Corrigenda for Mathematical Logic, Chiswell and Hodges

- p.20: Ex. 2.4.1(c): b (not a) unequal to 0.
- p.22: last displayed item, top right: should be D, not D'.
- p.23: Exercise 2.5.1(d) should read

  $$\vdash (((\phi \longleftrightarrow \psi) \longleftrightarrow \chi) \longrightarrow (\phi \longrightarrow (\psi \longleftrightarrow \chi)))$$

  It is not possible to do the exercise as given in the book using only the rules developed so far. For if it were, one could give a proof that

  $$\vdash ((\phi \longleftrightarrow (\psi \longleftrightarrow \chi)) \longrightarrow ((\phi \longleftrightarrow \psi) \longrightarrow \chi))$$

  and so a proof that

  $$\{(\phi \longleftrightarrow (\psi \longleftrightarrow \chi)), (\phi \longleftrightarrow \psi)\} \vdash \chi$$

  using just these rules. The method of Exercise 3.9.2 shows that this can't be done, as follows. Add to that exercise that the value of $(p \wedge q)$ is the minimum of the values of p and of q, and the value of $(p \longleftrightarrow q)$ is the same as that of $((p \longrightarrow q) \wedge (q \longrightarrow p))$. Then show that the conclusion of Exercise 3.9.2(c) remains true when the rules ($\wedge$I), ($\wedge$E), ($\longleftrightarrow$I) and ($\longleftrightarrow$E) can also be used in D.

  Therefore, it suffices to find values for ϕ, ψ, χ such that $(\phi \longleftrightarrow \psi)$ and $(\phi \longleftrightarrow (\psi \longleftrightarrow \chi))$ both have value 1 but χ has value less than 1. The following values work: ϕ and ψ have value 0, χ has value $1/2$.
- p.44: exercise 3.2.6, should be "if $D_1(\mu) = (\nu_1, \dots, \nu_n)$ then $D_2(f\mu) = (f\nu_1, \dots, f\nu_n)$."
- p.47: the penultimate sentence of Theorem 3.3.4 should read "Moreover in case (b) the formulas ϕ and ψ are uniquely determined segments of χ".
- p.54: in Definition 3.4.1(f), third line from bottom, "leaf" should be "node".
- p.62: in Definition 3.5.1, Line 3, "identity" should be "identify".
- p.72: Definition 3.7.1 should read "to each q_i $(1 \le i \le k)\dots$" .
- p.74: the last line should end "both $A^\star(\phi[S])$ and $A[S]^\star(\phi)$ are F."
- p.75: Theorem 3.7.6(b) should be labelled "Replacement Theorem".
- p.78: the expression (3.59) should be $(\cdots(\phi_1 \wedge \phi_2) \wedge \cdots) \wedge \phi_n)$.
- p.79: part (b) of Definition 3.8.1 should read "… these n formulas are called the *disjuncts* of the disjunction."
- p.84: at the end of exercise 3.8.2(a), it should be "…logically equivalent to ϕ.]".
- p.95: Exercise 3.10.1 should begin "If we kept the truth function symbols $\longrightarrow$, $\vee$ and $\longleftrightarrow$".
- p.115: Definition 5.3.6 should include the condition that, if a leaf is labelled by a term, then it has a mother. Equivalently, if a parsing tree has just a single node, then its label must be $\bot$.
- p.119: Definition 5.3.9, part (a), the complexity of a formula is the height of the parsing tree with leaves labelled by terms, and the corresponding edges, removed. This truncated parsing tree should also be used in part (c).
- p.144: parts (1) and (2) of Theorem 5.8.3 are referred to as (a) and (b) in the proof.
- p.146: in Definition 5.8.4, it should be $(k_1, \dots, k_n)$, not $(k_1, \dots, k_m)$.
- p.190: Definition 7.6.3, third line, $RL(\sigma)$ should be LR(σ).
- p.193: in the last sentence of the proof of the Compactness Theorem, it should be $\Gamma' \models \bot$, not $\Gamma \models \bot$.
- p. 202: proof of Theorem 7.8.3(a), third line, should be "Hence X is countable by Lemma 7.8.2."

- p.240: the definitions of t' and ϕ' in the solution of 5.4.8 should be as follows.

$$t' = \begin{cases} t & \text{if } t \text{ is a variable or a constant symbol in } \rho \\ x_0 & \text{if } t \text{ is a constant symbol not in } \rho \\ F(s'_1, \dots, s'_n) & \text{if } t \text{ is } F(s_1, \dots, s_n) \text{ for some terms } s_1, \dots, s_n \text{ and } F \text{ in } \rho \\ x_0 & \text{if } t \text{ is } F(s_1, \dots, s_n) \text{ for some terms } s_1, \dots, s_n \text{ and } F \text{ not in } \rho. \end{cases}$$

The definition of ϕ' needs modification only in the case that ϕ has the form $R(t_1, \dots, t_n)$, where R is a relation symbol in σ.

$$\phi' = \begin{cases} R(t'_1, \dots, t'_n) & \text{if } \phi \text{ is } R(t_1, \dots, t_n) \text{ and } R \text{ is in } \rho \\ \bot & \text{if } \phi \text{ is } R(t_1, \dots, t_n) \text{ and } R \text{ is not in } \rho \\ (s' = t') & \text{if } \phi \text{ is } (s = t) \\ \bot & \text{if } \phi \text{ is } \bot \\ (\psi' \Box \chi') & \text{if } \phi \text{ is } (\psi \Box \chi) \text{ with } \Box \in \{\wedge, \vee, \to, \leftrightarrow\} \\ (\neg \psi') & \text{if } \phi \text{ is } (\neg \psi). \end{cases}$$

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