

## Tarski through a wide-angle lens

Wilfrid Hodges

<http://wilfridhodges.co.uk/history21.pdf>

June 2012

1. Tarski's knowledge of history of logic.
2. The less known Aristotelian tradition.
3. Traditional semantics versus Tarski's.
4. Summary.

### 1. Tarski's knowledge of history of logic

What logicians < 1900 does Tarski mention  
(in Corcoran collection, 1936 book, 1944 paper)?

*Ancient Greeks:* Aristotle, Philo of Megara,  
Diodorus Cronus, Eubulides, Epimenides.

*17th c:* Pascal, Leibniz.

*19th c:* Bolzano, De Morgan, Boole, Dedekind, Peirce,  
Schröder, Cantor, Frege, Peano, Husserl, Hilbert.

Nothing in between — no Alexander, Abelard, Buridan,  
Ockham, obligationes literature, Kant.  
Also no Theophrastus, Chrysippus, Port-Royal, Peacock.

5

Nearly always, Tarski names these people as sources of specific ideas

(Aristotle for disjointness of sets, Frege for axioms for propositional logic, Husserl for semantic categories).

The list doesn't show much awareness of his own predecessors before late 19th century.

Ascription of the liar paradox to Epimenides is naive: the first person to link Epimenides with the logical 'liar' may have been Bayle in 18th c.



6

In his logical consequence paper 1936, Tarski recalls the 'common concept of consequence'. He says this is 'the old concept of consequence as commonly used by mathematical logicians', i.e. Frege and Peano onwards.

He contrasts it with 'the formalized concept of consequence, as it is generally used by mathematical logicians'; the definition in Hilbert and Ackermann 1928 fits his description.

He calls on 'logical, i.e. *formal*, consequence'. The identification of 'logical' and 'formal' here probably reflects Kant's *allgemeine Logik*, which Kant also called *formale Logik*. Tarski knew Frege's *Grundlagen* of 1882.



7

So far, no evidence of historical interests. But:

(a) 'materially adequate' (1944 Truth paper, translating Polish *trafny*, German *zutreffend*). The condition *adaequatus* on definitions goes back to Abelard in 12th century. It reappeared occasionally in 18th century.

(b) *suppositio materialis*, *suppositio formalis* mentioned in 1944 Truth paper, and added to 1941 edition of his book.

(c) Tarski seems to have been the first to speak of 'Leibniz's law' (of identity). After Tarski used this name in the 1941 edition of his book and in his 1944 paper on Truth, it quickly became standard.



8

(d) 'primitives', i.e. nonlogical constants. Tarski already refers to 'notion primitive' in the 1929 French abstract 'Les fondements de la géométrie des corps'.

The term traces back not to Leibniz (who only has primitive *propositions*), but to Pascal 'De l'esprit géométrique et de l'art de persuader', which was first fully published in 1844. Tarski cites the paper in his 1936 book, and clearly knew its contents.

Ladd-Franklin 1911 speaks of 'primitive concepts (terms)' in what is clearly an exposition of Pascal's paper, though she seems not to know the source.



## 2. The less known Aristotelian tradition

1930s Tarski gives a familiar view of the history of logic:

Aristotle and a few Stoics, then nothing interesting until the 19th century mathematicians, apart from anticipations in Pascal and Leibniz.

This leaves out the Aristotelian tradition established in 2nd and 3rd centuries AD, when logic became a part of general education. E.g. 2nd century AD logic textbooks by Galen, Alexander and Apuleius.

This tradition created very robust background assumptions, largely overturned during Tarski's career.

To understand the history of logic,  
one needs to describe this tradition.  
Not easy, and many historical pitfalls.

**Example One:** The square of opposition, often attributed to Aristotle but in fact first found in Apuleius' textbook.

Clear enough what it says,  
and how important it became in later logic.  
Perhaps a stimulus to thinking about dualities.  
But no direct link to Tarski's work.

**Example Two:** Reduction of some propositional logic to simple predicate logic, by reading  $(p \rightarrow q)$  as

Every situation in which  $p$  is true is a situation in which  $q$  is true.

Developed by Ibn Sīnā (11th c), Wallis (17th c) and Boole (19th c) in slightly different ways.

No common source known.

These are probably independent explications of an idea that the tradition makes semi-obvious through use of tensed sentences.

Rejected by Leibniz and Frege, and ignored by Tarski.

**Example Three:** Logical structure of sciences.  
Two parallel threads:

- (1) Primitive ideas + other ideas derived by definition.
- (2) Primitive assumptions + other propositions derived by logical inference.

Fundamental in both Ibn Sīnā and Pascal, though no common source known beyond Aristotle's *Posterior Analytics* (which doesn't mention the parallel threads).

Fundamental also in Tarski's 1936 book (§32, = §36 of 1994 edition).

First thread:

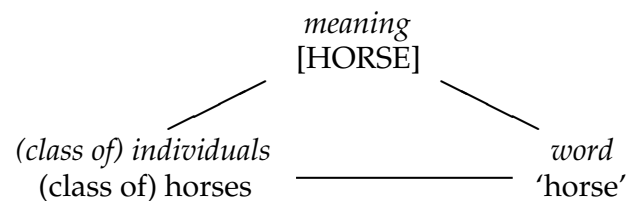
'We distinguish, first of all, a certain small group of expressions ... that seem to us to be immediately understandable; we call [them] PRIMITIVE TERMS ...'

Second thread:

'Some ... statements, whose truth appears to us evident, are chosen for the so-called PRIMITIVE STATEMENTS or AXIOMS ...'

Henceforth we concentrate on traditional semantics.

The **semantic triangle** (origins in Aristotle but the later tradition relies on Porphyry of Tyre, 3rd c AD):



In one presentation, Porphyry leaves out the top node.

### Comment A: Indexicals

The tradition generally takes indexical words as having a meaning that describes some part of the context of utterance. For example Ibn Sīnā:

' "I" signifies [THE PERSON WHO IS SPEAKING].'

Cf. similar remarks in Frege. This way indexical words are not logically different from other words.

A standard example sentence in the Roman period was 'The sun is up'.

### Comment B: Tying words to meanings

The tradition was in tension between two explanations:

(1) Words were tied to meanings by prehistorical contracts which involved 'imposing' the words on certain objects.

Old (cf. *Genesis*), but canonical form probably due to Porphyry.

(2) A word means what the speaker intends it to mean.

Attributed to Diodorus Cronus (c. 300 BC), who named his slave 'Nevertheless'.

Cronus' view reappears in Ibn Sīnā, and in Lewis Carroll:

'When *I* use a word, it means just what I choose it to mean, neither more nor less.'

(Humpty Dumpty in *Through the Looking-Glass*)

and in Hilbert:

'Wir denken drei verschiedene Systeme von Dingen: die Dinge des ersten Systemes nennen wir *Punkte* ...'. (First sentence of *Grundlagen der Geometrie* 1899)

Frege objected, not because we can't give words whatever meanings we want, but because Hilbert's use of the device is 'foreign to mathematics' and not governed by known laws. (*Grundlagen der Geometrie* 1906 p. 426).

### Comment C: Compositionality

Grammar combines words into phrases and sentences.

These compounds of words reflect corresponding compounds of meanings, except perhaps in a few interesting cases where they don't.

Explicit in Al-Fārābī (10th c), Abelard (12th c) and Frege. No common source known.

Corollary: meanings can have other meanings as parts. This idea is important in Frege and early Russell.

Don't confuse with Partee (etc.) compositionality, which we will trace back to Tarski.

In Partee compositionality meanings don't have parts.

## 3. Traditional semantics versus Tarski's

## A. Indexicals

In the early 1930s Tarski took the traditional view that indexicals have no special logical features.

Free use of indexicals in his big Truth paper 1933.

In §1 he uses *śnieg pada* 'it is snowing'. Later in same section he constructs a liar sentence involving *na tej stronie* 'on this page'. In §3 he uses  $x$  *widzy*  $y$  'x sees y'.

In 1935 Tarski read a paper 'The establishment of scientific semantics' at the International Congress of Scientific Philosophy in Paris. The German version of the paper has *es schneit* 'it is snowing' where the Polish has *tlen jest pierwiastkiem* 'oxygen is an element'.

With some help from David Hitchcock I think I can explain this change. Tarski spoke to the Congress in German and left his text there for the Proceedings. On his return to Poland he edited the Polish original for publication, removing the implicit indexical.

Something in Paris persuaded Tarski that the indexical would be a distraction. I can suggest what it was. Reichenbach was at the Congress. Three years later Reichenbach published a book *Experience and Prediction* discussing 'egocentric language'. This influenced Russell's 'egocentric particulars' 1940, and thus launched discussions in the 1950s which convinced people that indexicals have to be treated as a special case.

From 1936 onwards, Tarski soft-pedalled indexicals, though no evidence that his view of them changed.

The 1944 paper on Truth uses 'this paper' in its formulation of the liar.

But it changes Tarski's 1933 example 'it is snowing' to 'snow is white'

(which appeared in Boole in *Laws of Thought* p. 52, but also goes back to *Prior Analytics* i.4 and i.5).

Some people note that in Tarski's model theory a symbol  $R$  can be read as 'the relation labelled  $R$  in the structure', leaving it to the context to determine a structure.

This idea never appears in Tarski himself.

## B. Tying words to meanings

Tarski in the 1930s accepted both a version of the Porphyry 'imposition' theory and a version of Diodorus' 'speaker's intention' theory.

But he sharply distinguished where they apply.

(i) Natural languages and the meanings expressed in them are *potoczny* 'everyday' and *intuicyjny* 'intuitive' — two words that Tarski uses interchangeably. There is no implication that the meanings are pre-theoretical; e.g. he refers to the 'intuitive sense' of the phrase

'a finite sequence of objects satisfies a given sentential function'.

But he does assume that the 'everyday intuitive' meanings are objectively fixed in the usage of the relevant community. Thus 'material adequacy' of a formal definition means being extensionally equivalent to the salient term in its intuitive meaning.

(iii) In the truth definition and its applications, Tarski uses notion of a correlation between individuals and variables 'satisfying' a formula. He systematically avoids defining or explaining this notion in terms of the variables 'meaning' or 'referring to' the individuals.

For example he says that for any  $i$  and  $j$ , a sequence  $(a_0, a_1, \dots)$  satisfies ' $x_i R x_j$ ' if and only if  $a_i \subseteq a_j$ .

Also in the 1930s he avoids any notion of *changing* the meanings that have been given to primitive terms. Thus the rigmarole of *replacing* the primitives by variables, p. 416f in the Corcoran volume.

(ii) Formal languages are constructed by the mathematician or scientist, and their primitive terms are *given* meanings that make them 'seem to us to be immediately understandable'.

Tarski suggests we can give the meanings by regarding the terms as 'abbreviations' of natural language expressions (1936 book §32 = 1994 §36).

This idea of giving a meaning to  $X$  by regarding  $X$  as a notation for some given expression seems to be the best explanation of the puzzling clause in the big Truth paper:

A metalanguage which meets our requirements must contain ... expressions having the same meaning as all the constants of the language to be discussed ... (Corcoran vol. p. 210)

In the model-theoretic readjustments of 1950–1952, Tarski treats *all* non-logical symbols as kinds of variable, so that they are never given any meaning.

Joan Weiner, *Frege Explained* (2004) p. 164f:

'On the contemporary view [a formal] expression has meaning only on an interpretation [in a domain of objects].'

This is the Peacock-Boole view, not Tarski's.

Instead Tarski in model theory defines a set-theoretic relation between sentences and structures, and calls it 'true in', but there is no assignment of meanings at all.

Tarski's avoidance of 'meaning' *in definitions* is methodological. He will not use 'meaning' or 'designation' in the definitions of truth and satisfaction.

But he is happy to contemplate defining a notion of 'meaning' by the method of the truth definition (1944 §13). He cites with approval Carnap's definition of 'synonymous'; Carnap had defined it as 'having the same designation', so by implication Tarski is happy to see 'designation' defined by the same method.

His position is comparable to Porphyry, who postpones the theory of meaning until after logic has been set up.

### C. Compositionality

In 1922 Hilbert proposed 'a strict formalisation of the entire mathematical theory', which would involve a strict mathematical definition of the strings of symbols used as sentences of mathematics. Tarski was one of the first people to give a strict mathematical definition of the grammar of a language, around 1930.

Tarski's formal languages are defined inductively, as sets of strings closed under certain formation rules (though this terminology is later). Their syntax is *autonomous*, i.e. definable from 'the form of the expressions involved' (Tarski 1944).

In one early version of the truth definition (hidden in a paper on definable sets of real numbers, 1931) Tarski defines for each formula  $\phi$  the set  $\mu(\phi)$  (my notation) of sequences which satisfy it in a structure.

The definition of  $\mu$  is by induction on the construction of the language. It is compositional in the Partee sense, viz. the set  $\mu(\phi)$  for compound  $\phi$  is determined by the formation rule giving  $\phi$  and the sets  $\mu(\psi)$  for the immediate constituents  $\psi$  of  $\phi$ .

This is almost certainly the first nontrivial definition of semantic values that is (Partee-)compositional.

At this date, Bloomfield and others still defined syntactic well-formedness in terms of 'meaningfulness'. Tarski himself in 1944 said he was characterising the 'meaningful' expressions (not the grammatical ones).

But by the 1950s Zellig Harris, Noam Chomsky and others were arguing that natural languages have autonomous syntax. Chomsky noted that this raised the possibility of carrying across inductive semantic definitions to natural languages. Katz and Fodor introduced the term 'compositional' in this context.

This makes Tarski in effect the inventor of (Partee) compositionality.



Tarski-Partee compositionality is often confused with 'Frege compositionality', i.e. the traditional Aristotelian compositionality found in Al-Fārābī etc.  
 Unlike the Aristotelians, Tarski never used any notion of compound meanings, or of 'parts' of a meaning.

Once when Partee lectured on compositionality, Tarski told her his truth definition was not compositional.

We need more context to know what he meant.  
 My guess is that he meant his truth definition doesn't talk of meanings or semantic values for sentences; it gives necessary and sufficient conditions for truth.  
 His choice, to reduce the set-theoretic assumptions.

## 4. Summary

During the 1930s, and probably before that if we had the evidence, Tarski was a fairly traditional Aristotelian in his semantic assumptions.

In his formalising work of the 1930s he constantly invoked meanings (under the influence of Leśniewski). But (influenced by Hilbert and a spirit of positivism in the air) he avoided using the notion 'meaning' in definitions. Also he didn't in fact offer any formal definition of 'meaning', though he allowed the possibility.

This position itself had Aristotelian antecedents, e.g. Porphyry, though this is unlikely to have influenced him.

After 1950 Tarski worked mainly in mathematics. Here, as a result of his earlier methodological decisions, all use of the notion of meaning vanished.

During this period, mathematical logic and model theory in particular had a growing influence in linguistics and philosophy of language.  
 One consequence was the widespread loss of the Aristotelian semantic tradition.  
 (One still sees pieces of it popping up again.)

Thus Tarski became a revolutionary in semantics.

I doubt that Tarski himself intended this,  
or that he was always aware of it (e.g. compositionality).

His paper to the 1935 Paris Congress, claiming a  
programme to establish scientific semantics,  
was actually a piece of spin to help sell his 'semantical'  
truth definition.

Carnap reports that before the Congress '[Tarski] thought  
that most philosophers, even those working in modern  
logic, would be not only indifferent, but hostile to the  
explication of his semantical theory'.

In the late 1960s Tarski's ex-student Richard Montague,  
influenced by Reichenbach and Carnap, extended Tarski's  
semantic work while rowing back on some of his views:

- ▶ Distinction between natural and formal languages  
obliterated.
- ▶ Notion of meaning brought to centre stage,  
via intensions.
- ▶ Indexicals given special status.

Tarski kept in touch with Montague through this period,  
up to Montague's death in 1971.

I was told Tarski was strongly supportive of Montague,  
though more in personal than logical terms.