Logical rules at deep syntactic levels

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A very brief history of logic

4th c BC: Aristotle invents syllogisms

Roman Empire and Arab Empire: Logic developed through commentaries on Aristotle and then on Ibn Sīnā

12th-14th centuries: Western Scholasticism

15th–19th centuries: No global developments, but many individual insights e.g. Leibniz

Late 19th century: Peano, Peirce and Frege send logic in new directions, developed in 20th century

Widely agreed that logic after the late 19th century was

- less closely related to natural language,
- less accessible and intuitive,
- justified more by mathematical practice.

Technically there were two major changes:

1. Move away from *monotonicity inferences*.
2. Introduction of *deep rules*, i.e. rules operating at any syntactic level.

Monotonicity — a Scholastic innovation

Terms (roughly = noun phrases) occur in two kinds of place as N in $\phi(N)$.

1. If $\phi(A)$ and $(A \rightarrow B)$ then $\phi(B)$.
Socrates is a human, every human is rational,
so Socrates is rational.
(We say A is *upwards monotone* in $\phi(A)$.)

2. If $\phi(B)$ and $(A \rightarrow B)$ then $\phi(A)$.
Every rational being can laugh, every human is rational,
so every human can laugh.
(We say B is *downwards monotone* in $\phi(B)$.)

The Scholastic contribution has two parts.

First, *every term in a syllogistic sentence is either upwards or downwards monotone,*
and all valid syllogisms can be seen as applications of the appropriate monotonicity.
(Applications of downwards monotonicity were said to be instances of *dictum de omni et nullo*.)

Not known to the Roman or Arab Empire commentators, and probably Aristotle wasn't aware of it.

Second, *occurrences of terms are upwards monotone iff they are positive*
(i.e. in the scope of an even number of negations)
and downwards monotone iff they are negative
(… an odd number of negations).

This part starts to appear in the 13th c.
Pre-20th century versions are mostly very confused.

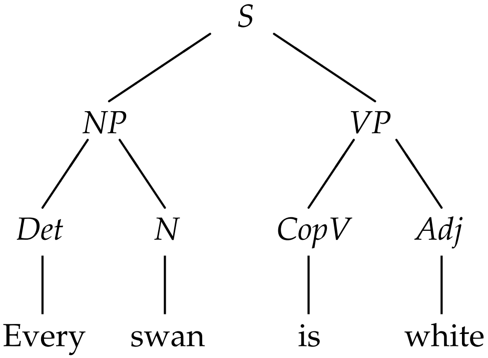
Only one instance of monotonicity survives in most modern logical systems, and it's upwards:

Modus ponens

> S.
> If S then T.
> Therefore T.

Deep rules, introduced by Boole 1847, Frege 1879

Syllogisms never go below a top level analysis of the syntax.



The Boole-Frege deep rule of identity:

$\phi(X)$.

X is the same thing/person as Y.

Therefore $\phi(Y)$.

(A rule like this appeared earlier in Leibniz.
The new feature here is that it's explicit that X can be at any syntactic depth.)

The boys who put the powder on the noses of the faces of the ladies of the harem of the court of King Caractacus are just passing by.

King Caractacus is the king of the Ancient Britons.

Therefore:

The boys who put the powder on the noses of the faces of the ladies of the harem of the court of the king of the Ancient Britons are just passing by.

(Apologies to Rolf Harris.)

Another common deep rule:
For every x, $\phi(x)$. Therefore $\phi(c)$.

Whatever vegetable you choose, Matilda can tell you how to make the fertiliser that's exactly right to get the best crop of the vegetable.

Rhubarb is a vegetable.

Therefore: Matilda can tell you how to make the fertiliser that's exactly right to get the best crop of rhubarb.

First question: Are the above deep rules hard? And if so, are they any harder than the parsing problems involved?

My guess: They are harder than syllogism rules,
but only because of the extra parsing.

Deep monotonicity rules

The monotonicity rules still work correctly when the positive or negative occurrences are at any syntactic depth.

This is a mid 20th century observation,
proved by induction on the complexity of formulas.

Since 1950 several logical calculi have been based on the deep monotonicity rules.
These calculi are sometimes claimed to be more 'natural' than first-order logic.

George Englebretsen, *Something to Reckon With: the Logic of Terms* (1996) p. 240:

> What *can* be claimed on behalf of a term logic such as the one outlined in this essay is that for those areas where both it and modern predicate logic claim to exhibit the forms of logical expression and inference—rational uses of natural language—it is simpler, more **natural**, and more powerful.

Similar claims of naturalness are in Fred Sommers and Johan van Benthem (for example).

At least two senses of ‘natural’:

(1) ‘Natural’ means using natural language syntax. Some deep monotonicity calculi, e.g. that of Sánchez Valencia, are very close to natural language.

(2) ‘Natural’ means the rules of inference are intuitive and follow most people’s usual reasoning patterns.

I strongly doubt that any deep monotonicity calculus is natural in sense (2).
So I ask:

Second question: Are the deep monotonicity rules harder or easier than the non-monotonicity deep rules above?

My guess: They are *much harder*, in fact so hard that they are normally impossible below two or three syntactic levels.

Sample of deep monotonicity:

If you've never met someone who can't name any writers, you lead a lonely life.

All novelists are writers.

Therefore, if you've never met someone who can't name any novelists, you lead a lonely life.

For comparison, an example of the deep rule of identity:

> If you've never met someone who has never eaten courgettes, you lead a lonely life.
>
> Courgettes are the same thing as zucchini.
>
> Therefore, if you've never met someone who has never eaten zucchini, you lead a lonely life.

Upshot

The relative hardness of deep monotonicity rules is an empirical question.

Asking around, I failed to find any experimental evidence bearing on it.

Obviously any experiment to test it would need careful design, and I don't have the expertise.

Is anyone game?