Abū al-Barakāt's diagram method in logic

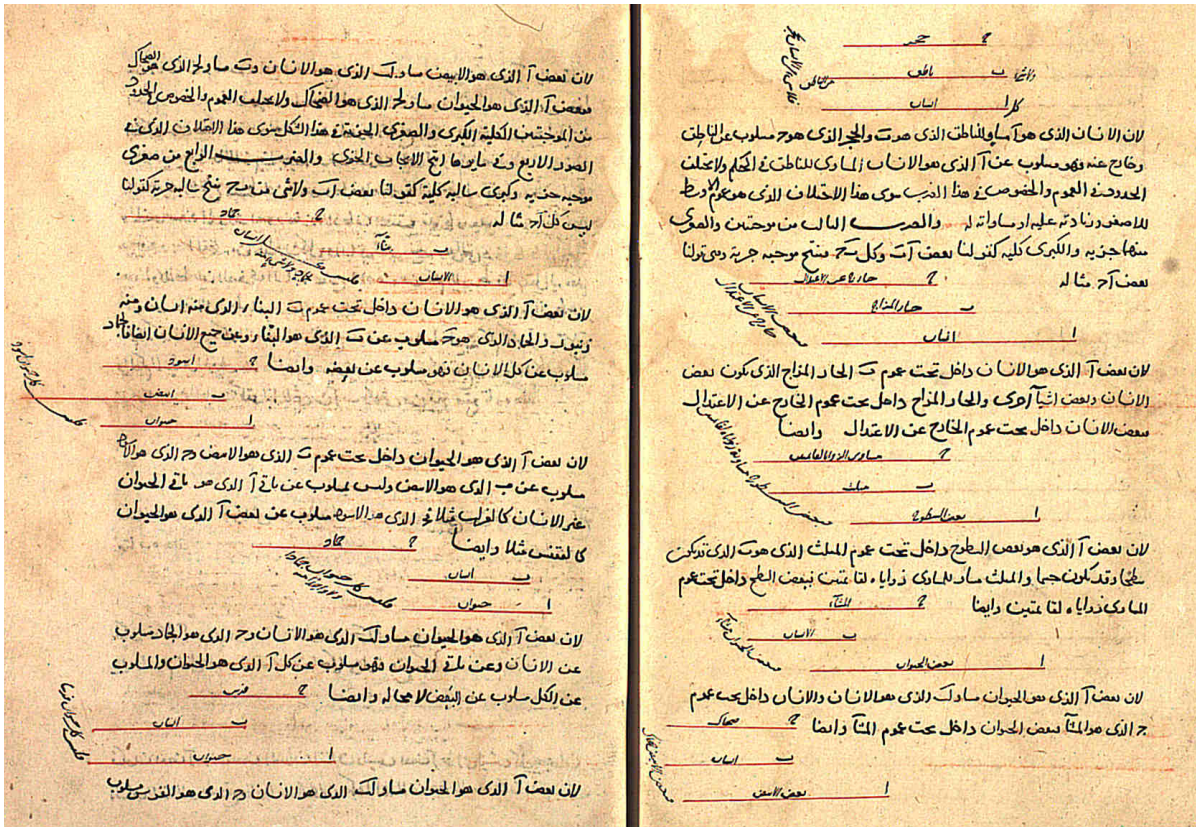
Wilfrid Hodges

ICHST Prague 2021
Symposium: Mathematical proofs and styles of reasoning: East vs. West

http://wilfridhodges.co.uk/arabic72.pdf

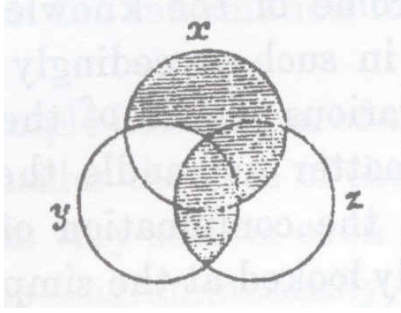
- Abū al-Barakāt bin Malkā al-Baghdādī, believed to be the same person as Rabbi Baruch ben Melekh.
- Lived 900 years ago in Baghdad, c. 1080–c. 1165.
- Fearsomely clever and original; nickname ‘Unique in his time’.
- In physics a pioneer in study of inertia and acceleration.
- In logic a follower of Ibn Sīnā (Avicenna) from 11th century, in fact probably one of our sources for Ibn Sīnā’s early logic. (Work of Jules Janssens.)

His main book, *Kitāb al-muᶜtabar*, is about more or less everything, but begins with a couple of hundred pages on logic, including a pictorial method for testing and proving syllogisms (simple argument forms due to Aristotle).



Worked examples of his diagram method are available on YouTube: `http://arabiclogic.com/youtube.pdf`

The diagrams look at first like an anticipation of Venn diagrams, which turn Aristotle's arguments into pictures. But in fact they are based on a different idea which reappeared in the 20th century as 'model-theoretic consequence' (Alfred Tarski 1936).



Al-Barakāt's contemporaries were bewildered by them.
Today we understand them, but we are still bewildered at how he thought they were helpful in his context.
The rest of this talk will explain this comment.

Basic idea of formal logic (Aristotle to modern textbooks):

- Given an inference 'p and q; therefore r', where p, q, r are meaningful sentences;
- we *formalise* p, q and r as p^F, q^F, r^F by turning some of their words or phrases into letters A, B or C.
- Then we show that r^F follows from p^F and q^F, *independently of* how A, B, C are read.
- Therefore our proof works for the original p, q and r too.

Example: Some human is white. Every human is an animal. Therefore something white is an animal.

Formalise: Some B is a C. Every B is an A. Therefore some C is an A.

Proof: Some B is a $C \Rightarrow$ Some C is a B.
Some C is a B, every B is an $A \Rightarrow$ Some C is an A.

The same proof works if we read A, B, C as 'animal', 'human', 'white'.

Q: Why not just prove 'If p and q then r' directly?
A: Because often the reason why the proof works is easier to see if we ignore the meanings of A, B, C.

Al-Barakāt's argument is different. Given p^F, q^F and r^F, there may be lots of relationships between sets A, B and C that make p^F and q^F true.
(The relationships that make p^F and q^F true are called 'models' of p^F, q^F.)
His method is to look through diagrams of all the models of p^F, q^F
and check that all these models are also models of r^F.

There are sixteen models of ‘Some B is a C. Every B is an A’. Two examples:

All the models make ‘Some C is an A’ true.

But only the first above is relevant to the original inference.
So why bother with the fifteen irrelevant models?

The ‘irrelevant models’ are relevant only for the formal inferences (with letters).
Al-Barakāt is making a division of labour, by separating the justification of the formal inferences from the justification of the meaningful inferences.

Though apparently pointless to medieval logicians, this separation has been standard in modern logic since Boole:

> *The formal process of . . . demonstration [is] conducted throughout in obedience to all the laws [of the combination of symbols], without regard to the question of the interpretation of the particular results obtained.*
> *The final result is . . . interpreted . . .*
> *(Boole, Laws of Thought (1854) ch. v p. 68)*

So why does al-Barakāt in the 12th century make this separation?

First suggestion (ideological):

> *There is an abstract world of logic which regulates both our world and our thinking about it.*

This fits with the fact that al-Barakāt puts logic at the beginning of his *Kitāb*, with no prior explanations. (Modern equivalents: Gödel, Łukasiewicz?)

Second suggestion (computational):

> *In principle we can reduce all thinking to computation.*
> *This would imply we can reduce all meaningful sentences to forms that allow computation.*

There are passages in al-Barakāt's treatment of temporal sentences (in his logic) that suggest he was moving in this direction.

Where to look next?

- Finding independent evidence of Ibn Sīnā's early logic might remove some of the bewilderment.
- Other than that, we need more people reading al-Barakāt's *Kitāb*. Unfortunately it is not translated into any western language.
- Any other suggestions welcome!

On al-Barakāt, recent books by Moshe Pavlov and older articles by Shlomo Pines, but none of them directly about formal logic.