Avicenna motivates two new logics

Wilfrid Hodges
wilfrid.hodges@btinternet.com

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Avicenna (Ibn Sīnā)

Who was he?

He is uniquely identified as the leading logician anywhere in the world west of India exactly a thousand years ago.

What did he do that is worth knowing about?

Here is one of his own answers.
(Sanserif text is quoted.)

In mathematical books we find many conclusions that have several clauses;
in particular we find them in the book of Euclid.

For example Euclid *Elements* i Prop. 27:
When (or if) a straight line falls across two straight lines
so that it makes the two alternate angles equal,
then the two lines are parallel.
$\forall L_1 \forall L_2 \forall L_3\ (\phi(L_1, L_2, L_3) \rightarrow \psi(L_2, L_3))$

Here $\phi(L_1, L_2, L_3)$ and $\psi(L_2, L_3)$ are two clauses.

Note the quantification over ordered triples of objects,
introduced c. AD 200 by Alexander of Aphrodisias.

But when logicians give inferences using sentences with several clauses,
these sentences are always premises and not the conclusion.

A typical example is modus ponens:

p. If p then q.
Therefore q.

So it is clear that we need to find those syllogisms [i.e. inference rules] that yield multiple-clause conclusions.

I myself have already done this.

(Avicenna, *Middle Summary in Logic*, 1013)

Avicenna claims here that he has discovered one or more new logics (where a logic has a vocabulary, a set of logical rules using this vocabulary, and some apparatus for justifying the rules that are not obvious).

The claim is true. Avicenna was the first logician since antiquity to create new logics in this sense.

In fact he created several, but we will concentrate on two ('Logic One' and 'Logic Two' for convenience)
from the first and less well documented half of his career, between c. 994 (age eighteen?) and 1013 (age thirty-seven?).

Logic One

Some A is a B.
No C is a B.
Therefore some A is not a C.

(Typical Aristotle syllogism with classes A, B, C.)

Avicenna c. 994 analyses the mental steps needed to get from the premises to the conclusiom.
For example we must detect that B occurs in both premises.
(Today we call this unification.)

Then we must cut out B and recombine A and C in a new sentence. (As in the resolution calculus.)

Avicenna claims that the same operations apply to syllogisms that have propositions instead of classes. (Wallis and Boole made the same claim centuries later.)

We can see this if we replace the classes A etc. by the classes of times when certain propositions are true:

Some A is a B.
No C is a B.
Therefore some A is not a C.

Some (time when p) is a (time when q).
No (time when r) is a (time when q).
Therefore some (time when p) is not a (time when r).

But this is not what Avicenna writes.
Instead he paraphrases

Some (time when p) is a (time when q).
No (time when r) is a (time when q).
Therefore some (time when p) is not a (time when r).

as:

(Sometimes when p), q.
Or: Sometimes p and q.
(Never when r) is it the case that q.
Therefore it is not the case that whenever p, r.

Note the two clauses p, r in the conclusion!

David Lewis 1975 called attention to these phrases ‘sometimes when p’, ‘always when p’ (= ‘whenever p’), ‘never if p’, ‘seldom before p’ etc. under the name ‘adverbs of quantification’.

The temporal adverb and the phrase with p together form a quantifier over times or cases.

Angelika Krätzer *Modals and Conditionals* pp. 88–91 has a good account of how the ‘if’ in these phrases is not part of a material implication. For example:

Sometimes if a man buys a horse, he pays cash for it.

This can be rewritten with ‘and’ (as Avicenna illustrates):

Sometimes a man buys a horse and pays cash for it.

Nicholas Rescher 1963 (before Lewis and Krätzer) got this wrong. He thought we can parse:

Sometimes: (if a man buys a horse, he pays cash for it.)

I.e. that a material implication holds at some time.

As a result his translations of this part of Avicenna's logic are largely nonsensical.
Unfortunately they were copied by Maróth, Shehaby, El-Rouayheb and others.

Krätzer's example is not ideal for Avicenna's 'sometimes when p, q', because it should be possible to switch around the p and the q. Contrast:

> !!Sometimes if a man pays cash for a horse, he buys it.

Possible alternative:

> Sometimes when I wake up, the duvet has fallen off.
> Sometimes when the duvet has fallen off I wake up.
> Sometimes I wake up and the duvet has fallen off.

Moral: The semantics of (Avicenna's and Lewis's) adverbs of quantification is linguistically tricky and you need more than an undergraduate degree in logic to handle it.

One last point: we saw how a clause p can be hidden inside a noun phrase 'time when p'.
In another work Avicenna suggests that there may be languages which use a device like this so that they never need subordinate clauses. An example is Abkhazian.

Logic Two

> Whenever r, every B is a C.
> Some B is not an A.
> Therefore whenever r, some C is not an A.

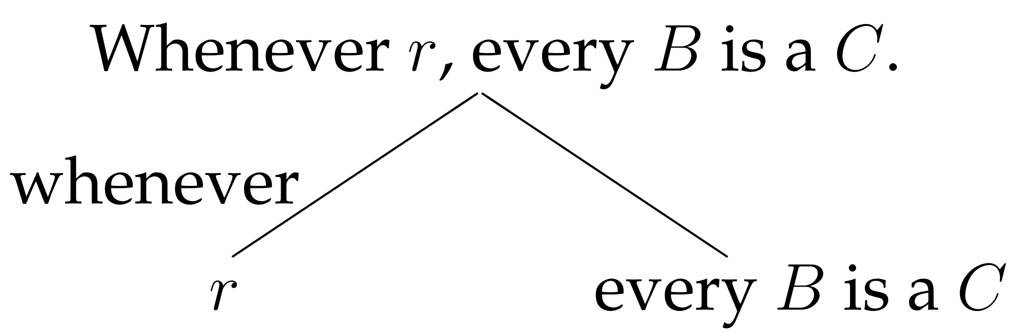
Note the two-clause conclusion.

> There are several ways of proving that these syllogisms do yield their conclusions, but for the sake of brevity we won't describe these ways in this book.
>
> (*Middle Summary in Logic*, 1013)

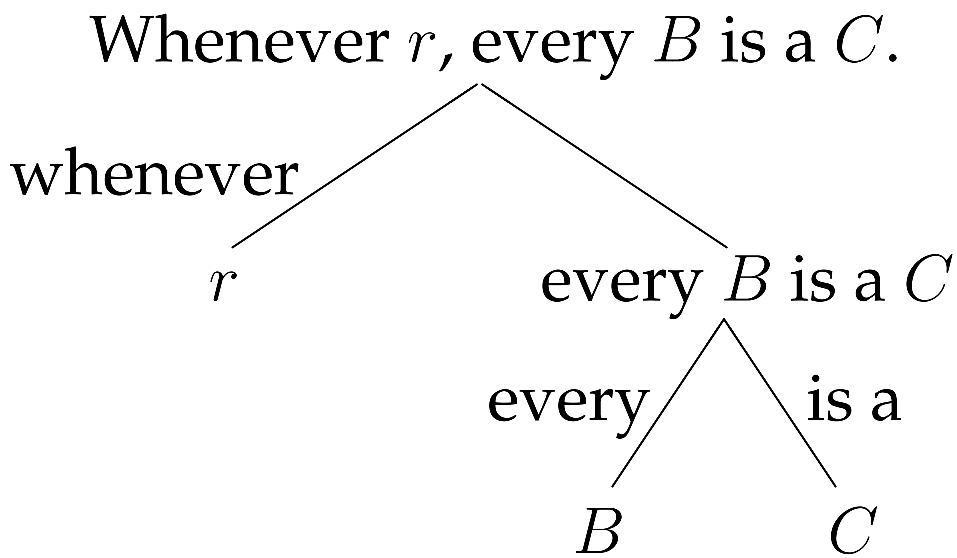
Probably he wrote up these justifications during the period before 1013; most of his work from this period is lost.

Avicenna describes these syllogisms as ‘in an incomplete part’. This name is important and we must explain it.

Avicenna believed that linguistic compounds are generally binary.
So the first premise analyses down to two ‘parts’ and a word ‘whenever’ for gluing the parts together:



One of the parts, ‘every B is a C’, is a binary compound too and can be analysed down into two parts:



By an ‘incomplete part’ Avicenna means a part of a part.

In the syllogism above, the incomplete part B in the first premise has to be unified with the part B in the second premise.
So the inference rule has to reach down two levels in the syntax of the first premise.

It seems Avicenna was the first person to notice that we need inference rules that reach down two or more levels in the syntax.
He was also the first person to provide examples of inference rules that do this.

Later logicians who also attacked the problem of inference rules going down more than one level are Walter Burley and Leibniz.

Leibniz saw the problem as one of linguistic analysis, not logic.
He looked for ways of paraphrasing sentences so as to bring deep components to the surface.
He was not very successful.

Work of Boole and Frege has removed the problem.

It's unfortunate that we have no clear idea of how Avicenna justified these inference rules.
But there is a prior problem: exactly what were the rules?
To see the problem, we paraphrase using a time variable:

For every time t when r, every B is a C at t.
Some B is not an A.
Therefore for every time t when r, some C is not an A at t.

QUESTION ONE: where is the time in the second premise?
Since the time is universally quantified in the conclusion, it has to be universally quantified in the second premise:

For every time t, some B is not an A at t.

QUESTION TWO: Where is the time in the first premise? Should we read:

For every time t when r, every B-at-t is a C-at-t.

or:

For every time t when r, every sometime-B is a C-at-t.

This question remains open.

Both readings can be supported from other texts of Avicenna.
Around 1013 he was experimenting with different ways in which time can appear in sentences.

Not knowing just how Avicenna read these syllogisms 'in an incomplete part', we can at least work out what he needed to say from the point of view of first-order logic. The following is from my 'Ibn Sīnā on reductio ad absurdum', *Review of Symbolic Logic* 10 (3) (2017) 583–601.

> Suppose T is a set of formulas and η, θ are formulas. Let $\delta(p)$ be a first-order formula containing a propositional variable p which occurs only positively in $\delta(p)$ and doesn't occur in $\delta(p)$ within the scope of any quantifier on a variable free in some formula of T. Then:
>
> $$\text{If } T, \eta \vdash \theta \quad \text{then} \quad T, \delta(\eta) \vdash \delta(\theta).$$

For example take $\delta(p)$ to be 'Whenever r then p',
or more precisely 'At every time t when $r(t)$, p'.
Then p occurs only positively in $\delta(p)$.

Suppose ϕ, ψ, χ are sentences, not dependent on t, such that

$$\phi, \psi \;\vdash\; \chi.$$

Since the quantifier is on a variable r not free in ψ,
the rule gives

$$(\text{Whenever } r \text{ then } \phi).\ \psi \vdash (\text{Whenever } r \text{ then } \chi).$$

Other cases can be checked.

Avicenna himself couldn't have stated the rule, because the required notion of scope became available only with Frege.

But he did continue to work with Logic Two, and he showed that it allows proofs by reductio ad absurdum to be expressed as direct proofs.

$$\dfrac{T \qquad \dfrac{T \qquad \dfrac{T \qquad \phi}{\psi}}{\chi}}{\bot} \qquad \Rightarrow \qquad \dfrac{T \qquad \dfrac{T \qquad \dfrac{T \qquad (\phi \to \phi)}{(\phi \to \psi)}}{(\phi \to \chi)}}{(\phi \to \bot)}$$

Note that in this procedure Avicenna uses 'If ϕ then ...' instead of 'Whenever ϕ then ...', thus ignoring time.

Frege independently gave the same argument as Avicenna in the more general setting of making and then resolving an assumption.
Frege's version allowed the assumption to contain quantifiers binding the following formula; in effect Frege used the 'whenever' version rather than the 'if' version.
Recall that the example from Euclid was quantified in this way.

Avicenna mentioned his application of ‘incomplete parts’ just once, in *Syllogism* viii.3 in the mid 1020s.

There are some signs that he became disheartened with formal logic in the late 1020s.
For example around 1030 he was advising his students in logic to concentrate on meaning rather than form.
Perhaps he saw that he had raised questions that he would never answer.

His late work shows a switch of interest to the metaphysics of the rational soul and the afterlife.

George Boole, *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities*, Dover, New York 1958 (original 1854); see Chapter XI ‘Of secondary propositions, and of the principles of their symbolical expression’.

Gottlob Frege, ‘Über die Grundlagen der Geometrie’, *Jahresbericht der Deutschen Mathematikervereinigung* 15 (1906) 293–309, 377–403, 423–430.

Kai von Fintel, *Restrictions on quantifier domains*, Dissertation, Department of Linguistics, University of Massachusetts 1994. (On adverbs of quantification.)

Wilfrid Hodges, ‘Proofs as cognitive or computational: Ibn Sīnā’s innovations’, *Philosophy and Technology* 31 (1) (2018) 131–153.

Wilfrid Hodges, *Identifying Ibn Sīnā’s hypothetical logics* (in preparation).

Text of this talk is at www.wilfridhodges.co.uk/arabic73.pdf.

Thanks for your attention