How did Avicenna understand the Barcan formulas?

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For the 85th birthday of John N. Crossley

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‘In the spring it was black
and the spring before it blue’
Zia Movahed, *The visits*

Zia Movahed, ‘Ibn-Sina’s anticipation of the formulas of Buridan and Barcan’, in *Logic in Tehran*, ed. Ali Enayat et al., Association for Symbolic Logic and A. K. Peters, Wellesley Mass. 2003, pp. 248–255.

In 2003 Zia Movahed pointed to a passage by Ibn Sīnā (known as Avicenna in the West), written probably in 1022, which Movahed claimed anticipated modal formulas named after Ruth Barcan and Jean Buridan.

Barcan formula	$\forall x \Box Fx \rightarrow \Box \forall x Fx$
Converse Barcan formula	$\Box \forall x Fx \rightarrow \forall x \Box Fx$
Buridan formula	$\Diamond \forall x Fx \rightarrow \forall x \Diamond Fx$

As Movahed says, these are written in Quantified Modal Logic (QML).

Movahed's claim raises a surprising number of issues about modal logic.

$\star$ Avicenna wrote in Arabic, not in QML.
So the claim must say that Avicenna wrote in Arabic sentences that mean the same as the Barcan formula etc.

$\star$ Avicenna never wrote any 'If … then …' sentences with modal operators.
So the claim must say that Avicenna wrote sentences S, T equivalent to $\forall x \Box Fx$, $\Box \forall x Fx$ respectively and said that S implies T.

$\star$ Avicenna, like Aristotelian logicians in general, always relativised his quantifiers.

This is a little trickier. We need versions of the Barcan formula etc. for relativised quantifiers, using only sentence forms equivalent to ones that Avicenna himself used.
For example we could write $\forall(x : B)\Box Ax$ for 'Every *B* is necessarily an *A*', and $\Box\forall(x : B)Ax$ for 'Necessarily every *B* is an *A*'.

Let that do for the moment. There are worse problems ahead.

Central problem: How do we tell if a QML formula is equivalent to an Arabic sentence?

Helpfully Avicenna uses letters for non-logical constants, following Aristotle. But what about the logical parts of his sentences?
Does his Arabic *ḍarūrī* mean the same as $\Box$?
How can we tell?
How to describe the meaning of $\Box$ anyway?

Recent study of logical works from the first half of Avicenna's career has opened up an unexpected approach to these and similar questions.

Epitome in Logic is a short work. By comparing its knowledge of sources with Avicenna's *Autobiography*, we can place it with high probability in the first half of the 990s, when Avicenna was in his late teens.

The next available logical work is much later, *Twenty Questions* in around 1012. This unusual work appears to be Avicenna's private notes to himself in preparation for the writing of:

Middle Summary in Logic, dated 1013, contains what is probably Avicenna's first systematic account of modal logic. The first authoritative Arabic edition was published in 2017 by Yousofsani (a colleague of Movahed in Tehran).

Features of *Epitome in Logic*:

1. Though *Epitome in Logic* refers to ‘necessary’ and ‘possible’, it has nothing intelligible to say about the meaning of either word.

2. Unlike all of Avicenna’s other writings on modal logic, *Epitome in Logic* makes no reference to sentences that quantify over time.

3. *Epitome in Logic* has a lot to say about the logical rules obeyed by ‘necessary’ and ‘possible’. Some of these rules contradict Aristotle, and apparently all other modal logicians before Avicenna.

4. Avicenna makes it clear that he is not ‘explaining’ the modal logic of Aristotle; he is attacking it. He continued to attack it at the same points, using much the same language, throughout his career. One sees this when one compares the early *Epitome in Logic* with *Pointers* written some thirty-five years later.

The programme of *Twenty Questions*, carried out in *Middle Summary in Logic*:

Aristotle made several mistakes about what logical laws hold for ‘necessary’ and ‘possible’ (cf. *Epitome in Logic*).
But also Aristotle gave many detailed justifications for his claims about modal laws.
It follows that these false modal laws rested either on false foundations (*ᶜuṣūl*), or on correct foundations wrongly applied.
Avicenna thought he could point to examples of both kinds.

So the programme set out in *Twenty Questions* is to introduce new foundations, or new applications of old foundations, which will justify all the claims in *Epitome in Logic* about modal laws.

Avicenna will find these new foundations by raiding the logical literature for clues and suggestions.

First new foundation: Themistian absolutes

Studying Aristotle's proofs of invalidity in categorical syllogisms, Avicenna found that Aristotle treated both the sentences

Every horse sleeps.
Every horse fails to sleep.

as true. How?

Answer: By reading them as

Every horse sometimes sleeps.
Every horse sometimes fails to sleep.

Thus Avicenna introduces into logic 'fluents' (McCarthy) which have two arguments, one for an object and one for a time.

Avicenna introduces these sentences with fluents and existential quantifiers over time as ‘Themistian absolutes’.

In the 1020s he will change this name to ‘broad absolute’, which is how they are generally known today.

Taking a hint from al-Fārābī’s long commentary on *Prior Analytics*,
Avicenna also argues that the relativised quantifier ‘Every horse’ should be read as ‘Everything that was, is or will be an actual horse’.
The same will apply to all relativised quantifiers in Themistian absolute sentences.

Second new foundation: strict necessity

In the 12th century Fakhr-al-Dīn al-Rāzī criticised Avicenna for using the word 'necessary' (*ḍarūrī*) in two different senses, 'unavoidable' and 'permanent'. He claimed that this led Avicenna into *khabṭ*, a state of stumbling in the dark.

Al-Rāzī is right that Avicenna used *ḍarūrī* in these two senses. The first sense refers to how Aristotle used 'necessary' in his modal syllogisms; following von Wright and Movahed I call it 'alethic necessity'. The second sense is Avicenna's second 'new foundation', introduced in *Twenty Questions*. In *Twenty Questions* Avicenna calls it 'strict necessity' (*al-ḍarūrī al-ḥaqīqī)*, though later he often shortens this to 'necessary'.

I don't believe Avicenna himself was confused about this.

Avicenna extracted strict necessity from an idea of Aristotle's student and successor Theophrastus in Theophrastus's *Prior Analytics* (now lost). Theophrastus believed that statements about permanence could be used to help classify statements about alethic necessity.

Avicenna found Theophrastus's examples unhelpful, and in place of them he proposed:

> *Everything that was, is or will be an actual B is, at every moment when it exists, an A.*

This can be shortened to

> *Every B is necessarily (or permanently) an A.*

Avicenna tells us it expresses 'strict necessity'.

Avicenna introduced Themistian absolute and strict necessary statements with the quantifier ‘Everything’, but he expects us to adapt to the quantifiers ‘Nothing’, ‘Something’ and ‘Not everything’.
In this way Avicenna constructed a new logical language (call it 2D logic) with eight sentence forms.

These sentences are interpreted by choosing words (including fluents) for the non-logical constants, and reading ‘sometimes’ etc. as quantifying over actual moments of time.

This unambiguously determines the logical relations between 2D sentences. Most are justified as Aristotle justified the categorical syllogisms. For those which aren’t, in *Middle Summary in Logic* Avicenna found new non-Aristotelian justifications.

Third new foundation: 'counting possible as absolute'

Finally the 2D logic is used to justify alethic syllogisms, as follows:

> *Read a possibility sentence as the corresponding Themistian absolute.*
> *Read an alethic necessity sentence as the corresponding strict necessary.*

This works: for example it justifies all the modal syllogisms that Avicenna claimed and Aristotle rejected.

So the programme of *Twenty Questions* and *Middle Summary in Logic* is successfully completed.

At this point another talk would explain why Avicenna's new justifications work, comparing his use of the real world and actual times with the modern use of Kripke frames.

But we will turn back to Movahed's claim about the Barcan formula.

We begin with the antecedent of the Barcan formula, using a relativised quantifier. Accepting Avicenna's 'counting the possible as absolute',

$$\forall(x : B)\Box Ax$$

translates to the 2D sentence

> *(1) Everything that was, is or will be an actual B is an A at all times when it exists.*

But what about $\Box\forall(x : B)Ax$?

In the 1010s Avicenna does write some modal sentences that begin ‘Necessarily’. But this is misleading; he explains that the place where ‘necessarily’ is written doesn’t have any effect on the meaning.

But in his *Commentary on De Interpretatione* (1022) he introduces a new kind of sentence where the operator ‘Necessarily’ is ‘on the quantifier’, i.e. includes the object quantifier within its scope. In this reading the alethic

> *Necessarily: Every B is an A.*

has to be translated to the 2D-like sentence:

> *(2) At every time τ, everything that is a B at time τ is an A at time τ.*

Now we confirm, not yet that Avicenna said (1) implies (2), but that (1) does in fact imply (2). We need one further assumption from Avicenna's logic:

> ($\star$) *Everything that has an affirmative property at time τ exists at time τ.*

To prove Barcan, assume (1). Let τ be any time, and *a* any object that is a *B* at time τ. Then *a* is a *B* at time τ, so by (1) it follows that *a* is an *A* whenever *a* exists. But at time τ, *a* has the affirmative property of being a *B*, so by ($\star$) it exists at τ. Hence *a* is an *A* at τ, proving (2). QED

This also proves the negative Barcan formula with $\neg A$ in place of *A*.

BUT: the Converse Barcan formula fails. (2) doesn't imply (1). I leave this as an exercise. Also we can find a way to express the Buridan formula, but again the implication fails.

The received wisdom is that the best way to get both Barcan and Converse Barcan formulas to hold is to work in a modal system where every Kripke frame has a single universe. Avicenna's 2D logic requires models with a single universe, the same for all times. But they also have an existence predicate which selects, for each time τ, those things that exist at time τ.

Is Movahed right that Avicenna claimed that (1) implies (2)?

I think he is, but not quite in the text that Movahed quotes. This involves some nontrivial questions about the interpretation of Avicenna's Arabic, I think not suitable for the end of a lecture. But details will be in the IGPL article reporting this lecture.

The outcome is that Avicenna can reasonably be read as claiming the Barcan formula (as Movahed said), but not the Converse Barcan formula or the Buridan formula (contrary to Movahed). Since the Barcan formula holds in the Arabic version, but the Converse Barcan and Buridan formulas don't, Avicenna does seem to have got it right.

My congratulations to John and to Zia,

and thanks to everybody else

Footnote 1

For validating a modal syllogism with a modeless premise, Avicenna translates the modeless premise to Themistian absolute.
This gives sensible results if the other premise was alethic necessary.

But if the other premise was alethic possible, then both premises are translated to Themistian absolute, and the effect is to erase the difference between possible and modeless.

Avicenna struggles to handle this case.

Footnote 2

Spencer Johnston found a Kripke-style semantics to validate Jean Buridan's divided modal syllogisms as reported in his *Summulae de Dialectica*.
There is a clear resemblance to the modal semantics of Avicenna, except that the condition for Buridan's 'necessarily an *A*' would read 'always an *A* and existing' in Avicenna's terms.

Recently Dagys et al. found in Buridan's *Treatise on Consequences* hints of a semantics even closer to Avicenna.
No line of transmission from Avicenna's logic to Buridan's is known.

Wilfrid Hodges and Spencer Johnston, 'Medieval modalities and modern methods: Avicenna and Buridan', *IfCoLog Journal of Logics and their Applications* 4 (4) (2017) 1029–1073.

Jonas Dagys, Živilė Pabijutaitė and Haroldas Giedra, 'Inferences between Buridan's modal propositions', *Problemos* 101 (2022) 31–41.