forms but not pursued their logical properties.

To plug these gaps, the later logicians had to accept a drop in standards of argument, relaxing the goals of a scientific theory—though elsewhere they rightly avoided some of Avicenna's own ambiguities.

Along the way, these later logicians missed key parts of Avicenna's logical system. These parts are simply not mentioned in the later texts. (This is equally true for Avicenna's logic of hypotheticals, but the details are different.)

The frames above form the ground plan for a talk to be delivered at a meeting in St Andrews in May 2019. For writing it up I will need to add a lot of references and specifics, and at that stage some things may change.

One gap in the present scheme is Rāzī's attempt to make sense of Avicenna's ill-starred approach to forming contradictory negations of predicates containing conjunctions. I think this work of Avicenna and Rāzī is interesting as an attempt to find a technique for solving a new kind of problem.

While I was writing this I heard the sad news of the death of Harold Simmons. We were good friends in our younger days as model theorists. He later went on to publish in modal logic. So I hope it is not inappropriate to dedicate this essay in the history of modal logic to his memory.