Architectural questions about theories of sentence and word meanings

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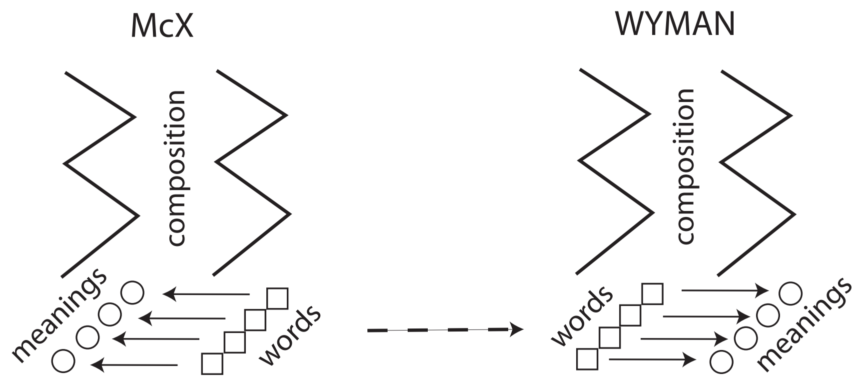
The *aristotelian theory*
probably put together by
Alexander of Aphrodisias (2nd c AD)
and Porphyry (3rd c AD)
to support use of Aristotle as textbook.

Dominated western thinking throughout
Middle Ages and up to Frege.

Van Vliet
A philosopher
and his pupil



Example Two: Gottlob Frege (1914)

> Our ability to understand sentences that we haven't heard before clearly relies on our building the sense (*Sinn*) of a sentence out of pieces that correspond to the words. [Letter to Jourdain]

- Words have occurrences in a sentence. What could an occurrence of a meaning be?
- The words of a sentence are arranged in temporal or spatial order. What dimension are (occurrences of) meanings arranged in?
- How do (occurrences of) meanings attach to each other in the construction? What are their surfaces?
- Etc.

1. McX has a thought T.
2. T is built up from meanings.
3. Each meaning is the meaning of a word,
4. and the arrangement of the meanings in T can be imitated by arranging the words in a sentence S.
5. McX speaks S, and Wyman hears it.
6. Wyman parses S into words, and recovers their meanings.
7. Wyman decodes the construction of S into the way that T is constructed from the meanings.
8. Wyman then has the thought T.

Example One: Al-Fārābī (10th c)

> Expressions signify thoughts . . . When someone hears them, the thing signified by the thought comes to his mind.

> . . . the imitation of the composition of meanings (*ma'ānī*) by the composition of expressions is by [linguistic] convention . . .

Test question (William of Champeaux, early 12th c):

> McX says to Wyman 'The hills are alive'. What happened?

William's answer:

> McX thinks 'The hills are full of activity'. Then he thinks 'The hills are alive' (literally), and conveys this thought to Wyman. Wyman receives the thought, and then thinks 'The hills are full of activity'.

First answer of Abelard (William's pupil):

> McX thinks 'The hills are full of activity'. When reporting this thought to Wyman, McX (improperly) says 'alive' instead of 'full of activity'. Wyman reconstructs 'The hills are alive', and then (improperly) decodes 'alive' as 'full of activity', so as to think 'The hills are full of activity'.

Later Abelard came round to William's view.
How could one tell? What would count as a right answer?

So is the construction of meanings a theoretical notion, to be supported by axioms?

Roger Bacon, *De signis* (mid 13th c)

> There is no difficulty about composite objects, because they have the composition indicated by the form of the noun [phrase], and universal principles (*rationes universales*) make their composition clear from the outset.

Or is it a metaphor?

Frege, *Logic* (unpublished 1897)

> The metaphors that underlie the expressions we use when we speak of grasping a thought . . . put the matter in essentially the right perspective. . . . Of course all metaphors go lame at some point.

A more perceptive view of 'Every':

It does have a meaning m, which takes other meanings and forms a sentence meaning from them.
(In the 13th c this kind of meaning was called an *officium*, 'function'—not in the modern mathematical sense.)

This puts some meat on the idea of composition: meanings compose when one acts on others as a function, 'binding' other meanings together in a chunk or 'constituent'.

Frege's 2nd Grundsatz (1884):

> Nach der Bedeutung der Wörter muss im Satzzusammenhange, nicht in ihrer Vereinzelung gefragt werden.
> One should look for the meanings of the words in the interconnections of the sentence, not in the words one at a time.

I.e. most words have a function-type meaning.

An early twist in the theory:
Some words don't correspond to meanings.
Instead they indicate features of the composition.

Ammonius (c. 500):

> Determiners . . . combine with the subject terms and indicate how the predicate relates to the number of individuals under the subject; . . .
> 'Every man is an animal' signifies that 'animal' holds of all individuals falling under 'man'.

Compare Bertrand Russell *On Denoting* (1905):

> *everything* [is] to be interpreted as follows:
>
> C(everything) means '$C(x)$ is always true'.
>
> . . . *Everything*, *nothing*, and *something* are not assumed to have any meaning in isolation,
> but a meaning is assigned to *every* proposition in which they occur.

BUT in general we can't recover a function f and its arguments $a_1, \ldots, a_n$ from the value $f(a_1, \ldots, a_n)$.

If we can't here, what remains of the idea that McX extracts the meanings from his thought?

If we can, why should that be?

This problem survives into many modern semantic theories.

1929: Tarski defines satisfaction and truth for formal languages.
His definition is recursive, regarding each compound formula as formed from others by a *fundamental operation* such as conjunction or universal quantification.

Thus the syntax of formal languages of logic is autonomous.
Meanings are added on 'homomorphically'
(Helena Rasiowa 1963).

1847 onwards: artificial languages constructed for logic.

1921 onwards: clear separation between syntax and semantics of these languages.
Emil Post's 1921 thesis distinguishes between *semantically equivalent* and *syntactically equal* formulas. (Boole, Frege, Wittgenstein had in effect confused these.)

1926–8: Tarski's Warsaw seminar on analysis of sentences of a formal theory, by recursion on complexity.

1950s: Noam Chomsky, under various influences, declares the *autonomy of syntax* for natural languages too.

1963: Jerrold Katz and Jerry Fodor,
'The structure of a semantic theory',
transfer the Tarski-Rasiowa view of meanings to natural languages, under the name *compositionality*.

Definition. By a *constituent structure* we mean an ordered pair of sets $(\mathbb{E}, \mathbb{F})$, where the elements of $\mathbb{E}$ are called the *expressions* and the elements of $\mathbb{F}$ are called the *frames*, such that the four conditions below hold.

(e, f etc. are expressions. F, $G(\xi)$ etc. are frames.)

1. $\mathbb{F}$ is a set of nonempty partial functions on $\mathbb{E}$.

('Nonempty' means their domains are not empty.)

2. (Nonempty Composition) If $F(\xi_1, \ldots, \xi_n)$ and $G(\eta_1, \ldots, \eta_m)$ are frames, $1 \leqslant i \leqslant n$ and there is an expression

$$F(e_1, \ldots, e_{i-1}, G(f_1, \ldots, f_m), e_{i+1}, \ldots, e_n),$$

then

$$F(\xi_1, \ldots, \xi_{i-1}, G(\eta_1, \ldots, \eta_m), \xi_{i+1}, \ldots, \xi_n)$$

is a frame.

Note: If $H(\xi)$ is $F(G(\xi))$ then the existence of an expression $H(f)$ implies the existence of an expression $G(f)$.

The view emerges that Al-Fārābī etc. had it exactly the wrong way round:

> The composition of meanings imitates the composition of expressions.

But 'composition of meanings' remains an uncashed metaphor,
so the status of this view is obscure.

We will see that 'composition of meanings' is a purely mathematical side-effect of the existence of syntax.

Sentences break down into *constituents*.

We can separate out

(a) a constituent C of a sentence S,

(b) the rest of S when C is removed.

The rest of the sentence is a *frame*,
i.e. an expression with a variable, that becomes a sentence when the variable is replaced by a suitable expression.

This is recursive;
we can also separate out constituents of constituents.

Warlpiri example: ‘child-DUAL-ERG PRES-DUAL dog-ABS chase-NONPAST small-DUAL-ERG’

$$\left[\begin{array}{lll}
\text{PRED} & \text{‘chase}\ \langle f_1, f_2\rangle\text{’} & \\
\text{SUBJ} & f_1: & \left[\begin{array}{ll} \text{PRED} & \text{‘child’} \\ \text{NUM} & \text{DUAL} \\ \text{CASE} & \text{ERG} \\ \text{ADJ} & [\text{PRED}\quad \text{‘small’}] \end{array}\right] \\
\text{OBJ} & f_2: & \left[\begin{array}{ll} \text{PRED} & \text{‘dog’} \\ \text{NUM} & \text{SG} \\ \text{CASE} & \text{ABS} \end{array}\right] \\
\text{TENSE} & \text{NONPAST} & \\
\text{ASPECT} & \text{PRESENT.IMPERFECT} &
\end{array}\right]$$

The lifting lemma

Let X be a set of expressions (for example the sentences) and $\mu : X \to Y$ any function (for example σ).

We will define a relation $\sim_\mu$ so that

$$e \sim_\mu f$$

says that expressions e and f make the same contribution to μ-values of expressions in X.

The fact that $\sim_\mu$ must be an equivalence relation more or less forces us to the following definition.

3. (Nonempty Substitution) If $F(e_1, \ldots, e_n)$ is an expression, $n > 1$ and $1 \leqslant i \leqslant n$, then

$$F(\xi_1, \ldots, \xi_{i-1}, e_i, \xi_{i+1}, \ldots, \xi_n)$$

   is a frame.

4. (Identity) There is a frame $1(\xi)$ such that for each expression e, $1(e) = e$.

We say e is a *constituent* of f if f is $G(e)$ for some frame G.

$F(e_1, f, e_3)$ is *the result of replacing the occurrence of* e_2 *in second place in* $F(e_1, e_2, e_3)$ *by* f.
(This notion depends on F, of course.)

Every bare grammar in the sense of Keenan and Stabler, *Bare Grammar*, CSLI 2003, has a constituent structure in an obvious way.

Assume $F(e)$ is an expression, $H(F(e))$ is in X and $e \sim_\mu f$.

Proof that $F(f)$ is an expression.
By Nonempty Composition $H(F(\xi))$ is a frame $G(\xi)$.
Since $e \sim_\mu f$ and $G(e)$ is in X, $G(f)$ is in X.
But $G(f)$ is $H(F(f))$, so $F(f)$ is an expression.

Proof that $F(e) \sim_\mu F(f)$.
Let $G(\xi)$ be any 1-ary frame such that $G(F(e))$ is an expression in X.
By Nonempty Composition $G(F(\xi))$ is a frame $J(\xi)$.
Since $e \sim_\mu f$ and $J(e)$ is in X,
$J(f)$ is in X and $\mu(J(e)) = \mu(J(f))$.
So $\mu(G(F(e)) = \mu(G(F(f))$ as required. $\square$

Definition We write $e \sim_\mu f$ if for every 1-ary frame $G(\xi)$,

- $G(e)$ is in X if and only $G(f)$ is in X;
- if $G(e)$ is in X then $\mu(G(e)) = \mu(G(f))$.

We say e, f have *the same $\sim_\mu$-value*, or for short *the same fregean value*, if $e \sim_\mu f$.
We write $|e|_\mu$ for this fregean value
(determined only up to $\sim_\mu$).

LEMMA. Suppose $F(e_1, \ldots, e_n)$ is a constituent of some expression in X, and for each i, $e_i \sim_\mu f_i$. Then:

(a) $F(f_1, \ldots, f_n)$ is an expression.

(b) $F(e_1, \ldots, e_n) \sim_\mu F(f_1, \ldots, f_n)$.

For the **proof**, by Nonempty Substitution we can make the replacements one expression at a time.
So it suffices to prove the lemma when $n = 1$.

$$|F(e_1,\dots,e_n)|_\mu = h_F(|e_1|_\mu,\dots,|e_n|_\mu).$$

We call h_F the *Hayyan function* of F.

Abu Ḥayyān al-Andalusī (Egypt, 14th c.)
argued that such functions must exist,
from the fact that we can create and use new sentences.

Using the Hayyan functions,
the values $|e|_\mu$ are definable by recursion on the syntax,
starting from $|f|_\mu$ for the smallest constituents f.

Definition Let ϕ be a function defined on expressions.
A definition of ϕ is called *compositional* if
for each expression $F(e_1,\dots,e_n)$,

$$\phi(F(e_1,\dots,e_n))$$

is determined by F and the values $\phi(e_i)$.

So fregean values are compositional.

We say that X is *cofinal* if every expression is
a constituent of an expression in X.

Basic example:

- L is a language,
- $(\mathbb{E},\mathbb{F})$ is the constituent structure of L,
- X is the set of sentences of L,
- for each sentence e, $\mu(e)$ is the class of contexts in which e is true.

Assume X is cofinal. Then by (b) of the Lemma,
if $e_i \sim_\mu f_i$ for each i then $F(e_1,\dots,e_n) \sim_\mu F(f_1,\dots,f_n)$
provided these expressions exist.
So F and the fregean values of the e_i
determine the fregean value of $F(e_1,\dots,e_n)$.

Hence there is, for each n-ary frame F, an n-ary map
$h_F : V^n \to V$, where V is the class of $\sim_\mu$-values,
such that whenever $F(e_1,\dots,e_n)$ is an expression,

$$|F(e_1,\dots,e_n)|_\mu = h_F(|e_1|_\mu,\dots,|e_n|_\mu).$$

PROPOSITION. The relation $\sim_\mu$ extends the relation $\mu(\xi_1) = \mu(\xi_2)$ if and only if:

For all e, f in X and every frame $F(\eta)$,

$$\begin{aligned} &\mu(e) = \mu(f) \text{ and } F(e) \in X \\ \Rightarrow\ &F(y) \in X \text{ and } \mu(F(e)) = \mu(F(f)). \end{aligned}$$

Proof again immediate from the definition. □

Abstract Tarski theorem

Let L be a language with a well-founded constituent structure, and μ a function whose domain is a cofinal set X of expressions of L.
Then μ has a definition of the following form.
A function ν is defined on all expressions of L by recursion on complexity. The basis clause is

- $\nu(e) = |e|_\mu$ for each atom e.

One can construct counterexamples to the converse implication:
there are compositional semantics that don't yield fregean values,
because they carry up redundant information.

E.g. game-theoretic semantics, where two different games can correspond to the same fregean value.

When is the lifting an extension?

PROPOSITION. Suppose $e \sim_\mu f$ and e is an expression in X. Then f is in X and $\mu(e) = \mu(f)$.

Proof. This is immediate from the definition,
by applying the identity frame $1(\xi)$. □

So on X the relation $\sim_\mu$ is a refinement of the relation $\mu(\xi_1) = \mu(\xi_2)$.

This guarantees there is a function p_μ so that for each e in X,

$$\mu(e) = p_\mu(|e|_\mu).$$

References 2

Relation to historical and contemporary linguistics

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- Marcus Kracht, 'Partial algebras, meaning categories and algebraization', *Theoretical Computer Science* 354 (2006) 131–141.

References 3

Possible application of lifting lemma to Quine's indeterminacy of translation

- Markus Werning, 'Compositionality, context, categories and the indeterminacy of translation', *Erkenntnis* 60 (2004) 145–178.
- Hannes Leitgeb, 'Hodges' theorem does not account for determinacy of translation: a reply to Werning', *Erkenntnis* 62 (2005) 411–425.

The recursion clause is

- $\nu(F(e_1,\ldots,e_n)) = h_F(\nu(e_1),\ldots,\nu(e_n))$
  for each complex expression $F(e_1,\ldots,e_n)$.

Then for each expression e in X,

$$\mu(e) = p_\mu(\nu(e)).$$

References 1

The lifting lemma and related mathematical results

- Wilfrid Hodges, 'Formal features of compositionality', *Journal of Logic, Language and Information* 10 (2001) 7–28.
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