Why modern logic took so long to arrive

Three lectures for Cameleon March 2009

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1 The main figures in traditional logic

The classical Greeks

Early 5th c BC Parmenides advocates reductio ad absurdum as a source of new information.

Early 4th c BC Plato discusses definition.

Late 4th c BC Aristotle invents syllogisms.
Theophrastus, Aristotle's successor as leader of the Peripatetic School, develops and extends syllogisms.

4th–3rd c BC The Megarian and Stoic schools of logic, led by Diodorus Cronus and Chrysippus.

1st c BC Aristotle's logical writings (the *Organon*) edited by Andronicus of Rhodes.

The Roman Empire commentators

2nd c AD Alexander of Aphrodisias provides commentaries to support the use of the *Organon* as a basis for liberal education.

Late 3rd c AD Porphyry proposes a programme to keep both Peripatetic logicians and Neoplatonist metaphysicians happy: restrict Aristotle's notions to what is needed for everyday logic.
Iamblichus, Dexippus etc.

c. 500 Ammonius leads School of Alexandria.

Early 6th c AD Stephanus, Philoponus, Olympiodorus at Alexandria; Boethius in Europe.

The Arabic translators and commentators

8th c AD Aristotle's logic summarised in Arabic by Ibn al-Muqaffa^c.

9th c AD Systematic high-quality translation of Aristotle into Arabic.

10th c AD Al-Fārābī writes commentaries on Aristotle's logic (partly lost).

11th c AD Ibn Sīnā.

12th c AD Ibn Rushd, Al-Ghazali.
Some logical work of Ibn Rushd translated into Latin.

13th c AD The Persian astronomer Tusi comments on Ibn Sīnā's logic.

The Scholastics

12th c AD Abelard, various anonymous e.g. author of 'Cum sit nostra'.

Early to mid 13th c AD William of Sherwood, Francis Bacon, Peter of Spain.

Late 13th c AD Logicians influenced by translations of Ibn Rushd (Averroes), such as Robert Kilwardby.
Modist linguists, mostly from Denmark, such as Boethius of Dacia.

Early 14th c AD Walter Burley, William Ockham, Jean Buridan.

Renaissance to 19th century

15th, 16th c AD Valla, Ramus and their followers emphasise use of logic for rhetoric and self-improvement.

17th c AD Antoine Arnauld and Pierre Nicole (Port-Royal Logic and Grammar), Joachim Jungius, Gottfried Wilhelm Leibniz, John Wallis.

18th c AD John Bernoulli, Euler.

Early 19th c AD Jeremy Bentham, Richard Whately, Bernard Bolzano, William Hamilton (of Edinburgh), Augustus De Morgan, George Boole.

Late 19th c AD Stanley Jevons, Charles Peirce, Gottlob Frege, Giuseppe Peano.

c. 1900 Aristotelian features still visible in David Hilbert, Bertrand Russell.

2 Texts

Aristotle

De Interpretatione (Oxford Translation ed. Barnes) §1.

First we must settle what a name is and what a verb is, and then what a negation, an affirmation, a statement and a sentence are.

Now spoken sounds are symbols of affections in the soul, and written marks symbols of spoken sounds. And just as written marks are not the same for all men, neither are spoken sounds. But what these are in the first place signs of — affections of the soul — are the same for all; and what these affections are likenesses of — actual things — are also the same. . . .

Just as some thoughts in the soul are neither true nor false while some are necessarily one or the other, so also with spoken sounds. For falsity and truth have to do with combination and separation. Thus names and verbs by themselves — for instance 'man' or 'white' when nothing further is added — are like the thoughts that are without combination and separation; for so far they are neither true nor false. . . .

Prior Analytics (Oxford Translation ed. Barnes) i.35, 36

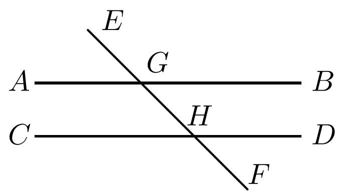
§35. We must not always seek to set out the terms in a single word, for we shall often have phrases to which no single name is equivalent. Hence it is difficult to reduce deductions with such terms. Sometimes too error will result from such a search, e.g. the belief that deduction can establish something immediate. Let A stand for two right angles, B for triangle, C for isosceles triangle. A then belongs to C because of B; but A belongs to B not in virtue of anything else (for the triangle in virtue of its own nature contains two right angles); consequently there will be no middle term for AB, although it is demonstrable. For it is clear that the middle must not always be assumed to be an individual thing, but sometimes a phrase, as happens in the case mentioned.

§36. . . . It happens sometimes that the first term is said of the middle, but the middle is not said of the third term, e.g. if wisdom is knowledge, and wisdom is of the good, then conclusion is that there is knowledge of the good. The good then is not knowledge, though wisdom is knowledge. Sometimes the middle term is said of the third, but the first is not said of the middle, e.g. if there is a science of everything that has a quality, or is a contrary, and the good both is a contrary and has a quality, the conclusion is that there is a science of the good — but the good is not a science, nor is that which has a quality or is a contrary, though the good is both of these. Sometimes neither the first term is said of the middle, nor the middle of the third, while the first is sometimes said of the third, and sometimes not; e.g. if there is a genus of that of which there is a science, and there is a science of the good, we conclude that there is a genus of the good. But nothing is predicated of anything. And if that of which there is a science is a genus, and there is a science of the good, we conclude that the good is a genus. The first term then is predicated of the extreme, but the terms are not said of one another.

The same holds good where the relation is negative. For 'that does not belong to this' does not always mean that this is not that, but sometimes that this is not of that or for that, e.g. there is not a motion of a motion or a becoming of a becoming, but there is a becoming of pleasure; so pleasure is not a becoming. Or again it may be said that there is a sign of laughter, but there is not a sign of a sign, consequently laughter is not a sign. This holds in the other cases too, in which a problem is refuted because the genus is asserted in a particular way in relation to it. Again take the inference: opportunity is not the right time; for opportunity belongs to God, but the right time does not, since nothing is useful to God. We must take as terms opportunity, right time, God; but the proposition must be understood according to the case of the noun. For we state this universally without qualification, that the terms ought always to be stated in the nominative, e.g. man, good, contraries, not in oblique cases, e.g. of man, of good, of contraries, but the propositions ought to be understood with reference to the cases of each term — either the dative, e.g. 'equal to this', or the genitive, e.g. 'double of this', or the accusative, e.g. 'that which strikes or sees this', or the nominative, e.g. 'man is an animal', or in whatever other way the word falls in the proposition.

Euclid, *Elements*, trans. Thomas Heath, i.29.

A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.



For let the straight line EF fall on the parallel straight lines AB, CD;

I say that it makes the alternate angles AGH, GHD equal, the exterior angle EGB equal to the interior and opposite angle GHD, and the interior angles on the same side, namely BGH, GHD, equal to two right angles.

For, if the angle AGH is unequal to the angle GHD, one of them is greater.

Let the angle AGH be greater.

Let the angle BGH be added to each; therefore the angles AGH, BGH are greater than the angles BGH, GHD.

But the angles AGH, BGH are equal to two right angles; therefore the angles BGH, GHD are less than two right angles.

But straight lines produced indefinitely from angles less than two right angles meet; therefore AB, CD, if produced indefinitely, will meet; but they do not meet, because they are by hypothesis parallel.

Therefore the angle AGH is not unequal to the angle GHD, and is therefore equal to it.

Alexander of Aphrodisias, *On Aristotle Prior Analytics* trans. Ian Mueller, Duckworth, London 2006, 260.18–261.19,

The syllogism showing that odds would turn out to be equal to evens if the diagonal were commensurable with the side is the following:

Let ABCD be assumed to be a square area, and let BC be its diagonal. If the diagonal BC is commensurable with the side AB, it will have to AB the ratio which a number has to a number. For we have this proved by Euclid in the tenth book of the *Elements*: 'Commensurable magnitudes have to one another the ratio which a number has to a number' — this is the fourth theorem in the tenth book. So let the number E be to the number F as the diagonal BC is to the side BA; and let the least numbers having the same ratio as these be taken; these are prime to one another, since this has also been proved in the seventh book of Euclid's *Elements*: the least numbers of those having the same ratio are prime to one another. But numbers are prime to one another if they are only measured by the monad. Let each of E and F be multiplied, and let G multiplied by itself be I, and H multiplied by itself be J. Therefore I and J are square numbers, and they are also prime to one another. For this has also been proved in the seventh book of the *Elements*: if two numbers are prime to one another and each of them being multiplied makes something, the numbers which come to be from them will also be prime to one another. So, since the number E is to the number F as the diagonal BC is to the side AB, but as E is to F, so is G to H, and as the diagonal BC is to the side AB so is the number G to the number H. And, therefore, as the square on the diagonal BC is to the square on the side AB, so will the square of G be to the square of H. But these latter are I and J. But the square on the diagonal is double the square on the side. Therefore, the number I is also double the number J. Therefore I is even, since every double of some number is even because it is divisible into equal parts. But half of I will also be even, since the halves of square numbers which are divisible into equal parts are also even. Therefore J is also even since it is half of I, which is square. But it is also odd, since I and J are prime to one another; but it is impossible for evens to be prime to one another, since evens are not only measured by the monad as common measure (and that is the specific characteristic of primes). So it is necessary that either both or one of them be odd. But both have also been proved to be even through the hypothesis. Consequently, when it is hypothesized that the diagonal is commensurable with the side, odds are equal to evens, which is impossible.

Proclus

Commentary on the First Book of Euclid's Elements, trans. Glenn Morrow, Princeton UP 1970, 255f.

Every reduction to impossibility takes the contradictory of what it intends to prove and from this as a hypothesis proceeds until it encounters something admitted to be absurd and, by thus destroying [i.e. refuting, not discharging] its hypothesis, confirms the proposition it set out to establish. In general we must understand that all mathematical arguments proceed either from or to the starting-points [i.e. the conclusions], as Porphyry somewhere says. Those that proceed from the starting-points are themselves of two kinds, as it happens, for they proceed either from common notions, that is, from self-evident clarity alone, or from things previously demonstrated. Those that proceed to the starting-points are either affirmative of them or destructive. But those that affirm first principles are called "analyses," and their reverse procedures "syntheses" (for it is possible from those principles to proceed in orderly fashion to the thing sought, and this is called "synthesis"); when they are destructive, they are called "reductions to impossibility," for it is the function of this procedure to show that something generally accepted and self-evident is overthrown. There is a kind of syllogism in it, though not the same as in analysis; for the structure of a reduction to impossibility accords with the second type of hypothetical argument. For example, if in triangles that have equal angles the sides subtending the equal angles are not equal, the whole is equal to the part. But this is impossible; therefore in triangles that have two angles equal the sides that subtend these equal angles are themselves equal. So much regarding reductions to impossibility.

Ammonius, *On Aristotle On Interpretation 1–8*, trans. David Blank, Duckworth, London 1996.

38.17–22: And if this is correct, it is clear that we shall not accept the opinion of Diodorus the Dialectician, who thought that every vocal sound is significant and as a proof of this claim called one of his servants 'Allamen' (i.e. *alla mên*, 'but in fact') and others by other conjunctions. For it is hard even to imagine what meaning such vocal sounds will have, that of some nature or person, as names do, or of an action or passion, as verbs do.

89.4–18: 'Determinations' are what we call certain designations which combine with the subject terms and indicate how the predicate relates to the multitude of individuals under the subject term, whether it is taken as holding or as not holding. Hence, they too are four in number, 'every' and 'none', 'some' and 'not every': two universal ('every' and 'none'), and two particular ('some' and 'not every'). And of the universal ones, 'every' is affirmative, e.g. 'Every man is an animal', signifying that 'animal' holds of all individuals under man, and 'none' is negative, e.g. 'No man is winged', indicating that the predicate term belongs to none of the singular men. And of the particular ones, again one is affirmative and one negative: 'some' is affirmative, e.g. 'Some man is pale', signifying that the predicate term belongs to at least some one of the individuals under the subject term, and 'not every' is negative, e.g. 'Not every man is just', which is destructive of 'every' and signifies that it is not true that the predicate term belongs to all the individuals under the subject term.

Ibn Sīnā

Autobiography tr. Gutas, *Avicenna and the Aristotelian Tradition*, Brill, Leiden 1988, pp. 26–28.

As for the *Elements* of Euclid, I read the first five or six propositions . . . , and thereafter undertook on my own to solve the entire remainder of the book. . . . The next year and a half I devoted myself entirely to reading Philosophy. I read Logic and all the parts of philosophy once again. During this time I did not sleep completely through a single night, or occupy myself with anything else by day. I compiled a set of files for myself, and for each argument that I examined, I recorded the syllogistic premisses it contained, the way in which they were composed, and the conclusions which they might yield, and I would also take into account the conditions of its premisses [i.e. their modalities] until I had Ascertained that particular problem. . . . So I continued until all the Philosophical Sciences became deeply rooted in me and I understood them as much as is humanly possible. Everything that I knew at that time is just as I know it now; I have added nothing more to it to this day. Having mastered Logic, Physics and Mathematics, I had now reached Theology. . . .

Madḵal

For there are accidents that are inherents of the whatness by a primary 35.18
and clear entailment that is not mediated by any other accident. So when
the entailment is not via some intermediate, it is impossible to negate the 35.20
accident from the whatness at the same time as affirming the whatness,
having them both enter the mental processor together. An example is [TRIANGLE] and [CAN IMAGINE A LINE OF THE TRIANGLE EXTENDED],
or some other meaning similar to this from among the accidents of [TRIANGLE]. It can sometimes happen that the holding of the accident is through 36.1
something intermediate, so when this intermediate thing doesn't come into the mental processor, one can negate [the accident] — for example [one can negate] that any two angles of a triangle are [together] less than two right angles.

^{c}Ibāra

[1.2.15] But someone might well say: You made it [part of] the defini- 12.7
tion of a noun that 'it signifies and no part of it signifies'. But there are nouns like the phrases 'non-human' and 'un-seeing'; there is no doubt that

these are nouns. How do these signify in the way that nouns [are sup-
posed to]? And how is it that the phrase ‘un-seeing’ can stand in place of 12.10
the phrase “blind” — and in the other case one finds the expression ‘un’
and the expression “human”, both of them signifying, and the meaning of
the whole is composed from the meanings of these two? We say: These
also are not really nouns, and insofar as they are [compounds], there is no
noun imposed on [their meaning] to signify [it]. Rather, they are the kind
of composite expression that can be used as an atomic expression, in the
same way as definitions or the expressions ‘herder of sheep’ and ‘thrower
of stones’. But they are like these expressions only subject to a qualification.
I say this because, for example, ‘non-human’ is not a compound of atomic
and autonomous expressions, but it’s a compound of a name and a nega- 12.15
tion particle. The fact that they correspond to nouns doesn’t indicate that
they really are nouns. Definitions and descriptions are natural examples of
the same phenomenon. Nevertheless you shouldn’t be misled by the occur-
rence of the negation particle in them, so as to think that there is a negation
like ‘not’ in [their meaning]. Rather, they are not affirmative or negative [in 13.1
themselves]; you can legitimately use them as affirmations or as negations,
so that they are imposed on an affirmative or negative [meaning]. Because
they are closely akin to nouns, let us call them ‘indefinite nouns’.

[1.2.19] In [the case of an inflected noun] the definition of noun has an 14.10
extra point attached, at least so far as Arabic is concerned. Namely, it is not
correct to connect to [the inflected noun] each the things that nouns are nat-
urally connected to. One doesn’t put ‘in’ before ‘Zayd’ in the nominative.
In the accusative, ‘Zayd’ is not [put before] ‘hit’ or ‘was’ or ‘is an animal’;
and the same holds for the genitive ‘Zayd’s’. In Greek, if you connect an
inflected noun to temporal verbs like ‘was’ or ‘is’ or ‘is now’, the result is 14.15
neither true nor false; but an uninflected noun connected to one of these
[gives something] true or false.

[1.2.20] Now worm-eaten wood is wood with worm-eating linked to it, 14.17
so it’s wood together with a feature that occurs to it, namely the worm-
eating, and it is in itself wood without any addition, but the combined
whole is not wood without reservation. I mean that it’s like a statue which
is both wood and a likeness. The wood is not described as being [just] its 15.1
matter, because the whole is not described as being one of its two compo-
nents. Likewise when we take the whole consisting of the noun and inflec-
tion attached to it, it becomes in effect a compound and not a noun. When
it is considered as subject to the inflection, it is an inflected name; when it

is considered independently, it is a noun without reservation. The difference between (1) considering it as an inflected noun and (2) considering the 15.5
whole [of which it forms a part] is like the difference between considering the tree-trunk that holds up the roof and considering the whole consisting of the tree-trunk and the roof. As in [the case of the tree-trunk], you should say that the inflected noun is an expression that has a meaning while no part of it has a meaning, and there is attached etc. etc. But you should not say that the whole arising from the noun and its inflection has a meaning while no part of it has a meaning. How could it be otherwise, considering that the noun is one of the two parts, and it has a meaning? This is a subtle rule; we should keep it in reserve for further discussion.

[1.2.21] There is a common kind of error about things that are joined 15.9
together. It occurs through not recognising that an idea taken with another 15.10
idea is not the whole arising from it and the thing taken with it; just as one added to six, when we consider it together with six, is not the sum of one and six, which is seven.

[1.3.5] Here we have [three things:] a posited name, a participle and 18.6
a verb. The posited noun signifies what is said and it doesn't signify an agent at all, whereas the participle signifies an indeterminate agent, which is a thing it inherits from the verb whose participle it is. So the participle signifies a meaning, and a thing which is the indeterminate agent [of the meaning], and a link between these two. For example the [participle] 'walking' signifies [THE ACT OF WALKING], and an indeterminate agent,
and the fact that [the act of] walking is an act of the agent. The verb sig- 18.10
nifies, besides this, the time at which [the agent] links to [the act]. Thus 'He-will-walk' signifies [THE ACT OF WALKING], and an indeterminate agent, and the fact that [THE ACT OF WALKING] belongs to the agent, and that this is in the future.

[1.3.6] Not everything called an 'action word' in Arabic is a verb. In fact 18.12
the words 'I-will-walk' and 'you-will-walk' are action words according to [the linguists], but they are not verbs in the absolute sense. The reason is that the hamza and the tau each signify a specific agent. Hence when you say 'I-will-walk' or 'I-walked', this makes a true or false statement, and
likewise 'You-will-walk' and 'You-walked'. 18.15

[1.5.6] Two kinds of composition are useful in the sciences. One is where 31.16
the composition is by restriction. This occurs when we acquire a concept by definitions, descriptions and the like. The other is composition by comment

(*ḥabar*). This occurs when we achieve assent through syllogisms and the 32.1
like. This [second] kind of composition gives rise to a kind of phrase called
a 'declarative [sentence]'.

Qiyās

II.4 Recombinant syllogisms

[2.4.1] These things that we have been discussing [(i.e. propositions)] 106.4
are referred to as 'premises' when one intends to study them as parts of
a syllogism. We assert that a [proposition] that follows from a syllogism 106.5
falls into one of two cases. The first case is that neither the proposition nor
its contradictory is mentioned explicitly in the syllogism [i.e. the premises];
syllogisms of this kind are called 'recombinant'. An example is when you
say: 'Every animal is a body, and every body is a substance, so every animal
is a substance'. The second case is that the proposition or its contradictory,
or more generally one of the two polarities of the conclusion, is mentioned
in it explicitly in some way. I call these [syllogisms] 'duplicative', though
the common name for them is 'propositional'. The reason I don't call them
propositional is that some propositional [syllogisms] are in fact recombi- 106.10
nant.

[2.4.2] Let us start with the recombinant [syllogisms]. Some of them 106.11
[are predicative, i.e. they] consist of predicative [propositions]. We as-
sert that every simple predicative recombinant syllogism is composed of
two premises which share a term, like the shared term 'body' the example
above. This term can be in one of the two [premises] as predicate and in the
other as subject; or it can be predicate in both; or it can be subject in both.
When this term is the subject in one and the predicate in the other, then 106.15
there are two cases. It can be predicated of [the term that is] the subject of 107.1
the goal and subject of [the term that is] the predicate of the goal; this case
is called 'the first figure'. Or else it can be predicated of the predicate [of
the goal] and subject of the subject of the goal. But when I come to discuss
it, I will eliminate this figure on grounds of deficiency, though it had to be
included in the classification

VIII.3 The syllogism of absurdity (On Aristotle *Prior Analytics* i.23, 41a21).

[8.3.1] The syllogism of absurdity is really a compound syllogism formed 408.4
from just two propositional syllogisms. Thus, if the goal is a predicative
[proposition] — this is the case which is investigated in the Analytics — 408.5

then the conclusion is this predicative [proposition]. But the [compound] syllogism will be a propositional one not containing a predicative syllogism, at least when it takes the natural straightforward path. Of the two propositional syllogisms in it, one is recombinant, with a premise consisting of a meet-like propositional compound whose first clause overlaps its second; and the other is a propositional meet-like duplicative syllogism. In this way the [syllogism of] absurdity in itself is completed. To find the completion of the syllogism of absurdity there is no need for the unconvincing
elaboration which [some people] attempt, or for it to be completed with 408.10
who knows how many syllogisms, or for the lengthy elaboration found in their books.

[8.3.2] The right way to look at it, which is how the the First Teacher 408.12
approached it, is as follows. Suppose for example that we take the goal to be 'Not every J is B'. Now we say:

(1) If the sentence 'Not every J is B' is false, then every J is B.

Then we add to it a true premise:

(2) Every B is A.

The recombinants which we have counted as propositional yield a consequence thus: 408.15

(3) If the sentence 'Not every J is B' is false then Every J is A.

Then we say: 409.1

(4) But not every J is A.

But that is a contradictory absurdity. Thus the contradictory of the second clause [of the compound] has been duplicated, so that the contradictory of the first clause follows, namely:

(5) Every J is B.

This is plain sailing.

[8.3.3] So this compound syllogism is completed by two syllogisms. In 409.3
these syllogisms there are two premises that are propositional compounds. One of these two [premises] takes the same form in all cases, I mean in the
sense that its first clause is the negation of the goal and its second clause is 409.5

the contradictory of the goal. But in the case of the second [premise], its first clause always takes the same form, but the form of its second clause varies. In fact its first clause is [always] the negation of the goal. But its second clause takes whatever form follows from the composition of the negation of the goal together with [the retained] true premise [i.e. $\bar{p}$ follows from $\bar{r}$ and q]. [The syllogism containing this second premise] is composed in such a way as to entail a predicative proposition if the goal was predicative, or a propositional compound if the goal was a propositional compound — as we said after the claim. 409.9

[8.3.4] Here is an example: 409.9

(6) If it is not the case that when ϕ then ψ, then it is not the case that when ϕ then ψ.

And then: 409.10

(7) Whenever χ then ψ.

There follows:

(8) If it is not the case that when ϕ then ψ, then it is not the case that when ϕ then χ.

But this is a contradiction, since

(9) it is not the case that whenever ϕ then χ.

There follows:

(10) Whenever ϕ then ψ.

This is the analysis of the syllogism known as 'by absurdity, [arguing] towards its premises'. 409.13

[8.3.5] There are people who try to posit the first propositional com- 409.14
pound, and then prove the absurdity from it, saying 'But its second clause 409.15
is impossible', which they take to be something that has to be proved. One of them goes to great trouble to find a syllogism which is a combination of the second clause and the absurdity, and then he says:

(11) The second clause and something true combine to make a syllogism that proves an impossibility; but [the impossibility] is not got by combining the second clause and the true [premise]. This is impossible.

Then he produces a syllogism that proves the minor premise, and he says:

(12) (1) The second clause combines with etc. etc. to make a syllogism which proves an impossibility; but (2) it doesn't combine with etc. etc. to make a syllogism which proves an impossibility. Therefore the second clause and a truth combine to make a syllogism that proves impossibility [namely the conjunction of (1) and (2)].

This is remote from [the normal] assimilation of premises. He goes to great 410.1
trouble to spin out his discussion to the point of impossibility.

[8.3.6] One of [the commentators] avoids this. He takes a premise-pair 410.3
consisting of the second clause and something true, which entails an impossibility. Then he reconsiders and says:

(13) This conclusion is an impossibility, so [the impossibility] comes either from the major premise, or from the minor, or from the premise-pair.

Then he uses a duplicative argument: it doesn't come from the premise-pair, and this implies that it comes either from the major premise or from the minor. Then he uses another duplicative argument: it doesn't come from the major, since the major premise is true, so this implies that it comes from the minor premise. Then he says: the minor is impossible, and this implies that the contradictory of the second clause is true and the contradictory of the first clause is true. But all of these kinds of mutilation, and [these] things that are hidden and not explicit, lengthen the discussion but
give us no new information. What he presents to us is exactly the same 410.10
absurdity syllogism [as we had in the first place], with nothing added or subtracted.

[8.3.7] The usual way to use absurdity is to use the recombinant [syl- 410.11
logism], and then to ignore its conclusion and not remember it, but rather to mention what is in reality a duplicate of the contradictory of its second clause. Then it entails the goal. For example the usual way [to present an argument from absurdity] is to say

(14) If [it's not the case that] not every J is B, then every J is B.
But every B is A, so every J is A, and this is absurd.
Hence [not] every J is B.

Thus when he says "so every J is A", this means 410.15

(15) If [it’s not the case that] not every J is B, then every J is A.

Thus if the circumstances are as we described, “then every J is A”. And his statement: “This is absurd” means

(16) Not every J is A.

— which duplicates the contradictory of the second clause. So the usual style agrees with our analysis of the absurdity syllogism. 411.1

[8.3.8] The phrase ‘syllogism of absurdity’ means a syllogism which reduces the discussion to an impossibility, so the word ‘absurdity’ (*k̲alf*) refers to impossibility. Some people say that the syllogism of absurdity is called *k̲ulf*. These people are out of line; *k̲ulf* is just about promises. Also some people have said that it is called syllogism of *k̲alf* just because it approaches the [goal] from behind it (*k̲alfih*) and not through the front door — since it approaches by way of the contradictory of the goal. But it seems to me that the most realistic [explanation] is that *k̲alf* is used here in the sense of impossibility, not in any other sense.

411.1 (beside the first line of [8.3.8]); 411.5 (beside "approaches by way of the contradictory of the goal. But it seems to me")

Notes on 8.3

[8.3.2]

408.12 ‘how the First Teacher approached it’: The First Teacher is Aristotle. There is no evidence in Aristotle’s text to support Ibn Sīnā’s attribution of this view to Aristotle. This is one of a number of places where Ibn Sīnā apparently assumes that Aristotle was such a good logician that he must have shared Ibn Sīnā’s own insights.

[8.3.4] I use the following abbreviations: ϕ = ‘J is D’, ψ = ‘H is Z’, χ = ‘I is U’.

[8.3.5]

410.2 ‘assimilation of premises’: The word translated ‘assimilation’ is *’idġām*. This is not known to be a technical term of logic. Probably it refers to Ibn Sīnā’s notion of how recombinant syllogisms proceed. The premises enter the mind (*bāl*), which notices an identity between a term in the first premise and a term in the second premise, and then reassembles the remaining terms into a conclusion. Probably Ibn Sīnā is saying that he thinks this commentator would have a hard time giving an an analogous explanation of the workings of the demonstration he has described.

410.3 'to the point of impossibility': I think this is meant as a pun. The commentator delays the point where his proof reaches the impossibility; but Ibn Sīnā makes it sound as if the commentator is taking his argument to impossible lengths. There is a similar joke at Aristotle's expense in ^{c}*Ibāra* 101.10, where Ibn Sīnā tells us that the only point of colloquial language to have attracted Aristotle's attention was 'repetitious gibberish' — the truth being that Aristotle did discuss a certain kind of repetitious gibberish.

[8.3.6] This paragraph could be reporting the view of al-Fārābī in his *Commentary on the Prior Analytics*. The commentary is lost except for a section on *Prior Analytics* ii. But in his discussion of reductio ad absurdum in *Kitāb al-mudḵal 'ilā l-qiyās* 76A al-Fārābī says that when the syllogism reaches a falsehood, 'that [i.e. the source of the falsehood] lies either in the two premises together or in one of them'. His subsequent comments ignore the first possibility, so very likely this is something that he dealt with in his longer *Commentary*.

[8.3.7]

410.12 'ignore the conclusion': The conclusion is not remembered, in the sense that the 'if' part of the first propositional premise is ignored. This makes the antecedent look a little like an assumption.

410.13 '[it's not the case that]': There are a couple of 'not's missing in all the manuscripts. This is unwelcome evidence of the logical incompetence of some very early copyist.

[8.3.8]

411.3 'promises': Ibn Manẓūr *Lisān al-^{c}arab* sv. *ḵlf* explains *ḵulf* as a verbal noun from *'aḵlafa* 'to default (on a promise)'.

411.4 'the front door': The reference is to an old Arabic saying (which I am told is still current, at least in Saudi Arabia) that anybody who comes into your house through any other route than the front door is up to no good.

4.3.6. (1) Confusa suppositio dividitur in distributivam et non distributivam, quae solet vocari 'confusa tantum'. (2) Distributiva est secundum quam ex termino communi potest inferri quodlibet suorum suppositorum seorsum, vel etiam omnia simul copulative, secundum propositionem copulativam, ut 'omnis homo currit', sequitur 'ergo Socrates currit', ergo Plato currit', vel etiam 'ergo Socrates currit et Plato currit . . . ' et sic de singulis.

5.8.2. Aliqui dicunt quod iste modus syllogizandi vere est in prima figura secundum descriptionem primae figurae datam a principio, scilicet quod prima figura est in qua medium subiicitur in maiore et praedicatur in minore. Nam cum dico

> cuiuslibet hominis asinus currit, omnis rex est homo; ergo omnis regis asinus currit,

iste terminus 'homo' est medium, et est subiectum in maiore, licet in obliquo, et praedicatum in minore, in recto. Sed alias dictum est quod in dicta maiore 'hominis' non est subiectum.

Unde notandum est quod non oportet in syllogizando ex obliquis vel ex terminis complexis eosdem esse terminos syllogisticos, scilicet medium et extremitates, et terminos praemissarum et conclusionis, scilicet subiecta et praedicata eorum. Unde non solum licet sumere sub aliquo distributo in principio propositionis posito, immo etiam ubicumque ponatur; verbi gratia, sequitur

> B est omne A et C est A; ergo B est C,

et est syllogismus perfectus, sicut in prima figura, quia tenet directe per sumptionem sub termino distributo. Ita etiam iste est bonus syllogismus, et perfectus,

> asinus omnem hominem videt, omnis rex est homo; ergo asinus omnem regem videt;

et est ibi medium syllogisticum 'homo', sumptum in maiore oblique et in minore recte, qui tamen nec est subiectum nec praedicatum maioris, et minor extremitas est 'rex', et maior extremitas est residuum, scilicet aggregatum ex 'asinus' et 'videt', quod eodem modo attribuitur huic obliquo 'regem' in conclusione sicut attribuebatur huic obliquo 'hominem' in maiore.

5.10.6. Primo sciendum est quod si possumus probare contradictorium conclusionis probandae esse falsum sequitur quod conclusio probanda est vera. Quia necesse est uno contradictorio exsistente falso alterum esse verum. Et ita si possumus probare contradictorium conclusionis probandae esse impossibile, sequitur quod conclusio probanda est necessaria, propter hoc quod necesse est si una contradictoriarum est impossibilis alteram esse necessariam.

Antoine Arnauld and Pierre Nicole
La Logique, ou l'Art de Penser (The Port-Royal Logic, 1662) iii.13.

Des Syllogismes dont la Conclusion est Conditionnelle

On a fait voir qu'un syllogisme parfait ne peut avoir moins de trois propositions: mais cela n'est vrai que quand on conclut absolument, et non quand on ne le fait que conditionnellement; parce qu'alors la seule proposition conditionnelle peut enfermer une des premisses outre la conclusion, et même toutes les deux.

Exemple. Si je veux prouver que la lune est un corps raboteux, et non poli comme un miroir, ainsi qu'Aristote se l'est imaginé, je ne le puis conclure absolument qu'en trois propositions.

Tout corps qui réfléchit la lumiere de toutes parts est raboteux;
Or la lune réfléchit la lumiere de toutes parts;
Donc la lune est un corps raboteux.

Mais je n'ai besoin que de deux propositions pour le conclure conditionnellement en cette maniere:

Tout corps qui réfléchit la lumiere de toutes parts est raboteux;
Donc si la lune réfléchit la lumiere de toutes parts, c'est un corps raboteux.

Et je puis même renfermer ce raisonnement en une seule proposition, ainsi:

Si tout corps qui réfléchit la lumiere de toutes parts est raboteux, et que la lune réfléchisse la lumiere de toutes parts, il faut avouer que ce n'est point un corps poli, mais raboteux.

Ou bien en liant une des propositions par la particule causale, *parce que*, ou *puisque* comme:

Si tout vrai ami doit être prês de donner sa vie pour son ami:
Il n'y a guere de vrais amis:
Puisqu'il n'y en a guere qui le soient jusques à ce point.

Cette maniere de raisonner est très-commune et très-belle; et c'est ce qui fait qu'il ne faut pas s'imaginer qu'il n'y ait point de raisonnement que lorsqu'on voit trois propositions séparéees et arrangées comme dans l'École: car il est certain que cette seule proposition comprend ce syllogisme entier:

Tout vrai ami doit être prêt de donner sa vie pour ses amis:
Or il n'y a guere de gens qui soient prêts de donner leur vie pour leurs amis:
Donc il n'y a guere de vrais amis.

Toute la différence qu'il y a entre les syllogismes absolus, et ceux dont la conclusion est enfermée avec l'une des prémisses dans une proposition conditionnelle, est que les premiers ne peuvent être accordés tout entiers, que nous ne demeurions d'accord de ce qu'on auroit voulu nous persuader; au lieu que dans les derniers on peut accorder tout, sans que celui qui les fait ait encore rien gagné; parce qu'il lui reste à prouver que la condition d'où dépend la conséquence qu'on lui a accordée est véritable.

Et ainsi ces arguments ne sont proprement que des préparations à une conclusion absolue: mais ils sont aussi très-propres à cela, et il faut avouer que ces manieres de raisonner sont très ordinaires et très-naturelles, et qu'elles ont cet avantage, qu'étant plus éloignées de l'air de l'école, elles en sont mieux reçues dans le monde.

Gottfried Wilhelm Leibniz

1. Grammaticae cogitationes (*Opuscules et Fragments Inédits de Leibniz*, ed. Couturat, Alcan, Paris 1903, p. 287; translation in Parkinson *Leibniz: Logical Papers* pp. 12–15).

Genitivus est adjectio substantivi ad substantivum quo id cui adjicitur ab alio distinguitur. Ensis Evandri, id est Ensis quem habet Evander. Pars domus, id est pars quam habet domus. Lectio poetarum, id est actus quo legitur poeta. ⟨Optime sic explicabitur⟩, ut Paris est amator Helenae, id est: Paris amat *et eo ipso* Helena amatur. Sunt ergo duae propositiones in unam compendiose collectae. Seu Paris est amator, et eo ipso Helena est amata. Ensis est ⟨ensis⟩ Evandri, id est Ensis est supellex quatenus Evander est dominus. Poeta est lectus quatenus ille vel ille est legens. Nam nisi obliquos casus resolvas in plures propositiones, nunquam exibis quin cum Jungio novos ratiocinandi modos fingere cogaris.

... In Grammatica rationali necessarii non sunt obliqui, nec aliae flexiones. Item careri etiam potest abstractis nominibus. Ad flexiones quidem vitandas circuitu opus est, sed tanti est ratiocinari compendiose, etsi non compendiose te enunties.

2. Proof that if painting is an art, then a person who learns painting learns an art. (Letter to Vagetius, in *Opera Omnia* ed. Dutens, vi.1.39. The translation in Parkinson seems rather loose; I haven't traced this to source.)

Graphice est ars.
Qui discit graphicen, discit rem quae est graphice.
Ergo, qui discit graphicen, discit rem quae est ars.
Qui discit rem quae est ars, discit artem.
Ergo, qui discit graphicen, discit artem.

3. Leibniz's working of part of Vagetius's reply (Mugnai, *Leibniz' Theory of Relations*, Steiner, Stuttgart 1992, p. 153).

Graphice est ars.
Ergo omne discens aliquam Graphicen est discens aliquam artem.
Si non, utique
Quidam discens aliquam Graphicen non erit discens aliquam artem.
Ergo si potest
Titius discens aliquam Graphicen, qui non sit discens aliquam artem.
...

Ergo Titius est discens quandam artem.

4. *Nouveaux Essais sur l'Entendement Humain* (published posthumously in 1765), iv.17

De plus il faut savoir qu'il y a des *conséquences asyllogistiques bonnes* et qu'on ne saurait démontrer à la rigueur par aucun syllogisme sans en changer un peu les termes; et ce changement même des termes est la conséquence asyllogistique. Il y en a plusieurs, comme entre autres *a recto ad obliquum*: par exemple: [si] Jésus-Christ est Dieu; donc la mère de Jésus-Christ est la mère de Dieu. Item, celle que les habiles logiciens ont appelée *inversion de relation*, comme par exemple cette conséquence: si David est père de Salomon, sans doute Salomon est le fils de David. Et ces conséquences ne laissent pas d'être démontrables par des vérités dont les syllogismes vulgaires mêmes dépendent.

John Wallis, *Institutio Logicae* (1702)

Thesis Secunda. Syllogismi Hypothetici, aliique Compositi, referendi sunt omnes ad Aristotelicos Categoricorum Modos.

Verum quatuor adhuc (nec plures) Modos esse volunt syllogismi *Compositi*, quos *ad nullam figurarum simplicium referendos esse* contendunt; Duos *Connexi*, totidemque *Disjuncti* syllogismi modos.

Quos causantur *Aristotelem vel non cognovisse vel saltem non agnovisse* ...

Ego vero hos modos *Aristotelem* (cum aliis) *et cognovisse et agnovisse* contendo, *usuque suo* (quod ipsi observant) confirmasse: (verum et *ex Aristotele doctrinam hanc collegisse et Ciceronem et Ramum ipsum*, diserte asserit ibidem *Dounamus* ...

Modorum *Connexorum* prior est, *qui assumit antecedens et consequens concludit, pag. 725*, qualis, verbi gratia, hic est.

> *Si Sol splendet, dies est;*
> *Sed Sol splendet;*
> *Ergo, dies est.*

At interim ipsi (ut jam ostensum est) pro syllogismis simplicibus (adeoque ad modos Aristotelicos referendos) sequentes hos habent syllogismos;

> *Ubi* (hoc est, *omni loco quo*) *sol splendet, dies est;*
> *Sed ubique (*h.e. *omni loco) sol splendet:*
> *Ergo ubique (*h.e. *omni loco) dies est.*

Vel,

> *Sed alicubi (*h.e. *aliquo loco) sol splendet:*
> *Ergo alicubi (*h.e. *aliquo loco) dies est.*

Vel,

> *Sed hic (*h.e. *hoc loco) sol splendet:*
> *Ergo hic (*h.e. *hoc loco) dies est.*

Augustus De Morgan

Formal Logic (1847) p. 114f.

There is another process which is often necessary, in the formation of the premises of a syllogism, involving a transformation which is neither done by syllogism, nor immediately reducible to it. It is the substitution, in a compound phrase, of the name of the genus for that of the species, when the use of the name is particular. For example, 'man is animal, therefore the head of a man is the head of an animal' is inference, but not syllogism. And it is not mere substitution of identity, as would be 'the head of a man is the head of a *rational animal*' but a substitution of a larger term in a particular sense.

Perhaps some readers may think they can reduce the above to a syllogism. If *man* and *head* were connected in a manner which could be made subject and predicate, something of the sort might be done, but in appearance only. For example, 'Every man is an animal, therefore he who kills a man kills an animal.' It may be said that this is equivalent to a statement that in 'Every man is an animal; some one kills a man; therefore some one kills an animal,' the first premise, and the second premise *conditionally*, involve the conclusion as *conditionally*. This I admit: but the last is not a syllogism: and involves the very difficulty in question. 'Every man *is* an animal; some one *is* the killer of a man': here is no middle term. To bring the first premise into 'Every killer of a man is the killer of an animal' is just the thing wanted. By the principles of Chapter III, undoubtedly the copula *is* might in certain inferences be combined with the copula *kills*, or with any verb. But so simple a case as the preceding is not the whole difficulty. If any one should think he can syllogize as to the instances I have yet given, let him try the following. 'Certain *men*, upon the report of certain other *men* to a third set of *men*, put a fourth set of *men* at variance with a fifth set of *men*.' Now every man is an animal: and therefore 'Certain *animals*, upon the report of certain other *animals*, &c.' Let the first description be turned into the second, by any number of syllogisms, and by help of 'Every man is an animal'.

The truth is, that in the formation of premises, as well as in the use, there is a postulate which is constantly applied, and therefore of course constantly demanded. And it should be demanded openly. It contains the *dictum de omni et nullo* (see the next chapter), and it is as follows. For every term used universally *less* may be substituted, and for every term used particularly, *more*. The species may take the place of the genus, when all the genus is spoken of: the genus may take the place of the species when some of the species is mentioned, or the genus, used particularly, may take the place of the species used universally. Not only in syllogisms, but in all the ramifications of the description of a complex term. Thus for 'men who are not Europeans' may be substituted 'animals who are not English.' If this postulate be applied to the unstrengthened forms of the Aristotelian Syllogism (page 17) it will be seen that all which contain A are immediate applications of it, and all the others easily derived.

Charles Sanders Peirce

'The reader is introduced to relatives', *The Open Court* 6 (1892) pp. 3416–3418.

But Mr. A. B. Kempe, in his important memoir on "Theory of Mathematical Forms," [Footnote: *Philosophical Transactions* for 1886, pp. 1–70. No logician should fail to study this memoir.] presents an analysis which amounts to a formidable objection to my views. He makes diagrams of spots connected by lines; and it is easy to prove that every possible system of relationship can be so represented, although he does not perceive the evidence of this. . . .

My position has been modified by the study of Mr. Kempe's analysis. For, having a perfect algebra for dual relations, by which, for instance, I could express that "A is at once lover of B and servant of C," I declared that this was inadequate for the expression of plural relations; since to say that A gives B to C is to say more than that A gives something to C, and gives to somebody B, which is given to C by somebody. But Mr. Kempe virtually shows that my algebra is perfectly adequate to expressing that A gives B to C; since I can express each of the following relations:

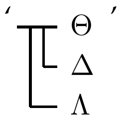
In a certain act, D, something is given by A;
In the act, D, something is given to C;
In the act, D, to somebody is given B.

This is accomplished by adding to the universe of concrete things the abstraction "this action." But I remark that the diagram fails to afford any formal representation of the manner in which this abstract idea is derived from the concrete ideas. Yet it is precisely in such processes that the difficulty of all difficult reasoning lies. We have an illustration of this in the circumstance that I was led into an error about the capability of my own algebra for want of just the idea that process would have supplied. The process consists, psychologically, in catching one of the transient elements of thought upon the wing and converting it into one of the resting places of the mind. The difference between setting down spots in a diagram to represent recognised objects, and making new spots for the creations of logical thought, is huge. To include this last as one of the regular operations of logical algebra is to make an intrinsic transmutation of that algebra. What that mutation was I had already shown before Mr. Kempe's memoir appeared.

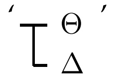
Gottlob Frege

Grundgesetze der Arithmetik I, Jena 1893.

§12 Es mögen in



'—Θ' Oberglied, '—Δ' und '—Λ' Unterglieder heissen. Wir können aber auch



als Oberglied und '—Λ' allein als Unterglied auffassen.

§15. Wenn dieselbe Zeichenverbindung in einem Satze als Oberglied und in einem andern als Unterglied auftritt, so kann man auf einen Satz schliessen, in welchem das Oberglied das zweiten als Oberglied und alle Unterglieder beider ohne das genannte als Unterglieder erscheinen Doch brauchen Unterglieder, die in beiden vorkommen, nur einmal geschrieben zu werden.

'Über die Grundlagen der Geometrie', *Jahresbericht der Deutschen mathematiker-Vereinigung* 15 (1906), quoting from pp. 379–392

Die Einsicht in die logische Natur einer mathematischen Theorie wird oft dadurch erschwert, dass in scheinbar selbständige grammatische Sätze zerissen wird, was sich eigentlich als einheitliches Satzgefüge darstellen sollte. Dies geschieht vielfach aus stilistischen Gründen, um ein Satzungetüm zu vermeiden; aber man darf sich dadurch die Einsicht in das Wesen der Sache nicht verbauen lassen. Man fängt z. B. so an: "Es sei $a \dots$", dem man freilich oft das unrichtige "Es bedeute a" vorzieht. Solche Sätze können mit verschiedenen Buchstaben teils der Ableitung vorhergehen, teils in sie eingeschoben sein. So gelangt man endlich zu einem Ergebnisse, ausgesprochen in einem Satze, der die Buchstaben enthält, die vorher scheinbar erklärt worden sind; denn Sätze wie "Es bedeute $a \dots$" sehen aus wie Erklärungen, die den Buchstaben Bedeutungen verleihen sollen. …

Man lasse sich also dadurch nicht täuschen, dass zuweilen aus stilistischen Gründen ein uneigentlicher Bedingungssatz in einer Form auftritt, in der er, flüchtig betrachtet, als Erklärung eines oder mehrerer Buchstaben erscheint. Aber weder diese scheinbaren Erklärungen, noch der Satz, in dem das Endergebnis ausgesprochen wird, sind eigentliche Sätze, sondern sie gehören als uneigentliche Bedingungssätze und uneigentlicher Folgesatz untrennbar zusammen, so dass erst das aus ihnen bestehende Ganze ein eigentlicher Satz ist. Die Einsicht in den logischen Bau gewänne sehr, wenn das, was sachlich ein einziger eigentlicher Satz ist, sich auch sprachlich als einheitliches Satzgefüge darstellte und nicht in selbständige Sätze zerfiele. Freilich nähme ein solches Satzgefüge in unsern Wortsprachen manchmal eine ungeheuerliche Länge an, während die Begriffsschrift durch ihre Übersichtlichkeit zur Wiedergabe des logischen Gewebes besser befähigt ist. ...

Wir dürfen unsre Pseudoaxiome nicht als selbständige Sätze behandeln, die wahre Gedanken enthalten und so als Grundsteine unsres logischen Aufbaues dienen können, sondern wir müssen sie als uneigentliche Bedingungssätze mitführen. Statt unsres uneigentlichen Satzes A haben wir nun zu schreiben:

Falls allgemein hinsichtlich A und α gilt

> wenn A zu α in der p-Beziehung steht, so ist A ein Π,

und falls allgemein hinsichtlich A und B gilt

> wenn A ein Π ist und wenn B ein Π ist, so gibt es etwas, zu dem sowohl A, als auch B in der q-Beziehung steht,

so gilt allgemein hinsichtlich A, B und α

> wenn sowohl A, als auch B zu α in der p-Beziehung steht, so gibt es etwas, zu dem sowohl A, als auch B in der q-Beziehung steht.

Hierin haben wir einen Satz, der einen Gedanken ausdrückt; aber wir haben auch nur einen einzigen Gedanken darin; die Teile, die sich grammatisch als Sätze darstellen, sind nur uneigentliche Sätze. Die Buchstaben 'Π', 'p', 'q' verleihen dem ganzen Satze Allgemeinheit des Inhalts, während die durch die Buchstaben 'A', 'B', 'α' bewirkte Allgemeinheit sich immer auf einen der drei uneigentlichen Teilsätze bezieht, die eingerückt sind. Hieraus wird sich klar erkennen lassen, wie die uneigentlichen Teilsätze, obwohl vereinzelt sinnlos, doch einen Satz bilden können, der einen Gedanken ausdrückt.

Logik in der Mathematik (unpublished) p. 264.

Nehmen wir an, wir haben einen Satz von der Form: "Wenn A gilt, so gilt B". Nehmen wir nun noch den Satz "A gilt" hinzu, so können wir aus diesen beiden Prämissen schliessen: "B gilt." Aber damit der Schluss möglich sei, müssen beide Prämissen wahr sein. Und deswegen müssen auch die Axiome, wenn sie als Prämissen wahr sein. Und deswegen müssen auch die Axiome, wenn sie als Prämissen dienen sollen, wahr sein. Denn aus etwas Falschem kann man nichts schliessen. Aber, könnte man vielleicht sagen, kann man nicht doch Folgerungen ableiten aus einem Satze, der vielleicht falsch ist, um zu sehen, was sich ergibt, wenn er wahr wäre? Ja, in gewissem Sinne ist es möglich. Aus den Prämissen

Wenn Γ gilt, so gilt Δ
Wenn Δ gilt, so gilt E

kann man schliessen:

Wenn Γ gilt, so gilt E.

Hieraus und aus der weiteren Prämisse

Wenn E gilt, so gilt Z

schliesst man weiter:

Wenn Γ gilt, so gilt Z.

Und so kann man weitere Folgerungen ziehen, ohne zu wissen, ob Γ wahr ist oder falsch. Aber der Unterschied ist zu beachten. In dem vorigen Beispiele fiel die Prämisse "A gilt" ganz aus dem Schlusssatze weg. Hier behalten wir immer die Bedingung "Wenn Γ gilt". Wir können sie als Bedingung nur los werden, wenn wir erkannt haben, dass sie erfüllt ist. In diesem Falle kann man "Γ gilt" gar nicht als Prämisse ansehen, sondern als Prämisse haben wir

Wenn Γ gilt, so gilt Δ,

also etwas, wovon "Γ gilt" nur ein Teil ist. Diese ganze Prämisse muss natürlich wahr sein; aber dies ist möglich, ohne dass die Bedingung erfüllt ist, ohne dass Γ gilt. Genau genommen kann man also gar nicht sagen, dass aus einem falschen oder zweifelhaften Gedanken hier Folgerungen gezogen werden; denn dieser tritt nicht selbständig als Prämisse auf, sondern ist nur ein Teil einer Prämisse, die als solche zwar war sein muss, aber auch wahr sein kann, ohne dass der Teilgedanke wahr ist, der als Bedingung in ihm enthalten ist.

Solche scheinbaren Folgerungen aus etwas Falschem haben wir beim *indirekten Beweise*.

David Hilbert

Grundlagen der Geometrie 1899, opening

Erklärung. Wir denken drei verschiedene Systeme von Dingen: die Dinge des ersten Systems nennen wir *Punkte* und bezeichnen sie mit A, B, $C, \ldots$

Grundzüge der Theoretischen Logik (with W. Ackermann, 1928)

ii Introduction: … Z. B. sucht man vergebens nach einer formalen Darstellung der logischen Beziehung, die in den drei Sätzen:

> "Alle Menschen sind sterblich;
> Cajus ist ein Mensch;
> folglich ist Cajus sterblich."

zum Ausdruck kommt. Der Grund hierfür ist, dass es bei Schlüssen dieser Art nicht nur auf die Aussagen als Ganzes ankommt, sondern dass die innere logische Struktur der Aussagen, die sich sprachlich durch die Beziehung zwischen Subject und Prädikat ausdrückt, eine wesentliche Rolle spielt. Durch diese Erwägungen werden wir dazu veranlasst, den Kalkül oder wenigstens seine inhaltliche Bedeutung zu ändern und den sogenannten *Prädikatenkalkül* einzuführen.

iii.1: Zur Verdeutlichung des hier vorliegenden Sachverhalts möge noch ein weiteres, übrigens nicht der Mathematik angehöriges Beispiel angeführt werden. Es ist gewiss eine logisch selbstverständliche Behauptung: "Wenn es einen Sohn gibt, so gibt es einen Vater", und von einem logischen Kalkül, der uns befriedigt, können wir verlangen, dass er diese Selbstverständlichkeit in Evidenz setzt, in dem Sinne, dass der behauptete Zusammenhang vermittels der symbolischen Darstellung als Folge von einfachen, logischen Prinzipien kenntlich wird. Davon ist aber bei unserem bisherigen Kalkül keine Rede. Wir können hier zwar (unter Anwendung des kombinierten Kalküls) die betrachtete Behauptung symbolisch ausdrücken durch die Formel: $\overline{|X|} \to \overline{|Y|}$, worin X, Y bezüglich die Prädikate "ist ein Sohn",. "ist ein Vater" bedeuten. Doch vermag uns diese Formel gewiss nicht zur Einsicht in die Richtigkeit der Behauptung zu verhelfen, da sie ja bei anderer Einsetzung für X und Y auch falsche Sätze ausdrücken kann. Es kommt in der Formel nicht dasjenige zur Darstellung, worauf der logische Zusammenhang zwischen Vordersatz und Nachsatz beruht, dass nämlich die Prädikate des Sohn-Seins und des Vater-Seins eine Beziehung eines

Gegenstandes zu einem anderen enthalten. Die entsprechende Sachlage findet sich bei fast allen komplizierteren Urteilen.

Stanisław Jaśkowski

'On the rules of suppositions in formal logic', *Studia Logica* 1 (1934) 5–32.

In 1926 Professor J. Lukasiewicz called attention to the fact that mathematicians in their proofs do not appeal to the theses of the theory of deduction, but make use of other methods of reasoning. The chief means employed in their method is that of an arbitrary supposition. The problem raised by Mr Lukasiewicz was to put those methods under the form of structural rules and to analyse their relation to the theory of deduction. The present paper contains the solution of that problem.

Footnote: The first results on that subject obtained by the author in 1926 at Professor Lukasiewicz's seminar were presented at the First Polish Mathematical Congress in Lwów in 1927 and were mentioned in the proceedings of the Congress: *Księga pamiątkowa pierwszego polskiego zjazdu matematycznego*, Kraków, 1929.