

Some raw material on mathematical and musical beauty

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Deryck Cooke, *The Language of Music* (1959)

- The purely 'architectural' aspect is found in a limited number of contrapuntal works built out of material which is not emotionally expressive. (p. 33)
- Form ... is simply expression, and ... technique on any scale is simply the means of achieving this expression. (p. 220)
- If music does express spiritual or mystical intuitions, it must do so through the emotional terms of musical language. (p. 272)

Cooke's view is that apart from 'a limited number of contrapuntal works', music achieves its aims through using certain configurations of notes to express emotions.

He sees little role for mathematical patterns in music.

Beauty is in the ear of the listener.

My ear is totally different from Cooke's.

For me, geometry is part of the fabric of music, whether or not it expresses emotions.

This talk will present some examples. (For copyright reasons some scores are missing in this file.)

1. Brahms, 'Ihr habt nun Traurichkeit', Ein Deutsches Requiem, The Schutz Choir of London, The London Classical Players, cond. Roger Norrington, soprano Lynne Dawson

Figure repeated at half speed in lower voice, for narrative reason

Soprano solo

Musical score for Soprano and Tenors. The Soprano part is written on a treble clef staff in common time (C). The Tenors part is written on a treble clef staff in common time (C). The lyrics are: "A - ber ich will euch wie - der se - hen und eu - er Herz soll sich freu - en, und".

Tenors

Ich will — euch — trô — sten, ich

Strings

Musical score for Strings. The score is written on two staves: a treble clef staff and a bass clef staff, both in common time (C). The strings play a melodic line in the treble and a supporting bass line in the bass.

2. Elgar, Violin Concerto, St Louis Symphony Orchestra
cond. Leonard Slatkin, violin Pinchas Zukerman

Motif geometrically transformed in various ways, some
iterated, giving a unity to the work

Score

Elgarvc

Elgar

$\text{♩} = 100$

Violin

6

10

15

Detailed description: The image shows a musical score for a violin part. It consists of four staves of music. The first staff starts with a treble clef, a key signature of two sharps (D major), and a 4/4 time signature. A tempo marking above the first staff indicates a quarter note equals 100 beats per minute. The first staff contains measures 1 through 5. A pink box highlights measures 2 and 3. The second staff starts with measure 6 and contains measures 6 through 9. Two pink boxes highlight measures 7 and 8, and measures 8 and 9. The third staff starts with measure 10 and contains measures 10 through 14. A pink box highlights measures 10 and 11. The fourth staff starts with measure 15 and contains measures 15 through 18. Two pink boxes highlight measures 16 and 17, and measures 17 and 18. The score ends with a double bar line and repeat dots.

3. Messiaen, 'Regard du Fils sur le Fils', Vingt Regards sur l'Enfant-Jésus, John Ogdon

Rhythm repeated in ratio 2:3, almost inaudible and significant chiefly for the composer

4. Byrd, 'Non vos relinquam', Motets, Durham Cathedral
Choir cond. James Lancelot

Motif repeated several times rising in pitch, to represent
ascension; clear to performers if not to listener

Musical staff 1: Treble clef, starting with a whole rest, followed by a half note G4, a quarter note A4, a quarter note B4, a quarter note C5, and a quarter note D5. A slur covers the last four notes. A sharp sign is above the C5 note.

Va - - - - do

Musical staff 2: Treble clef, starting with a whole rest, followed by a half note G4, a quarter note A4, a quarter note B4, a quarter note C5, and a quarter note D5. A slur covers the last four notes. A sharp sign is above the C5 note.

Va - - - - do

Musical staff 3: Bass clef, starting with a half note G3, followed by a quarter note A3, a quarter note B3, a quarter note C4, and a quarter note D4. A sharp sign is above the C4 note.

Va - - - - do

Musical staff 4: Bass clef, starting with a half note G3, followed by a quarter note A3, a quarter note B3, a quarter note C4, and a quarter note D4. A slur covers the first two notes. A sharp sign is above the C4 note.

Va - - - - do

5. Josquin Desprez, 'Huc me sydereo', Orlando Consort

The earliest unambiguous example of pictorial use of the musical time-pitch plane, here to depict a meteor shower

de - scen - - de - re ius - sit O - lym - - po

The image shows a single staff of music in G major (one sharp) and 4/4 time. The melody consists of the following notes: G4 (quarter), A4 (quarter), B4 (quarter), A4-G4 (beamed eighth notes), F#4 (quarter), E4 (quarter), D4 (quarter), C4 (quarter), B3 (quarter), A3 (quarter), G3 (quarter), F#3 (quarter), E3 (quarter), D3 (half). The lyrics are written below the staff, with hyphens indicating syllables across multiple notes. The final note, D3, is a whole note.

6. Schubert, 'Der Tod und das Mädchen', Bryn Terfel, acc.
Malcolm Martineau

Pincer movement in the musical plane

Moderato

und rüh - re mich nicht an, und rüh - re mich nicht

p

Detailed description: This system contains the first two measures of a musical piece. The vocal line is in a single treble clef with a key signature of one flat and a common time signature. The lyrics are 'und rüh - re mich nicht an, und rüh - re mich nicht'. The piano accompaniment consists of three staves: a right-hand treble clef and a left-hand bass clef. The piano part begins with a piano (*p*) dynamic. The first measure shows a piano introduction with chords in the right hand and block chords in the left hand. The second measure continues the accompaniment. The third measure features a vocal entry with a slur over the notes. The fourth measure continues the piano accompaniment with a slur over the notes.

an.

pp *dim*

Detailed description: This system contains the next two measures of the piece. The vocal line continues with the lyric 'an.' in the first measure, followed by a whole rest in the second measure. The piano accompaniment continues with chords in the right hand and notes in the left hand. The second measure features a piano (*pp*) dynamic and a *dim* (diminuendo) marking. The piano part concludes with a final chord in the right hand and a whole rest in the left hand.

7(a). Paganini, 24th Capriccio for Violin, Salvatore Accardo

7(b). Rachmaninov, Variation 18, Rhapsody on a Theme of Paganini, St Louis Symphony Orchestra cond. Leonard Slatkin, piano Abbey Simon

The later and more popular tune is the earlier one with pitches reversed in the musical plane (plus some slight adjustments)

8. Berio, Cries of London, Swingle II

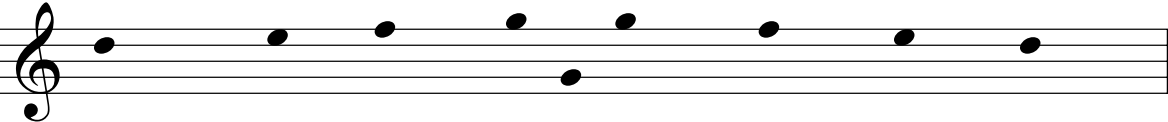
Though rotations in the musical plane are musically meaningless, horizontal transvections occur naturally

9. Mozart, Clarinet Quintet, synthesizer

The opening bars are almost perfectly symmetrical under pitch reversal

10. Handel, Hallelujah Chorus, Messiah, The Scholars
Baroque Ensemble

Motif symmetrical under time reversal, used as symbol of
power as in large symmetrical civic buildings



11. Janacek, 'Veruju', Glagolitic Mass, Royal Philharmonic
cond. Rudolf Kempe

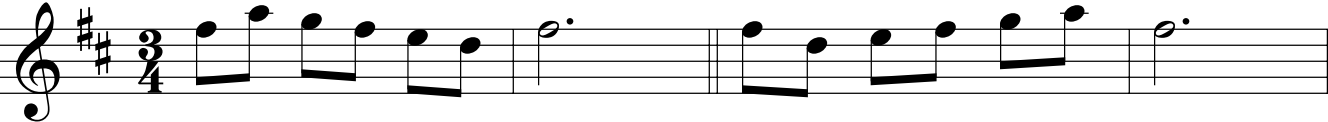
One of the very few examples of a figure symmetrical under
the full Klein 4-group. Why here, and why so rare
elsewhere?

12. Rimsky-Korsakov, Suite from the Golden Cockerel,
Armenian Philharmonic Orchestra cond. Loris Tjeknavorian

For narrative reasons the composer uses the contrast
between a motif with an orientation and its mirror image

DANGER

PEACE



Reference

- Wilfrid Hodges, 'The geometry of music', in *Music and Mathematics: From Pythagoras to Fractals*, ed. John Fauvel et al., Oxford University Press 2003, pp. 91–111.

The proceedings of a conference on music and mathematics, Oxford 1991. The book contains many further references.