Buridan and the Avicenna-Johnston semantics

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Avicenna, c. 980-1037

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Fakr al-Dīn Rāzī a century and a half later commented on Avicenna's discussions of modal logic.

He accused Avicenna of *kabt*—'stumbling around':

"The logical literature has found itself stumbling around as a result of using the expression 'necessary' sometimes for what is inevitable and sometimes for what is permanent.

This has led logicians to stumble around in the dark, about the parts and contradictory negations of propositions, not least about the 'non-permanent' propositions." (*Mulakkas* 150.2-4)

Two examples picked at random from Avicenna's relatively early *Muktasar* (available only in manuscript):

"Know that 'impossible' means permanently absent, either absolutely or under a condition so that the absence persists for as long as that condition holds."

"... it has been stated that A is true of everything fitting the description B for as long as it fits the description B, and [that] C is permanently a B. So [the thing] will be with necessity an A; and this necessity is proved of it not by the major premise [alone] but from the fact that the argument proves it."

Four modern reactions to this kabt

Reaction One denies its existence. We can tell in each case when Avicenna means 'permanent' and when he means 'necessary'.

My comment: it doesn't work. Nobody who adopts this view has presented any convincing account of the logical content.

Also there are a number of key passages where Avicenna is obviously making a deliberate conflation of 'permanent' and 'necessary'.

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Reaction Two: Each of Avicenna's sentences has two independent modalities, one temporal and one alethic.

My comment: This attributes to Avicenna precisely the same scheme that Rāzī invented to replace Avicenna's one. As often in Avicenna research, ideas of later scholars are being read back into Avicenna.

In any case the modalities are not independent. Avicenna's *kabt* never conflates 'temporary' and 'necessary', or 'permanent' and 'contingent', though he does state that sentences with these mixes occur.

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Reaction Three, alleged to go back to Avicenna's student Bahmanyār: All through, Avicenna is talking about a metaphysically-based kind of necessary permanence.

My comment: When he first introduces the 'permanent' and 'temporary' sentences, notably in *Qiyas* and *Mašriqiyyan*, Avicenna includes no notion of necessity in them at all. Also this reaction overrides Avicenna's many comments that his sentences are about what is 'factual' (bil fi^cl).

Also you should see what Avicenna said privately about the logical aptitudes of Bahmanyār.

Reaction Four: Ahmed (2003) acknowledges that Avicenna deliberately writes sentences that allow 'different manners of construing'. He thinks Avicenna was making the point that different construals give different logical consequences.

My comment: Thank goodness for a commentator who doesn't distort the text. But what Ahmed says about different logical consequences is true for the possible/contingent ambiguity, and *not* for the permanent/necessary ambiguity. In no case does Avicenna give a syllogism where one conclusion follows if we construe as 'permanent' and another conclusion if we construe as 'necessary'.



Antikythera mechanism, plates fused together

The solution is partly known and partly speculative.

Known part: Two-dimensional logic

Avicenna introduces some sentence forms for a kind of temporal logic. I call it 'two-dimensional logic'; Peirce's student Mitchell in the 1880s gave this name to a similar logic.

The sentences quantify both over objects and over times.

'Nothing that is sometimes a B is ever an A during the time during which it's a B'.

'Something that is sometimes a B is an A for all the time that it exists'.

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These sentence forms contain no modal operators (apart from the time quantification) either in themselves or in the explanations that Avicenna gives for them. As Avicenna says, they talk only about what is 'factual' (bil $f_i^c l$).

In the modern jargon, they are *extensional*. Roughly this means that their semantics is purely set-theoretic, so that it's completely objective and uncontroversial what laws they obey.

Two-dimensional logic is a new form of logic intermediate between categorical syllogisms and first-order logic.

The first four books of *Qiyas*, if you disregard references to necessity, form an accurate, scholarly and in places highly original textbook of two-dimensional logic.

For example they include a detailed proof theory for a significant part of two-dimensional logic, based on *Prior Analytics i.5,6* but making adjustments (some very clever) in precisely the places where Aristotle's methods don't transfer.

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system.

Speculative part

At least we know what the book is doing, as a logical text. But why on earth does Avicenna keep glossing 'permanent' as 'necessary', thus creating the *kabt*?

Key fact: The logical laws that Avicenna claims for his temporal sentences with the pair 'permanent/temporary' are exactly the same as the ones that he claims for modal sentences with 'necessary/possible'.

He comments that the difference between the two pairs 'is not a thing that the logician knows about as a logician'.

syllogisms.

In fact this is exactly where Ibn Sīnā concentrates his efforts

when he talks about possibility.
Also his arguments at this point are logically empty and partly based on banter and verbal jingles.

The two-dimensional logic is presented as an axiomatic

So if Avicenna wants to prove that the alethic modal logic

obeys the laws of the two-dimensional logic, he need only

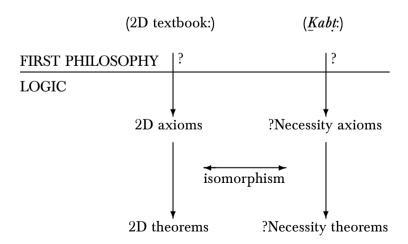
These are some conversions, ectheses and first-figure

prove that modal logic obeys the right *axioms*.

That's because axioms aren't derived logically—otherwise they wouldn't be axioms.

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Remark One: Avicenna regards 'possible' as derivative from 'necessary'.

Remark Two: If I have Avicenna right, he regards 'necessary' as a logical notion, not as a metaphysical one.

There are many references to necessity in his *Metaphysics*, but they are always to *necessary existence*.

His word for 'necessary' here is different from his logical word for 'necessary' (*wajib* instead of *darart*).

Now to Spencer Johnston's PhD thesis.

Spencer proposes a Kripke semantics for the divided modal logic of Jean Buridan.

He shows that the semantics validates all Buridan's claims of validity or invalidity of syllogisms in this logic.

This involves translating the modal syllogisms into set-theoretic statements about Kripke structures, and checking which of these statements are true.

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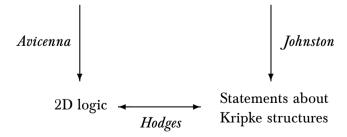
Example: Buridan $A \stackrel{L}{a} B$ 'Necessarily every A is a B'

~ In Kripke structure K, $M^K(A) \subseteq L^K(B)$ and $M^K(A) \neq \emptyset$. 'The somewhere-As are a subset of the everywhere-Bs'.

(We continue:) \sim In a world of objects and times, 'Every sometime-A is a B all the time it exists'. This is Avicenna's two-dimensional 'necessary' proposition. (Actually the Buridan version says 'always exists and is always a B', but in the overlap of Avicenna's and Buridan's logics this makes no difference to the valid syllogisms.)

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 $same \; \textit{(nearly)} \\ \text{Avicenna's Modal Logic} \; \sim \; \text{Buridan's Modal Logic}$



I think that as in an Ellery Queen detective novel, we have now seen enough information to solve the \underline{kabt} problem, especially in the light of Johnston's work.

Over to you, dear Reader.