

## Abū al-Barakāt and his 12th century logic diagrams

Wilfrid Hodges  
CLMPST Prague 2019  
wilfridhodes.co.uk/arabic66.pdf

Abū al-Barakāt bin Malka al-Baghdādī  
lived c. 1085–c. 1170

A Baghdad Jew who converted to Islam late in life,  
apparently under duress.

Book: *Kitāb al-Muʿtabar*, 'Book of things I considered'.  
It contains the earliest statement that bodies fall with  
constant acceleration.

### Aristotle's categorical syllogisms

Four kinds of sentence:

- Every  $B$  is an  $A$ .
- No  $B$  is an  $A$ .
- Some  $B$  is an  $A$ .
- Not every  $B$  is an  $A$ .

(Or with other letters.)

A 'premise-pair' is two categorical sentences with one  
letter in common.

Aristotle lists 48 premise-pairs, grouped into three  
'figures'.

Each premise-pair has four 'candidate conclusions',  
which are the four categorical sentences using the two  
letters that each occur just once in the premises.  
(The order of the letters is fixed by the figure.)

A premise-pair is 'productive' if it entails one or more of  
the candidate conclusions.  
(Its 'conclusion' is the strongest entailed candidate.)

Otherwise it is 'sterile'.

Aristotle determined which of the 48 premise-pairs are productive and which are sterile.

For the *productive* pairs, he gave a proof theory: four productive premise-pairs plus conclusion are taken as axioms, and the remaining ten productive premise-pairs are derived from the axioms.

For each *sterile* premise-pair, he proved sterility by giving *two* interpretations (i.e. words to put for the letters) that make the premises true, but each of the candidate conclusions comes out false under at least one of the interpretations.



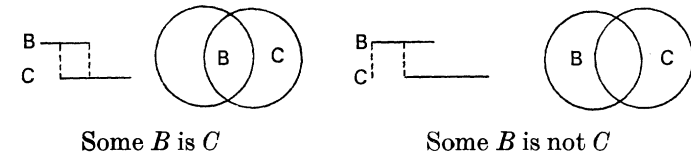
They may still be the earliest. But we now know that five hundred years earlier, Barakāt used diagrams in a different way, to represent *interpretations* or *models* rather than sentences.

For each *sterile* premise-pair he gave three interpretations making the premises true (i.e. models of the premises) so that each candidate conclusion is false in at least one of the interpretations.

If he had read Aristotle—as he pretended he had—he would have known that only two are needed.



Leibniz (late 17th century), followed by Euler, Venn and Lewis Carroll, showed that Aristotle's proofs for the productive case can be replaced by pictures that represent the sentences.



Till two years ago, Leibniz's diagrams for categorical sentences were the earliest known.

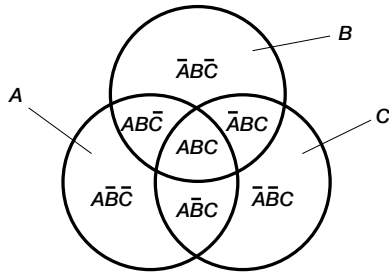


For each *productive* premise-pair he gave between two and four interpretations, all of which are models of the premises and of the conclusion. How does this work?

We can assume that:  $\phi, \psi$  entail  $\theta$  if and only if *every* model of  $\phi$  and  $\psi$  is a model of  $\theta$ .

But there are indefinitely many models, because there are indefinitely many words we could put for the letters. How to cut down to a small finite number that we can check?





To know which sentences using  $A, B, C$  are true under a given interpretation, it suffices to know which of the seven labelled areas are empty and which are nonempty. So  $2^7 = 128$  possibilities. Also no loss in assuming all the circles are nonempty. This leaves 109 possibilities.

Still too many for practical calculations.



Barakāt has a further idea: start by giving just the interpretations that make both premises true. This cuts down to at most 16 interpretations, which is manageable.

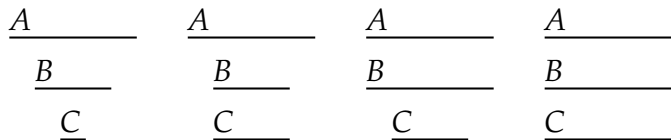
In fact Barakāt never gives more than four interpretations, but read carefully they are enough to show the pattern.

Sadly no evidence that anybody did read them carefully. Most later records of Barakāt’s method are *very* inaccurate, including the report by al-Ṭūsī in his famous 13th century Persian logic textbook *Asās al-iqtibās*.



**First example (productive, one of Aristotle’s axioms):**

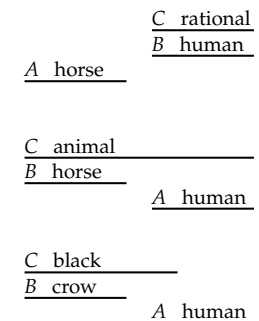
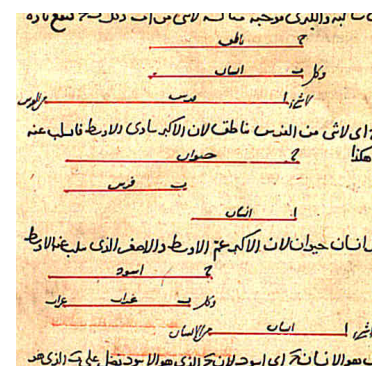
Every  $C$  is a  $B$ . Every  $B$  is an  $A$ .

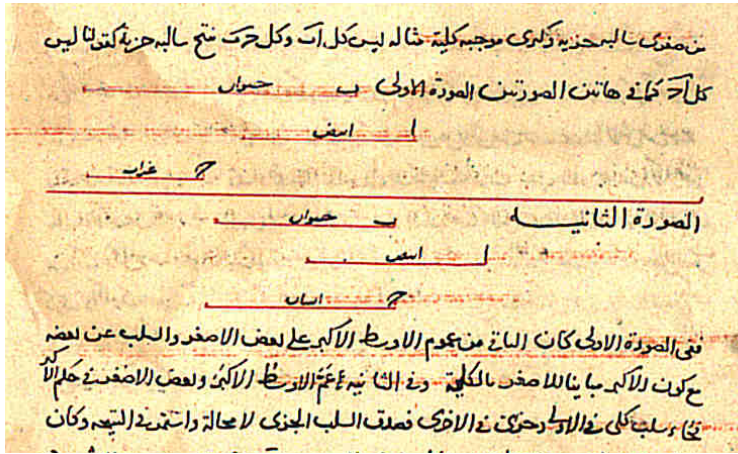


In each of the four interpretations, Every  $C$  is an  $A$ . So this is the conclusion.

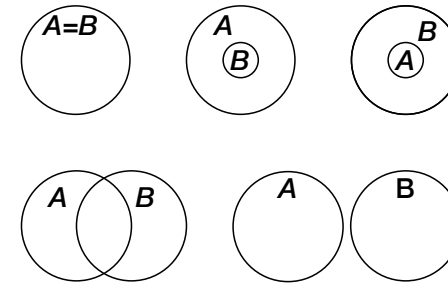


**Second example, sterile: No  $A$  is a  $B$ , Every  $B$  is a  $C$ .**





Barakāt's diagrams are not Leibniz-Euler-Venn diagrams representing sentences. They are Gergonne (1816/7) diagrams representing situations or structures:



Except that Barakāt has three labelled sets while Gergonne had two.

- Gergonne (2 sets): 5 cases.
- Barakāt (3 sets): 109 cases.
- Next step up (4 sets): 32,297 cases.

Barakāt just managed to squeeze in under the combinatorial explosion!

Margaret E. Baron, 'A note on the historical development of logic diagrams: Leibniz, Euler and Venn', *The Mathematical Gazette* 53 (384) (1969) 113–125.

Wilfrid Hodges, 'Two early Arabic applications of model-theoretic consequence', *Logica Universalis* 12 (1–2) (2018) 37–54.

Moshe M. Pavlov, *Abu 'l-Barakat al-Baghdadi's Metaphysical Philosophy*, *The Kitab al-Mu'tabar*, Routledge, Abingdon 2017.