Composition of meaning
A class at Düsseldorf

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1.1

QUESTION ONE
What role has a mathematician in the study of meaning?

1.2

- A **statistician** provides mathematical tools for analysing data.
- An **applied mathematician** gives formal descriptions of (possible) phenomena.
- An **pure mathematician** proves consequences of formal descriptions.

1.3

Statistics are important but not our present concern.

As an applied mathematician I will offer formal descriptions of some features of grammar and meaning.

As a pure mathematician I will prove some theorems, about grammatical structure
and its relationship with the structure of meanings.

For example an extension theorem:
Under certain conditions, a partial semantics can be extended to a total semantics with certain good properties.

1.4

Basic definitions

A *semantics* for a language L is a function μ which assigns to some (possibly all) grammatical expressions e of L an object $\mu(e)$.

We call $\mu(e)$ the μ-*meaning*, or for short just the *meaning*, of the expression e.

We abbreviate 'grammatical expression' to 'expression'.

1.5

We say that expressions e and f are *μ-synonymous*, in symbols $e \equiv_\mu f$, if

$$\mu(e) = \mu(f).$$

Then $\equiv_\mu$ is an equivalence relation.
Equivalence relations $\equiv_\mu$ are called *synonymies*.

1.6

If $\equiv$ is any equivalence relation on a set of grammatical expressions,
we can make $\equiv$ a synonymy $\equiv_\mu$
by defining $\mu(e)$ to be the $\equiv$-equivalence class of e.

Should we make further restrictions
on the possible values of μ?

The great variety of semantics in the literature suggests not.

1.7

Husserl, *Logische Untersuchungen* i 'Ausdruck und Bedeutung' §6 (1900):

> Man pflegt in Beziehung auf jeden Ausdruck . . .
> einen gewissen Belauf von psychischen Erlebnissen,
> die, an den Ausdruck assoziativ geknüpft,
> ihn hierdurch zum Ausdruck von etwas machen.
> Meistens werden diese psychischen Erlebnisse als
> Sinn oder Bedeutung des Ausdruckes bezeichnet . . .

1.8

Robin Cooper, *JoLLI* 12 (2003) p. 369:

> Lots has happened in semantics since [the seventies]. It is now standard to take a dynamic perspective on semantics where meaning is regarded as representing update potential rather than merely truth conditions.

(Note: ‘representing’.)

2.1

QUESTION TWO
Is ‘meaning’ a function from grammatical expressions to their meanings?

2.2

Our definition of ‘semantics’ is not neutral.
It implies an attitude to semantics
which many disagree with.

We shall gather up several relevant issues,
starting with Chomsky’s general warning
that it may be inappropriate to think of meanings
as objects that correspond to expressions.

2.3

Noam Chomsky, ‘Language and Thought: some reflections on venerable themes’, p. 22f of *Powers and Prospects* (1996).

> The study of speech production and analysis
> postulates no [relation between words and molecular motions],
> but rather asks how the person’s mental representations enter into articulation and perception.
> The study of the meaning of expressions should proceed along similar lines, I believe.

2.4

Unconvincing analogy. We *do* study speech production and analysis through phonological matrices assigned to words (e.g. as in Chomsky and Halle 1968).

A more revealing analogy:
we no longer study physical phenomena by associating to each physical event a 'cause'.
Maybe the notion of 'meaning' will likewise be found to be too naive for a satisfactory theory.

Pessimism is premature. Let's wait a century and see.

2.5

Warning from history

The ‘classical’ European view of semantics
(say, middle ages to beginning of 19th century)
assigned meanings of one kind to words,
meanings of another kind to sentences,
but nothing in between.

(There were other traditions, e.g. in Europe the Modists,
and semanticists in the medieval Arab world.)

2.6

Georg Meier, *Versuch einer allgemeinen Auslegungskunst* (1757) §§104, 106:

> Der Sinn der Rede ist also der Inbegriff aller einzelnen Bedeutungen derjenigen Worte, welche die Rede ausmachen . . .
>
> Ein Ausleger [muss] erkennen 1) die Rede samt allen Worten, woraus sie besteht; 2) den Sinn der Rede, folglich a) die Bedeutungen aller einzeln Worte, woraus die Rede besteht, b) den Zusammenhang dieser Bedeutungen untereinander . . .

2.7

In late life Frege placed himself in this classical tradition.
Letter to Jourdain (1914):

> Die Möglichkeit für uns, Sätze zu verstehen, die wir noch nie gehört haben, beruht offenbar darauf, dass wir den Sinn eines Satzes aufbauen aus Teilen, die *den Wörtern* entsprechen.

My italics. Note there is no reference to understanding more complex parts of a sentence.

(Likewise the opening paragraph of *Gedankengefüge* (1923), a paper about compound *sentences*.)

2.8

The move to associate meanings with compound expressions had two sources, both in Germany:

(a) Growing awareness of syntactic constructions, as a result of comparative linguistics (Franz Bopp, Wilhelm von Humboldt etc.);

(b) A strongly mentalistic account of language, according to which syntactic constructions reflect mental constructions.

2.9

Johann Herbart, *Systematische Pädagogik* (1800s)

We form compound concepts by recursively associating simpler concepts, a process called ‘Besinnung’ or ‘Vertiefung’:

> Der Fortschritt einer Vertiefung zur anderen *assoziiert* [sic] die Vorstellungen.

These two streams come together in Lotze, Wundt, Husserl.

2.10

There is a suspicious psychology in the background.
A modern psychologist would note that we understand sentences word by word as we hear them.
So naively one should attach interpretations to *initial segments* of sentences rather than to component expressions.
(This is just as true for formal languages that we read from left to right.)

At the least, any assignment of meanings to expressions should interlock with left-to-right interpretation.

2.11

Objection: The meaning of an expression is not unique

Functions are single-valued.
But on some accounts of meaning, the ‘meaning’ of an expression need not be a single object.

For example a word can have several meanings.

We can respond
(a) by distinguishing words written the same with different meanings,
(b) by allowing complex meanings (e.g. for ‘door’).

2.12

Also (c) by allowing that different people give different meanings to the same word (and hence use different semantics).

This is clearest for proper names.
Speaking of ‘Schultze’, Husserl (*LU* iv §3):

> inhaltlich wechselnde Vorstellungsbestände,
> ohne welche die aktuelle Bedeutung die Richtung
> auf die bedeutete Gegenständlichkeit nicht gewinnen
> [. . .] kann.

Similarly for natural kinds (Putnam).

2.13

I’ve nothing else useful to add about multiplicity of meaning.

Often we will try to avoid problems in this area by concentrating on the relation

e and f have the same meaning.

But of course for a linguist or a psychologist there are further questions about the psychological reality of meanings.

2.14

Objection: Some ungrammatical expressions have meanings.

(Higginbotham, 'On semantics', *Linguistic Inquiry* 1985)

Example. In Icelandic the following is acceptable:

> John wishes Mary would visit himself.

Higginbotham: 'Do children learning Icelandic grasp a rule of interpretation that English-speaking children do not? This seems implausible.'
So this sentence, ungrammatical in English, has the same meaning as if it was grammatical.

2.15

This argument is vulnerable to other examples.
Consider Russian, which has

> U menya tysiacha rublei.
> I have a thousand orubles.

> U menya rublei tysiacha.
> I have roughly a thousand roubles.

Consider a language Subrussian, viz.
Russian without this construction.

2.16

In Subrussian the ungrammatical expression

U menya rublei tysiacha.

has to mean ‘I have a thousand roubles’.

So the Russian child does indeed grasp a rule of interpretation that Subrussian children do not.

2.17

Higginbotham’s view is a modern version of Husserl’s assumption that (*LU* iv §12)

> Fragen wir nach den Gründen, warum in unserer Sprache gewisse Verknüpfungen gestattet sind und andere verwehrt, so werden wir allerdings zu einem sehr erheblichen Teil auf zufällige Sprachgewohnheiten und überhaupt auf Tatsächlichkeiten der bei einer Sprachgenossenschaft so, bei einer andern anders vollzogenen Sprachentwicklung hingewiesen. Zum andern Teil stossen wir aber auf den wesentlichen Unterschied der selbständigen und unselbständigen Bedeutungen, sowie auf die innig damit zussamenhängenden apriorischen Gesetze der Bedeutungsverknüpfung und Bedeutungsverwandlung . . .

2.18

Same objection, but with Arabic data

Meanings attach to strings of three consonants (‘bases’), which never occur as expressions.

E.g. basis *nfḍ* ‘tremble’ gives rise to

- *nafaḍa* ‘he trembled’
- *nāfaḍa* ‘he shook (a thing)’
- *tanaffaḍa* ‘he was shaken’
- *intafaḍa* ‘he shuddered’

Conclusion: Allow a broadened notion of ‘expression’.

3.1

QUESTION THREE
What grammatical structure is available to us when we assign meanings?

3.2

Answer: All of it.

But what does ‘grammatical structure’ consist of?

We shall go as far as we can with two notions:

- the notion of an expression *occurring in* another expression,
- the notion of an occurrence of an expression *being replaced by* another expression.

3.3

Roman Jakobson, *Parts and Wholes in Language* (1960):

> The comparison of incomplete and explicit messages, the fascinating problem of fragmentary propositions, challengingly outlined in Charles Peirce's perusal of "blanks" and in the semiotic studies of Frege and Husserl, strange as it may seem, have found no response among linguists.

(!)

3.4

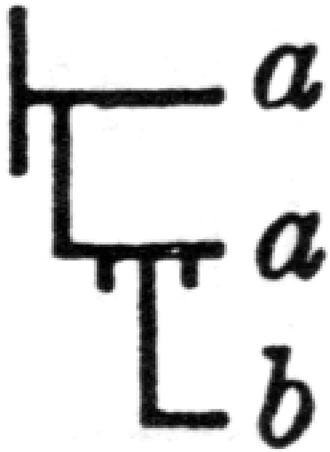
Jakobson is referring to Peirce 'The reader is introduced to relatives' (1892, and in Vol. iii of the Hartshorne-Weiss collection).

> ... if in any written statement we put dashes in place of [demonstratives], the professedly incomplete representation resulting may be termed a ... *rhema*. ... A rhema is somewhat closely analogous to a chemical atom or radicle with unsaturated bonds.

The same paper anticipates the Kenny-Davidson analysis of action statements.

3.5

In fact Frege’s languages, and later languages of logic, have well-defined notions of occurrence and substitution, through phrase markers.



3.6

BUT they don’t allow for overlapping constituents.

John Donne:

the flood did, and fire shall overthrow [them].

So the parsing

(the flood did) overthrow them

must be available as well as the usual parsing

the flood (did overthrow them)

3.7

The part-whole theories of Husserl, Leśniewski etc.
allow overlapping parts,
BUT they say nothing useful about substitution.

So we start afresh.
Later we will relate our notions to more conventional term algebras.

3.8

Basic idea

If expression e occurs in expression f,
then removing this occurrence gives a function F
such that $F(e')$ is f with e replaced by e'.

We define a *constituent structure* for a language L.

A constituent structure is a pair of sets $\mathbb{E}, \mathbb{F}$, called respectively the set of *expressions* and the set of *frames* of L, satisfying the following six axioms.

3.9

AXIOM ONE. Each frame F is a nonempty partial function

$$F : \mathbb{E}^n \to \mathbb{E}$$

for some positive integer n.

($\mathbb{E}^n$ is the set of lists of n elements of $\mathbb{E}$.
A nonempty partial function $F : \mathbb{E}^n \to \mathbb{E}$ assigns elements $F(e_1, \ldots, e_n)$ of E to some (possibly all) such lists $(e_1, \ldots, e_n)$.
So F is a function of n variables.)

3.10

We write an n-ary frame F as $F(\xi_1, \dots, \xi_n)$ with n distinct variables.

Then $F(e_1, \dots, e_n)$ (if defined) is the expression f got by substituting each e_i into the blank ξ_i in F.

We say that each e_i *occurs in* f, in symbols $e \preccurlyeq f$.

(The different occurrences are distinguished by
the variables ξ_i.)

3.11

An occurrence need not be the literal occurrence of one string in another string.

Example: Welsh.

pen	head
y mhen i	my head
ei ben e	his head
ei phen hi	her head

Changes like these, when an expression is joined to others, are called *sandhi* (Sanskrit for ‘junction’).

3.12

What about

ein	a
eine Frau	a woman
eines Kind	a child

I guess Husserl would have allowed these as occurrences of 'ein'.

This illustrates that the notion of one expression occurring in another is not a purely empirical notion.

3.13

> AXIOM TWO. There is a frame $1(\xi)$, such that $1(e) = e$ for every expression e.

If $e \preccurlyeq f$ and e is not f, then we say that e *occurs properly* in f, in symbols $e \prec f$.

So every expression occurs in itself, but not properly. (A technical convenience.)

3.14

AXIOM THREE (Well-foundedness) There are no infinite sequences

$$e_1 \succ e_2 \succ e_3 \succ \ldots .$$

In particular no expression has a proper occurrence in itself.

An expression e is called *atomic* if there is no f such that $f \prec e$.

An expression that is not atomic is called *compound*.

3.15

Husserl, *LU* iv §1:

> Unseren Ausgang nehmen wir von der zunächst selbstverständlichen Einteilung der Bedeutungen in einfache und zusammengesetzte. Sie entspricht der grammatischen Unterscheidung der einfachen und zusammengesetzten Ausdrücke oder Reden. . . . Finden wir nun in einer Teil-Bedeutung abermals Teil-Bedeutungen, so mögen auch in diesen wieder Bedeutungen als Teile auftreten; aber offenbar kann dies nicht *in infinitum* fortgehen.

3.16

Notation

Each frame is a partial function from some $\mathbb{E}^n$ to $\mathbb{E}$. So when we use different variables ξ, ζ, these are just a notational convenience.

Suppose for example that $F(\xi_1, \ldots, \xi_n)$ and $G(\zeta_1, \ldots, \zeta_k)$ are frames. Then we can compose them by substituting G for ξ_1 in F. The standard notation for this is

$$F(G(\zeta_1, \ldots, \zeta_k), \xi_2, \ldots, \xi_n).$$

This is a function of $n + k - 1$ variables, but we don't yet know that it is a frame in $\mathbb{F}$.

3.17

AXIOM FOUR (Substitution) Suppose

$$f = F(e_1, \ldots, e_n) \text{ and } e_1 = G(d_1, \ldots, d_k).$$

Then there is a frame $H(\zeta_1, \ldots, \zeta_k, \xi_2, \ldots, \xi_n)$ such that

$$H(\zeta_1, \ldots, \zeta_k, \xi_2, \ldots, \xi_n) = F(G(\zeta_1, \ldots, \zeta_k), \xi_2, \ldots, \xi_n).$$

And likewise for each of $\xi_2, \ldots, \xi_n$ in place of ξ_1.

(Note the closure property: If $H(d_1, \ldots, e_n)$ is an expression then $G(d_1, \ldots, d_k)$ must also be an expression.)

3.18

PROPOSITION. $\preccurlyeq$ is a well-founded partial ordering.

PROOF. Transitivity: suppose $e \preccurlyeq f$ and $f \preccurlyeq g$. For example suppose $f = F(e)$ and $g = G(f, d)$. Then
$g = G(F(e), d) = H(e, d)$ with H as in Axiom Four, so $e \preccurlyeq g$.

Antisymmetry: If $e \preccurlyeq f$ and $f \preccurlyeq e$ then by Axiom Three we must have $e = f$.

Well-foundedness is Axiom Three. $\square$

3.19

AXIOM FIVE (Instantiation) Suppose

$$f = F(e_1, \ldots, e_n),$$

and $n > 1$. Then there is a frame $J(\xi_2, \ldots, \xi_n)$ such that

$$F(e_1, \xi_2, \ldots, \xi_n) = J(\xi_2, \ldots, \xi_n)$$

and similarly for other choices of variable.

3.20

The next axiom is for natural languages. Some formal languages obey it, some don't.

> AXIOM SIX (Finite length) If e is an expression, then either e is atomic or there is a largest positive integer n such that e can be written as
>
> $$E(f_1, \ldots, f_n).$$

Such a term $E(f_1, \ldots, f_n)$ with n maximal is called a *complete analysis* of e if all the shown expressions f_i are atomic.

3.21

PROPOSITION. Every compound expression has at least one complete analysis.

The PROOF uses Substitution, Well-foundeness and Finite Length. □

3.22

Other axioms suggest themselves but are not needed. For example:

> NONAXIOM (Church-Rosser) Each compound expression has a unique complete analysis.
>
> NONAXIOM (Unique parsing) If $e = E(f_1, \ldots, f_n)$, then $f_1, \ldots, f_n$ are determined by e and E.

4.1

QUESTION FIVE

Does L have a term grammar?

4.2

We shall prove a Normal Form Theorem which shows a close relationship between term algebras and constituent structures.

Until further notice let $(\mathbb{E}, \mathbb{F})$ be a fixed constituent structure.

(Grammatical) expressions e are assumed to be in $\mathbb{E}$, / / and frames $F(\xi)$ in $\mathbb{F}$.
$F(e)$ is well-defined if and only if it is an expression.

4.3

We define an equivalence relation on expressions:

> $e \sim f$ iff in every frame $E(\xi)$, $E(e)$ is well-defined if and only if $E(f)$ is well-defined.

The equivalence classes of $\sim$ are called *categories*.
We write $(e)^{\sim}$ for the $\sim$-equivalence class of e.

4.4

Here we are following Husserl, *LU* iv §10.

1. Since it is determined a priori what combinations of words are meaningful, there must be a priori laws desdribing which combinations are allowed.

> ... die Bedeutungen unter apriorischen Gesetzen stehen, welche ihre Verknüpfung zu neuen Bedeutungen regeln.

4.5

2. Two expressions are defined to have the same category if and only if the a priori laws don't distinguish between them.
(This is a little vague, but I believe our definition of $\sim$ captures it.)

I ignore the question whether these laws are really a priori.

4.6

Husserl seems in §13 to be aware of the following, though I couldn't find it explicitly.

LEMMA. If $e \sim f$ and $E(e)$ is well-defined then $E(e) \sim E(f)$.

PROOF. By definition of $\sim$, $E(f)$ is well-defined too.
We must show that if an expression $G(E(e))$ is well-defined then so is $G(E(f))$.
Using Substitution, put $H(\xi) = G(E(\xi))$.
Then assuming $H(e)$ is well-defined, so is $H(f)$ since $e \sim f$.
But $G(E(f)) = H(f)$, so $G(E(f))$ is well-defined. $\square$

4.7

Likewise Husserl seems to be aware of the following:

LEMMA. Suppose $F(e_1, \dots, e_n)$ is well-defined and for each i, $e_i \sim f_i$. Then $F(f_1, \dots, f_n)$ is well-defined and of the same category as $F(e_1, \dots, e_n)$.

PROOF. Do it in n steps, changing e_1 to f_1, then e_2 to f_2, etc. For the first step, use Instantiation to find the frame

$$H(\xi) = F(\xi, e_2, \dots, e_n).$$

Then $H(e_1)$ is well-defined and so by the previous lemma, $H(f_1)$ is well-defined and $H(f_1) \sim H(e_1)$. $\square$

4.8

Thus if $F(e_1, \dots, e_n)$ is an expression of category δ, and each e_i is of category γ_i, then we can associate to F a law

$$(\gamma_1, \dots, \gamma_n \ \Rightarrow \ \delta)$$

saying that if for each i, f_i is an expression of category γ_i, then $F(f_1, \dots, f_n)$ is well-defined and has category δ.

This presumably is what the Existenzialgesetz for F is supposed to say.

4.9

In particular if F has a law

$$(\delta, \delta \;\Rightarrow\; \delta)$$

then we can build up arbitrarily complicated expressions of category δ:

$$e_1, F(e_1, e_2), F(e_1, F(e_2, e_3)), \ldots$$

4.10

Husserl §13:

> Substituert man nun in den herausgestellten primitiven Formen schrittweise und immer wieder für einen einfachen Terminus eine Verknüpfung von eben diesen Formen, und wendet man dabei allzeit das primitive Existentialgesetz an, so resultieren neue, in beliebiger Komplikation ineinander geschachtelte Formen . . .

4.11

Noam Chomsky, *Syntactic Structures* (1962) p. 14f:

> On what basis do we actually go about separating grammatical sequences from ungrammatical sequences? . . . Any grammar of a language will *project* the finite and somewhat accidental corpus of observed utterances to a set (presumably infinite) of grammatical utterances. In this respect, a grammar mirrors the behavior of the speaker who, on the basis of a finite and accidental experience with language, can produce or understand an indefinite number of new sentences.

4.12

The passage just quoted from Husserl §13, together with his earlier discussion of Unsinn, makes Chomsky's point explicitly and describes probably the simplest method for generating nontrivial infinite languages by finite means.

Nothing remotely comparable appears in the passages from von Humboldt, Darwin or Frege which are usually quoted as anticipations of Chomsky.

(But Husserl would have disagreed that the speaker learns the grammar from 'accidental experience'.
He regarded the basic grammar as given a priori.)

4.13

Husserl limits himself to 'primitive' frames.
We can pick these out as follows.

A frame $E(\xi_1, \dots, \xi_n)$ is called a *construction* if it is not the frame 1 but it can't be decomposed into other frames by substitution or instantiation,
e.g. it can't be written as

$$F(\xi_1, \dots, G(\xi_i, \xi_{i+1}, \dots), \dots, \xi_k, e_{k+1}, \dots, e_n)$$

for any frames F and G both distinct from 1.

4.14

LEMMA. If $E(e_1, \ldots, e_n)$ is a complete analysis of a compound expression e, then e can be written as $F(e_1, \ldots, e_n)$ where F is a compound of constructions.

PROOF. Since we can't introduce further variables to $E(\xi_1, \ldots, \xi_n)$, any counterexample would have to involve an infinite sequence

$$G(\xi) = H_1(H_2(\xi)), H_2(\xi) = J_1(J_2(\xi)), \ldots .$$

So taking an appropriate expression e, we would have

$$G(e) \succ H_2(e) \succ J_2(e) \succ \dots$$

contradicting well-foundedness. $\square$

Note that a construction (or any frame) can have more than one law associated with it.
For example the frame 1 has all the laws $(\gamma \Rightarrow \gamma)$, where γ is any category.

This is why we couldn't directly decompose E in Lemma 4.14 into a compound of constructions.

4.16

We could if we wanted split each frame $\neq 1$ into the union of other frames, each with a single law.
I see no issue of principle here.

For example one might be happy to have a single frame $E(\xi_1, \xi_2)$ that joins two expressions by putting 'and' between them, provided both expressions come from the same (appropriate) category.

4.17

NORMAL FORM THEOREM. Let $(\mathbb{E}, \mathbb{F})$ be a constituent structure. Then there are a many-sorted term algebra $\mathbb{A}$ and a surjective function $m : \mathbb{A} \to \mathbb{E}$ such that

(a) $m(s)$ and $m(t)$ have the same category if and only if s and t have the same sort.

(b) If for each i, s_i and t_i are terms of $\mathbb{A}$ with $m(s_i) = m(t_i)$, and $\mathbb{A}$ contains $r(s_1, \ldots, s_n)$, then it also contains a term $r(t_1, \ldots, t_n)$, and $m(r(s_1, \ldots, s_n)) = m(r(t_1, \ldots, t_n))$.

(c) $e \prec f$ if and only if there is a term t with a subterm s such that $m(s) = e$ and $m(t) = f$.

4.18

We sketch the construction.

The sorts are the categories.

For each construction E and each associated law $(\gamma_1, \ldots, \gamma_n \;\Rightarrow\; \delta)$ we introduce an n-ary function symbol which takes arguments of sorts γ_1, ... , γ_n and delivers a value of sort δ.

For each atomic expression e, say of category γ, we introduce a constant term of sort γ.

4.19

There is a unique term algebra $\mathbb{A}$ built on the data above.

We define the map m from $\mathbb{A}$ to $\mathbb{E}$,
by induction on the complexity of terms.

Each constant symbol of $\mathbb{A}$ comes from an atomic expression e; put $m(c) = e$.

4.20

If the term t of $\mathbb{A}$ is not a constant, then by the construction of term algebras there are a unique function symbol r and unique terms $s_1, \ldots, s_n$ such that $t = r(s_1, \ldots, s_n)$.
By induction hypothesis we can suppose the expressions $m(s_i)$ are defined.
The function symbol r comes from some construction E.
We define $m(t) = E(m(s_1), \ldots, m(s_n))$.

4.21

The map m is surjective. Suppose for example that e is an expression. If e is atomic then $e = m(c)$ where c is the corresponding constant.

If e is not atomic, then by Proposition 3.21, e has a complete analysis $E(e_1, \ldots, e_n)$. Hence by Lemma 4.14, e can be written as $F(f_1, \ldots, f_n)$ where F is a compound of constructions. This description of e corresponds to a term t with $m(t) = e$.

The other properties are left to the reader. $\square$

5.1

QUESTION FIVE

Must a semantics be a refinement of the function $(\ )^{\sim}$?

5.2

This mathematical jargon means the following.
Given a semantics μ defined on $\mathbb{E}$, is it true that
$\mu(e) = \mu(f)$ implies $e \sim f$?

When the answer is Yes,
we say that the semantics μ is *husserlian*.

5.3

For mathematicians, 'husserlian' means we can define a function f from the set of meanings to the set of categories, so that both routes round this diagram get you to the same place:

$$
\begin{tikzcd}
\mathbb{E} \arrow[r, "(\ )^{\sim}"] \arrow[dr, "\mu"'] & \text{(Meanings)} \arrow[d, "f"] \\
& \mathbb{E}/\sim
\end{tikzcd}
$$

5.4

For naive ‘meanings’ in English the answer is No:

> in spite of
> notwithstanding

are synonyms, but one is a preposition and the other a postposition:

> I loved her in spite of her irritating laugh.
> I loved her, her irritating laugh notwithstanding.

(Though this difference could be absorbed into the sandhi.)

5.5

However, there are deep arguments in the other direction, beginning with one from Frege, *Grundlagen der Arithmetik* (1884).

Frege is investigating (§18):

> dem allgemeinen Begriffe der Anzahl. . . . Dabei werden voraussichtlich auch die Eins und die Vermehrung um eins erörtert werden müssen und somit auch die Definitionen der einzelnen Zahlen eine Ergänzung zu erwarten haben.

5.6

Frege §46:

> Um Licht in die Sache zu bringen, wird es gut sein, die Zahl im Zusammenhange eines Urtheils zu betrachten, wo ihre ursprüngliche Anwendungsweise hervortritt.
> To throw light on the matter it will be helpful to consider how number connects to the rest of a judgement which illustrates its basic use.

5.7

I translate cautiously.
'Zusammenhang' has two English translations:

(1) surroundings or context,

(2) connection to surroundings.

English commentators on Frege, following Austin (see table 5.8 below), commonly choose (1) when the surroundings are a sentence; so 'im Zusammenhange eines Urtheils' would mean 'occurring in a judgement'.

ix	Zusammenhang mit der Logik	connexion
ix	Zusammenhänge zwischen Sachen	connexions
x	Satzzusammenhange	context of a proposition
x	Zusammenhange mit einander	connexion with each other
40	aus dem Zusammenhange zu ergänzen	supplied from the context
42	einen gewissen Zusammenhang	some sort of connexion
42	der innere Zusammenhang	its connexions
43	innern Zusammenhang	internal cohesion
59	im Zusammenhange eines Urtheils	in the context of a judgement
71	Zusammenhang mit dem Gedachten	connexion
72	Zusammenhange eines Satzes	context of a proposition
73	Zusammenhange eines Satzes	context of a proposition
99	Zusammenhang zwischen	connexions

5.9

One of Frege's sample sentences (§46):

> Der Wagen des Kaisers wird von vier Pferden gezogen.

The relevant form is

$$\phi(four X)$$

which paraphrases to

> There are four X such that ϕ(they).

5.10

In short, ‘four’ in its basic use requires an argument which expresses a property, as X in the sentence

> There are four Xs.

Likewise ‘number’ requires a property argument, as in

> The number of X is Y.

5.11

Frege's conclusion:
'number' is a function word.
We should define a function word f by stating a schematic sentence of the form

$$Y \text{ is the } f \text{ of } u.$$

where Y is an expression containing the symbol 'u'.

5.12

Frege, *Grundgesetze I* (1893) §33:

> Wenn ein Name einer Function erster Stufe mit einem Argumente definirt wird, müssen die Argumentstellen auf der linken Seite der Definitionsgleichung mit einem lateinischen Gegenstandsbuchstaben ausgefüllt werde, der auch rechts die Argumentstelle des neuen Functionsnamens kenntlich macht.

5.13

The definition of 'number' in the Grundgesetze §40 follows this recipe:

> (roughly) The number of u is the class of all classes equipollent to the class u.

(Tarski in 1953 gave a closely similar normal form for definitions of functions.)

5.14

In the Grundlagen, after establishing that ‘number’ requires a property argument, Frege moves to other questions about the concept of number.

He returns in §60 to his point about arguments.
If we ignore how a word fits into sentences,
we are reduced to thinking of the meaning of the word
as a Vorstellung.

5.15

> So scheint ein Wort keinen Inhalt zu haben, für welches uns ein entsprechendes inneres Bild fehlt. Man muss aber immer einen vollständigen Satz ins Auge fassen.
> Nur in ihm haben die Wörter eigentlich eine Bedeutung. . . . Es genügt, wenn der Satz als Ganzes einen Sinn hat; dadurch erhalten auch seine Theile ihren Inhalt.

When a word has arguments, its relation to these arguments is an essential ingredient of its meaning.

5.16

So Frege asserts that the meaning of a word depends essentially on its argument structure.

He seems also to assume that this argument structure is the same in all sentences containing the word.
(Chomsky's Projection Principle asserts this for thematic arguments.)

Since the argument structure is reflected in the category, this suggests that words of different categories should have different meanings.

5.17

To measure the distance between the opposing arguments, consider James Pustejovsky, *The Generative Lexicon* (1995) p. 135:

> Mary likes watching movies.
> Mary likes movies.
> Mary likes John to watch movies with her.
> Mary likes that John watches movies with her.
>
> Mary enjoys watching movies.
> Mary enjoys movies.

5.18

Pustejovsky argues that there is a difference of meaning: 'enjoy' 'selects an event function'.
Unclear whether this is more than a restatement of the syntactic difference.

There is another kind of argument that 'enjoy' and 'like' have different meanings.
Namely, if we say 'enjoy' where 'like' is expected,
we don't get a sentence which automatically carries the meaning that the sentence with 'like' would have had.

5.19

Suppose someone said:

> Mary enjoys that John watches movies with her.

We could interpret this in any of three ways:

> Mary likes that John watches movies with her.
> Mary enjoys having John watch movies with her.
> Mary enjoys thinking about the fact that John watches movies with her.

(This is similar to the argument against Higginbotham above.)

5.20

Pustejovsky's example illustrates a common phenomenon: e means the same as f, but replacing one by the other can lead from grammatical to ungrammatical.

This is not the phenomenon of hyponymy,
as between 'spoon' and 'teaspoon'.
They have the same category,
but replacing one of them by the other can lead from truth to falsehood.

5.21

Another common phenomenon:
The psychologist Cyril Burt described a boy who knew the meaning of ‘vertical’ and ‘horizontal’, but only for pencils. He was unable to say whether objects of other shapes were horizontal or vertical.

Clearly this boy had only partial knowledge of the meaning of ‘horizontal’.

But for other words like ‘acid’ most of us are in a similar position.

5.22

Husserl §5 introduces a distinction between selbständige and unselbständige Bedeutungen.

The distinction is obscure, particularly when he maintains (§6) that a Bedeutung can be 'teils selbständige und teils unselbständige',
or (§8) that an unselbständige Bedeutung can become the object of a selbständige Bedeutung.

5.23

But broadly an unselbständige Bedeutung is one that requires an argument. Thus (§6)

> grösser als ein Haus (§6),
> rot (§8)

both require an object argument, while

> oder (§9)

needs two sentence or property arguments.

5.24

Assuming this is right, then by Husserl's general theory, two words with different argument structures have different Bedeutungskategorien and hence different Bedeutungen.

Hence the name *husserlian*.

Husserl might say that two expressions can have the same Bedeutung and hence the same argument structure, but express the argument structure by different syntax.

5.25

Tentative conclusions

1. If two words have different categories, this is prima facie evidence that they have different meanings. But where it is clear that the only difference between them is how the syntax expresses the argument structure, one normally reckons their meanings are the same. One moral of Pustejovsky's examples, and many others in the recent literature, is that this is a hard distinction to draw.

5.26

2. Frege's advice, to distinguish the meanings of words by distinguishing the grammaticality and truth conditions of sentences containing them, is now a standard method.

6.1

QUESTION SIX

How far do the grammaticality and truth of sentences determine the meanings of other expressions?

6.2

As warned, we restrict to the question how far grammaticality and truth of sentences determine whether expressions have the same or different meanings.

Grammaticality we have already considered.
Now we ask what can be done with grammaticality and truth together.

6.3

Some expressions, like ‘Please may I’, don’t occur in sentences which can be true or false.

More generally, all performative sides of meaning are lost in this approach.

Most formal languages—at least those with a well-defined semantics—contain no imperatives.

6.4

This should puzzle people concerned about the formalisation of mathematics.
Informal mathematics is full of imperatives, often to do impossible things:

- Suppose now that V has finite dimension, and let n be its dimension. (*Serre*—we aren't told what V is.)
- Write the sequences of the values of the successive functions one below another, as the rows of an infinite matrix. (*Kleene*)

6.5

Notation

Let μ be a partial semantics, i.e. a function defined on some subset of the set $\mathbb{E}$ of expressions. We write

$$\mu(E(e)) \simeq \mu(F(f))$$

('$\mu(E(e))$ is strictly equivalent to $\mu(F(f))$') means that

- $\mu(E(e))$ is well-defined if and only if $\mu(F(f))$ is well-defined, and
- if they are well-defined, then they are equal.

6.6

Suppose given a partial semantics μ on $\mathbb{E}$.
There is a canonical way of defining a synonymy $\equiv$ on all of $\mathbb{E}$ so that $e \equiv f$ if and only if e and f make exactly the same contribution to the μ-meanings of expressions containing them.

Namely, $e \equiv f$ if and only if for every frame $E(\xi)$,

$$\mu(E(e)) \simeq \mu(E(f)).$$

For reasons below, we call $\equiv$ the *fregean cover* of μ.

6.7

The definition of $\equiv$ unpacks as follows:

(a) If $e \equiv f$ then for all $E(\xi)$, $\mu(E(e))$ is well-defined if and only if $\mu(E(f))$ is well-defined.

(b) If $e \equiv f$ and $\mu(E(e))$, $\mu(E(f))$ are both well-defined then they are equal.

(c) If $e \not\equiv f$ then there is $E(\xi)$ such that either exactly one of $\mu(E(e))$, $\mu(E(f))$ is well-defined, or both are well-defined and they are different.

6.8

Consider

(a) If $e \equiv f$ then $\mu(E(e))$ is well-defined if and only if $\mu(E(f))$ is well-defined.

Write $e \sim_X f$ if for all frames $E(\xi)$,
$E(e)$ is in X if and only if $E(f)$ is in X.
Then $\sim_X$ is an equivalence relation. Also $\sim_{\mathbb{E}}$ is just $\sim$, and (a) above is $\sim_{\mathrm{dom}\mu}$; for short we write it $\sim_\mu$.

We said $\equiv$ is husserlian if $e \equiv f$ implies $e \sim f$.
So we read (a) above as: $\equiv$ is *μ-husserlian*.

6.9

We compare $\sim$ with $\sim_X$ where X is a set of expressions. We say that X is *cofinal* (in $\mathbb{E}$) if for every expression e there is some expression f in X with $e \preccurlyeq f$.

LEMMA. If X is cofinal and $e \sim_X f$ then $e \sim f$.

PROOF. Suppose $e \sim_X f$ and let $E(\xi)$ be any frame such that $E(e)$ is well-defined.
By cofinality there is a frame $G(\zeta)$ such that $G(E(e))$ is in X.
Use Substitution to define $H(\xi) = G(E(\xi))$.
Then $H(e)$ is in X, so $H(f)$ is in X since $e \sim_X f$.
It follows that $H(f)$ is well-defined, and hence so is $E(f)$. $\square$

6.10

LEMMA. Assume X is cofinal. If $e \sim_X f$ then for all frames $E(\xi)$, if $E(e)$ is well-defined then $E(e) \sim_X E(f)$.

PROOF. Assume $e \sim_X f$ and suppose $G(E(e))$ is in X. Define $H(\xi) = G(E(\xi))$. Then $H(e)$ is in X, and so $H(f)$ is in X since $e \sim_X f$, so $G(E(f))$ is in X. This proves $E(e) \sim_X E(f)$ provided we know that $E(f)$ is an expression; that would follow if we knew there was such a G. But there is, by the cofinality of X. $\square$

6.11

Consider:

(b) If $e \equiv f$ and $\mu(E(e))$, $\mu(E(f))$ are both well-defined then they are equal.

This says (to put it more loosely) that the μ-meaning of $E(e)$ depends only on the $\equiv$-class of e.
Or, the contribution that e makes to the meaning of $E(e)$, given that $E(e)$ has a meaning, depends only on the meaning of e.

This is a weak form of *compositionality*.
We read it as '$\equiv$ is partially 1-compositional over μ'.

6.12

Consider:

(c) If $p \not\equiv q$ then there is $E(\xi)$ such that either exactly one of $\mu(E(e))$, $\mu(E(f))$ is well-defined, or both are well-defined and they are different.

Because of an analogous notion in computer science, we read this as '$\equiv$ is *fully abstract* over μ'.
This is a 'no garbage' condition: it says that $\equiv$ makes no distinctions that aren't justified by the meanings of expressions in X.

6.13

Properties of the Fregean cover

PROPOSITION. $\equiv$ is an equivalence relation.

PROOF. Reflexive and symmetric are immediate from the definition.

Suppose now that $e \equiv f \equiv g$ and $\mu(E(e))$ is well-defined. Then by (a) and (b), $\mu(E(f))$ is well-defined and equal to $\mu(E(e))$. So by (a) and (b) again, $\mu(E(g))$ is well-defined and equal to $\mu(E(f))$, hence to $\mu(E(e))$.
This proves transitivity. $\square$

6.14

PROPOSITION. If e and f are in X, and $e \equiv f$, then $\mu(e) = \mu(f)$.

PROOF. Recall $e = 1(e)$ and $f = 1(f)$.
Since $1(e)$ and $1(f)$ are both in X and $e \equiv f$, we infer

$$\mu(e) = \mu(1(e)) = \mu(1(f)) = \mu(f)$$

as claimed. $\square$

Later we shall ask about the converse to this proposition.

6.15

PROPOSITION. Assume the domain of μ is cofinal in $\mathbb{E}$. Then $\equiv$ is husserlian (i.e. $e \equiv f$ implies $e \sim f$).

PROOF. Suppose $e \equiv f$ and let $E(\xi)$ be any frame such that $E(e)$ is well-defined. By the cofinality, there is a frame $F(\xi)$ such that $F(E(e))$ is in the domain of μ.
By Substitution there is a frame $G(\xi)$ such that $G(\xi) = F(E(\xi))$. Then $\mu(G(e))$ is well-defined, so $\mu(G(f))$ is well-defined, so $G(f)$ is well-defined. Thus $F(E(f))$ is well-defined, so $E(f)$ is well-defined. $\square$

6.16

PROPOSITION. (Compositionality) Assume the domain of μ is cofinal in $\mathbb{E}$. Suppose $E(\xi_1, \dots, \xi_n)$ is a frame and $E(e_1, \dots, e_n)$ is well-defined, where $e_1 \equiv f_1$, ..., $e_n \equiv f_n$. Then $E(f_1, \dots, f_n)$ is well-defined and $E(e_1, \dots, e_n) \equiv E(f_1, \dots, f_n)$.

PROOF. Making one substitution at a time, we prove $E(e_1, \dots, e_n) \equiv E(f_1, \dots, e_n) \equiv \dots \equiv E(f_1, \dots, f_n)$.

6.17

Suppose $E(e_1, \ldots, e_n)$ is well-defined. Using Instantiation, let $G(\xi_1)$ be the frame $E(\xi_1, e_2, \ldots, e_n)$.
Then $G(f_1)$ is well-defined by the husserlian property.

Now to prove that $E(e_1, \ldots, e_n) \equiv E(f_1, e_2, \ldots, e_n)$, again the task is to show $G(e_1) \equiv G(e_2)$.
Let $H(\xi)$ now be any frame such that $\mu(H(G(e_1)))$ is well-defined, and let $J(\xi)$ be the frame $H(G(\xi_1))$. Then

$$\mu(H(G(e_1))) = \mu(J(e_1)) = \mu(J(f_1)) = \mu(E(f_1, e_2, \ldots, e_n)$$

and hence $E(e_1, \ldots, e_n) \equiv E(f_1, e_2, \ldots, e_n)$ as required. $\square$

6.18

Definition of compositionality

Let μ be a semantics for a language with constituent structure $(\mathbb{E}, \mathbb{F})$.

We say that μ is *compositional* if the μ-meaning of any compound expression $E(e_1, \ldots, e_n)$ is determined by E and the meanings $\mu(e_1)$, . . . , $\mu(e_n)$.

6.19

Equivalently, for every pair of compound expressions $E(e_1, \ldots, e_n)$ and $E(f_1, \ldots, f_n)$, if $\mu(e_i) = \mu(f_i)$ for each i then

$$\mu(E(e_1, \ldots, e_n)) = \mu(E(f_1, \ldots, f_n)).$$

Note that this definition can be rewritten using the synonymy $\equiv_\mu$ rather than μ.
So it makes sense (as in 6.16) to call a synonymy compositional.

6.20

PROPOSITION. In the definition of compositionality, it suffices to consider frames E that are constructions.

The PROOF uses an induction on the distance from being a construction.
The chief case is where, say, $E(\xi_1, \xi_2, \xi_3) = F(G(\xi_1, \xi_2), \xi_3)$ where compositionality for F and G has been proved.
If $E(e_1, e_2, e_3)$ and $E(f_1, f_2, f_3)$ are both well-defined then $G(e_1, e_2)$ and $G(f_1, f_2)$ are well-defined.
Use induction hypothesis to show $G(e_1, e_2) \equiv_\mu G(f_1, f_2)$, then substitute in F. $\square$

6.21

Warning

In the definition of compositionality,
it is not enough to consider frames with one variable.
This is because $E(e_1, e_2)$ and $E(f_1, f_2)$ can be well-defined
but $E(e_1, f_2)$ and $E(f_1, e_2)$ not well-defined.
A German example:

> Sie haben; Ihr habt;
> NOT Ihr haben; Sie habt.

For a husserlian semantics, compositionality for one variable implies compositionality in general.

6.22

EXISTENCE THEOREM. Let X be a cofinal set of expressions in a language with constituent structure $(\mathbb{E}, \mathbb{F})$, and μ a semantics defined on X.
Then there is a husserlian, μ-husserlian, compositional and fully abstract (over μ) synonymy $\equiv$ defined on all expressions, with the property that
for all e, f in X, $e \equiv f$ implies $\mu(e) \neq \mu(f)$;
$\equiv$ is unique with these properties.

6.23

PROOF. We have shown that $\equiv$ has these properties.
Suppose $\equiv'$ has them too, and $e \not\equiv f$.
Then by full abstraction there is $E(\xi)$ such that
either (1) just one of $E(e)$, $E(f)$ is in X,
or (2) both are and $\mu(E(e)) \neq \mu(E(f))$, so $E(e) \not\equiv' E(f)$.
If (1) and $\equiv'$ is μ-husserlian then $e \not\equiv' f$.
If (2) and $\equiv'$ is compositional then $e \not\equiv' f$. $\square$

Almost trivial mathematically, but one often sees it denied.

6.24

COROLLARY (Zadrozny, unpublished). Let μ be any semantics defined on all expressions of a language L. Then there is a compositional semantics ν for L such that

(a) If $\nu(e) = \nu(f)$ then $\mu(e) = \mu(f)$;

(b) if $\nu(e) \neq \nu(f)$ then there is a frame $E(\xi)$ such that $\mu(E(e)) \neq \mu(E(f))$.

PROOF. Apply Theorem 6.22 to μ, and define ν so that $\equiv$ is $\equiv_\nu$. $\square$

7.1

QUESTION SEVEN
What does the compositionality of the fregean cover tell us?

7.2

We can construct a fregean cover whenever we have a semantics μ for all expressions in a class X of expressions.

Typical cases:

- (the default case:) X is the set of sentences, $\mu(e)$ describes when e is true.
- X is the set of sentences and common nouns, μ is as above on sentences and $\mu(e)$ for a common noun e is a description of what things would count as instances of e.

7.3

Philip Larkin, *High Windows*:

> I know this is paradise
>
> Everyone old has dreamed of all their lives

This is a garden-path sentence, split between two stanzas. To make it grammatical we have to understand

> I know *that this is the paradise which* everyone old has dreamed of through all their lives.

7.4

Here at least some of the syntax of the first line *must be* available to us when we interpret the second.
In fact all of the syntax *is* available,
making it hard to tell exactly what is needed.

A fregean cover for initial segments would pick up exactly the features of the syntax that must be available,
and incorporate them into the meaning.

7.5

I haven’t worked out a fregean cover for initial segments.
It would have to assign to each initial segment a range of possible meanings corresponding to different possible syntaxes, to be sieved as the sentence proceeds.
(Cf. Kempson, Meyer-Viol, Gabbay, *Dynamic Syntax* 2001.)

Apart from this important complication,
I doubt there would be any new issues of principle.

7.6

So the fregean cover assigns to an expression e

> whatever is needed from e to interpret sentences containing e.

This ‘whatever’ might contain things you prefer to regard as syntax.

The key point is that the quoted phrase doesn’t in general name any one object (recall the example of proper names). Our construction of fregean covers entitles us to assume that there is one such object—this is a kind of unique existence proof.

7.7

Some expressions carry an obvious candidate for meaning, e.g.

- for nouns e, the mental image of 'an e';
- for sentences, the circumstances in which they are true.

In both these cases (but on different occasions) Frege warns us that the obvious candidate may not determine how the expression contributes to larger expressions containing it.

In other words, if μ is the 'obvious candidate' semantics and $\equiv$ its fregean cover, we can have $\mu(e) = \mu(f)$ but not $e \equiv f$. (Contrast Proposition 6.14.)

7.8

Example: Janssen's numerals.

An arabic number $n_1 \dots n_k$ is parsed as $n_1 \ (n_2 \dots n_k)$.
We can write this construction as $E(\xi_1, \xi_2)$ so that

$$n_1 \dots n_k \text{ is } E(n_1, n_2 \dots n_k).$$

We allow a number to start with a string of 0's.
So there are two categories: the arabic numbers of length 1 (they can go in the first slot of E), and the others.

7.9

The obvious semantics is $\mu(e) = |e|$, the numerical value of e.

We have

$$|n_1 \ldots n_k| = 10^{(k-1)}.|n_1| + |n_2 \ldots n_k|.$$

So the information needed from n_1 is $|n_1|$,
and the information needed from $n_2 \ldots n_k$ consists of
$|n_2 \ldots n_k|$ together with k.

7.10

Thus in all cases we can take the fregean cover semantics ν to be

$$\nu(e) = (|e|, \text{length of } e).$$

Now 01 and 001 have the same value but different lengths, so

$$\mu(01) = \mu(001), \quad \nu(01) \neq \nu(001).$$

7.11

This English example from Barbara Partee (in Landman and Veltman 1984) is similar:

> Anyone can solve that problem.
> If anyone can solve that problem, I suppose John can.

If μ reports truth values,
then its fregean cover has to do more than that.

Note Husserl *LU* iv §13: 'zu bemerken ist, dass auch volle Sätze zu Gliedern in andern Sätzen werden können'.

7.12

Frege, *Grundlagen* §62:

> Nur in Zusammenhange eines Satzes bedeuten die Wörter etwas.

Taken out of context (!), this could be read as the bizarre statement that words don’t have dictionary meanings. But in context it clearly recalls Frege’s §60, and in particular

> Es genügt, wenn der Satz als Ganzes einen Sinn hat; dadurch erhalten auch seine Theile ihren Inhalt.

7.13

Partee's example illustrates this, though with a sentence rather than a word.
You get the meaning of the sentence wrong if you fail to consider how it contributes to larger sentences.

Frege's point is obviously sound
(and is the reason for the name *fregean cover*).

The matter of historical interest is not whether he ceased to believe it, but how he chose to express it in his more mature years.
The statement in Grundlagen §62 is quite unnecessarily gnomic (Dummett's word).

7.14

But Frege's point is often ignored.
E.g. a recent preprint asserts that

> autobiography
> history of the life of its own author

are synonymous, and then promptly refutes this by considering

> John's autobiography
> John's history of the life of its own author

Fortunately computational lexicography is making it harder to operate with this inadequate notion of synonymy.

7.15

We say that the fregean cover $\equiv$ *extends* μ if for all expressions e, f in the domain of μ,

$$e \equiv f \quad \text{if and only if} \quad \mu(e) = \mu(f).$$

When and only when this condition holds,
we can choose ν so that $\equiv$ is $\equiv_\nu$ and ν is an extension of μ.

7.16

EXTENSION THEOREM. For $\equiv$ to extend μ, it is necessary and sufficient that:

1. If $\mu(e)$ and $\mu(f)$ are well-defined and equal, then $e \sim_\mu f$.
2. If $\mu(e)$ and $\mu(f)$ are well-defined and equal, and $E(\xi)$ is any frame such that $E(e)$ and $E(f)$ are in the domain of μ, then $\mu(E(e)) = \mu(E(f))$.

PROOF. Immediate from the definition of $\equiv$. $\square$

7.17

In Janssen's numerals, both 1 and 2 fail:

$$\mu(0) = \mu(00), 0 \not\sim_\mu 00,$$
$$\mu(1) = \mu(01), 11 \neq 101.$$

7.18

In Partee's case, 2 fails:

> If anyone can solve that problem, I suppose John can.
> If everyone can solve that problem, I suppose John can.
> If nobody can fail to solve that problem, I suppose John can.

Note how the hypothesis makes part of its syntax available to the conclusion.

7.19

What happens to the fregean cover if we extend the language?

Two examples:

1. Suppose we removed from English all ways of forming negative or conditional sentences.
Then the following sentences would be $\equiv$ to each other:

   (a) Anyone can solve that problem.
   (b) Everyone can solve that problem.

Here, extending the language causes one meaning to split into two.

2. ‘likely’ and ‘probable’.

As of August 2003, these words have different categories:

(a) Sandan is likely to miss the Gophers game.
(b) ⋆Sandan is probable to miss the Gophers game.

I took (b) off a sports website.
If ‘likely’ and ‘probable’ come to have the same category, this will clinch that they have the same meaning.
(At the moment I think it is debatable.)
Here, adding new grammatical sentences causes two meanings to become one.

8.1

QUESTION EIGHT
How can we make the fregean cover informative?

8.2

Mathematically the problem is, given a fregean cover $\equiv$ of a partial semantics μ, to find a semantics ν so that $\equiv_\nu$ is $\equiv$.

There obviously can't be one right way to do it, since we may be after different kinds of information:

- How can the language user understand the sentence?
- What mathematical properties does the language have?
- How to check efficiently whether a sentence is true?

Etc.

It will be convenient to write $|e|$ for $\nu(e)$.

8.3

Worked example

Syllogistic consists of all sentences of the forms

$$\left\{\begin{array}{c}\text{All}\\\text{Some}\\\text{No}\end{array}\right\} x \left\{\begin{array}{c}\text{are}\\\text{are not}\end{array}\right\} y.$$

where x and y are plural noun phrases.

8.4

There are two main parsings;
in our setting we can accept them both.

1. The standard linguistic parsing:

   [[All x $_{NP}$] [[are (not) $_V$] y $_{VP}$] $_S$].

2. The standard logical parsing:

   Frames 'All ξ_1 are(n't) ξ_2'.

   Terms x, y.

8.5

Categories:

- τ Terms.
- α Quantifiers ‘All’ etc.
- β Quantifier phrases ‘Some x’ etc.
- κ Copulas ‘are’, ‘are not’.
- ϕ Verb phrases ‘are (not) y’.
- σ Sentences.

8.6

Constructions:

1. F, $(\alpha, \tau \; \Rightarrow \; \beta)$
2. G, $(\kappa, \tau \; \Rightarrow \; \phi)$
3. H, $(\beta, \phi \; \Rightarrow \; \sigma)$
4. Six frames with category sequence $(\tau, \tau \; \Rightarrow \; \sigma)$

8.7

For our partial semantics μ we use the truth values T or F of sentences in some real or imagined world, in which the collection of all objects forms a set Ω and the terms give a reasonable spread of subsets of Ω.

Since sentences can't occur in other sentences,
the fregean cover ν extends μ by Theorem 7.16.

8.8

PROPOSITION. If x and y are terms, then $x \equiv y$ if and only if the class of x is the same as the class of y.

PROOF. Suppose the classes are different;
then either there is an object which is among the x but not among the y, or vice versa.
In the first case

> 'All x are x' is true.
> 'All x are y' is false.

So $x \not\equiv y$; similarly the other case.
Converse: left to the reader. $\square$

8.9

There is an obvious set of objects in one-one correspondence with the classes of the terms x,
namely those classes themselves.
So we write $|x|$ for the class of x.

On the logician's analysis the rest is easy.
Write $\chi(p)$ for the characteristic function with value T when p is true and F when p is false.

8.10

If $E(\xi_1, \xi_2)$ is the frame 'Some ξ_1 are not ξ_2', then we associate to E the characteristic function

$$\chi(|\xi_1| \cap (\Omega \setminus |\xi_2| \neq \emptyset).$$

Likewise for the other five logician's frames.

Then

$$E(e, f) = \chi(|e|, |f|).$$

Note that we have assigned to E a function which we can think of as its meaning, viz. the function taking pairs $(|e|, |f|)$ to $\mu(E(e, f))$.

8.11

We turn to the linguists' parsing, which was not intended for semantic purposes and hence may give us more work.

We could proceed as before: establish ν-values for all the quantifier phrases and verb phrases, and then define a function as a value for the functor $H(\xi_1, \xi_2)$ combining these phrases.

8.12

But we have another option.

Namely if we can regard the quantifier phrase as an argument of the verb phrase, we can start by assigning ν-values to the quantifier phrases, and then assign to the verb phrases f the functions θ_f such that for all quantifier phrases e,

$$|H(e, f)| = \theta_f(|e|);$$

or the same the other way round.

8.13

Are there principled ways of choosing between these possibilities?

Everybody has their own pet theory here.
But I think in this case there is a good argument for choosing one particular way.

First we need to describe the equivalence classes of $\equiv$ on quantifier phrases.

8.14

PROPOSITION. The ν-values of quantifier phrases can be assigned as follows:

1. $|\text{All } x| = (all, |x|)$ when $|x|$ is not empty;
2. $|\text{Some } x| = (some, |x|)$ when $|x|$ is not empty;
3. $|\text{No } x| = (no, |x|)$ when $|x|$ is not empty;
4. $|\text{Some } x| = neut1$ when $|x|$ is empty.
5. $|Qx| = neut2$ when $|x|$ is empty and Q is 'All' or 'No'.

(Use the truth values given by the simplest rules in 8.10.)

8.15

PROPOSITION. The ν-values of verb phrases can be assigned as follows:

1. $(\phi, |y|)$ when the phrase is 'are y';
2. $(\phi, \Omega \setminus |y|)$ when the phrase is 'are not y'.

The ϕ is to ensure the husserlian property;
$|y|$ must be different from $|\text{are } y|$.
We write $(a, b)_1 = a$, $(a, b)_2 = b$.

8.16

Heim and Kratzer *Semantics in Generative Grammar* (1998) p. 140, quoting Geach quoting 'a reputable textbook':

> "If there are no dragons, the phrases 'all dragons' and 'no dragons' both refer to one and the same class—a null or empty class. Therefore 'All dragons are blue' and 'No dragons are blue' say the same thing about the same class; so if one is true, the other is true. But if there are no dragons to be blue, 'No dragons are blue' is true; therefore, 'All dragons are blue' is also true." I know the argument sounds like bosh; but don't you be fooled—it *is* bosh.

Exercise (H&K): 'What is wrong with the logician's argument?'

8.17

First answer: The logician has correctly described the fregean cover.
He seems to be justifying the intuitive truth values from the fregean cover, which is odd since the truth values are data and the fregean cover is derived from them.
If his purpose is to confirm that the fregean cover does lead to the right truth values, then it seems harmless.
I don't know the book, but I don't think one should take on trust Geach's account of its intentions.

8.18

Since I have no idea whether Heim and Kratzer would accept this answer, here is another one.

Second answer: Syllogistic is a tiny fragment of English. We should assign semantic values that work in general. Extending the language, we find the pair of sentences

> There are no dragons in Kraków.
> There are all dragons in Kraków.

The second sentence makes no sense, and hence 'no dragons' $\neq$ 'all dragons'.

8.19

In case that was no good either, here is a third attempt.

Third answer: A value function θ for the frame H has to be as follows:

$$\theta(e, f) = \begin{cases} \chi(|e|_2 \subseteq |f|) & \text{when } |e|_1 = all; \\ \vdots & \vdots \\ \mathrm{F} & \text{when } |e| = neut1; \\ \mathrm{T} & \text{when } |e| = neut2. \end{cases}$$

8.20

Unlike the function in 8.10, this function is calculated in different ways according to whether $|x|$ is empty or not. This is psychologically unreal, since one can understand the sentence without knowing whether $|x|$ is empty.

The same problem arises if we take the quantifier phrase as argument of the verb phrase.

The moral is that we should go for the third option and make the verb phrase the argument of the quantifier phrase. The value of a quantifier phrase will then be a function, so the values used by the quoted logician are not available.

8.21

To clinch this, here is the definition of the function θ_e for a quantifier phrase e:

$$\theta_e(f) = \begin{cases} \chi(|e|_2 \subseteq |f|) & \text{when } |e|_1 = all; \\ \chi(|e|_2 \cap |f| \neq \emptyset) & \text{when } |e|_1 = some; \\ \chi(|e|_2 \cap |f| = \emptyset) & \text{when } |e|_1 = no. \end{cases}$$

We remark finally that there is no problem about taking the second term as argument of the copula.

9.1

QUESTION EIGHT

How does the fregean cover respond to variables or indices?

9.1

Our working example is first-order logic.

We assume given a signature, i.e. a set of symbols of the following forms:

- individual constant symbols;
- relation symbols, each with fixed arity ≥ 1.

Variables are the symbols

$$v_0, v_1, v_2, \ldots$$

The *terms* are the variables and individual constants.

9.3

The *atomic formulas*:

- If s and t are terms then $s = t$ is an atomic formula;
- if R is a relation symbol of arity n, and $t_1, \ldots, t_n$ are terms, then $R(t_1, \ldots, t_n)$ is an atomic formula.

9.4

The *formulas*, or for Tarski *sentential functions*:

- Every atomic formula is a formula;
- if ϕ is a formula then $\neg\phi$ is a formula;
- if ϕ, ψ are formulas then $(\phi \wedge \psi)$ is a formula;
- if ϕ is a formula and v a variable, then $\forall v\phi$ and $\exists v\phi$ are formulas.

9.5

In the last clause, $\forall v$ and $\exists v$ bind the shown occurrences of the variable v,
and any occurrences of v in ϕ that are not already bound by quantifiers further in.

A formula in which all variables are bound is called a *sentence*.

9.6

We have some choice of constituent structure. I propose the following constructions:

1. For each n, $F_n(R, t_1, \ldots, t_n)$ is $R(t_1, \ldots, t_n)$.
2. $G(t_1, t_2)$ is $t_1 = t_2$.
3. $H(e)$ is $\neg e$.
4. $I(e, f)$ is $e \wedge f$.
5. $J(v, f)$ is $\forall v f$.
6. $K(v, f)$ is $\exists v f$.

9.7

The categories are as follows:

- υ Variables.
- κ Individual constants.
- ρ_n Relation symbols of arity n.
- ϕ Formulas.

9.8

One interprets the constant symbols, the relation symbols and the quantifiers in a structure A of appropriate signature.
The *sentences* then have truth values, T or F.
The *logical equivalence class* of a sentence ϕ is the class of structures A in which ϕ is true (the *models* of ϕ).
One normally takes this class as the 'meaning' of ϕ.
But for simplicity we shall consider a single structure A;
so $\mu(\phi)$ is the truth value of ϕ in A.

9.9

The conditions of the Extension Theorem 7.16 are satisfied, since all combinations of sentences are by truth functions. Hence the fregean cover of μ is an extension of μ. The fregean cover is known as the *Tarski truth definition* (for this first-order language interpreted in A).

Immediate problem: The class S of sentences is not a category. So $\sim_S$ will be a refinement of S.

9.10

PROPOSITION. The $\sim_S$ categories are the same as the $\sim$ categories, except that the category ϕ of formulas splits into infinitely many families:

ϕ_W Formulas in which the free variables are those in W

for each finite set W of variables.

PROOF. If x is a free variable of e and not of f, consider the sentence

$$\forall x_1 \ldots \forall x_n\ f$$

where $x_1, \ldots, x_n$ are the free variables of f. $\square$

9.11

The fregean cover assigns to each formula f of category ϕ_W the pair (W, X) where X is the set of functions a from W to the domain of A which satisfy f (i.e. intuitively make f true when each free variable v is read as a name of $v(a)$).

Normally one mentions only X, which determines W. But in the one case where f is logically false, X is empty and W is undetermined;
so omitting W would violate the husserlian condition in this one extreme case.

9.12

Tarski obscured this point, and with it the fact that his semantics gives a fregean cover.

He was aware that formulas with different free variables have different $\sim_S$-categories (*Concept of Truth* §4):

> ... the functors of two primitive sentential functions belong to the same category if and only if the number of arguments in the two functions is the same, and if any two arguments which occupy corresponding places in the two functions also belong to the same category.

But he argued (for a reason not clear to me) that there are different notions of satisfaction, with no common definition, according to which variables are involved. (*CT* §4)

> In order to avoid this ambiguity . . . we had recourse to an artifice which is used by logicians and mathematicians in similar situations. Instead of using infinitely many concepts of satisfaction . . . we tried to operate with the semantically uniform, if somewhat artificial, concept of the satisfaction of a function by a sequence of objects.

9.14

Thus he always assumed that assignments are to *all* variables whether they occur free in the formula or not.

Montague (Tarski's student) followed suit; *English as a Formal Language* §4:

> Thus a model should assign to a basic expression not a denotation but a *denotation function*, that is, a function that maps each infinite sequence of individuals onto a possible denotation of the expression.

Roughly, variables correspond to pronouns.

9.15

The protests against this procedure for handling pronounsinclude Pauline Jacobson, 'Towards a variable-free semantics', *Linguistics and Philosophy* 22 (1999). There are two issues:

(a) Do we need numbered variables, bearing in mind that pronouns don't carry numbers?

(b) Do we need assignments to variables/pronouns that don't occur unbound?

The answer to (a) has to be No; but without the numbering we need another way of linking the variables.

9.16

Jacobson wishes to maintain (in our notation):

> Consider any expression $E(e)$ where E contains no pronouns unbound within C, and e is a noun phrase. Then if f is a pronoun which is unbound in $E(f)$, then $E(f)$ is also grammatical.

She wishes to reject the conclusion (which looks to me an offshoot of Tarski's unfortunate convention) that $E(e)$ and $E(f)$ must have the same 'semantic category'.

9.17

Natural languages contain other things not catered for in standard Tarski semantics.

Consider, from mathematical English (but the students have no trouble understanding it):

For every pair of numbers x and y there is a number z depending only on x, such that $\ldots$.

On Tarski's analysis this is true if and only if for all numbers m and n there is a number p such that

- p depends only on m and
- $\ldots$

9.18

The notion that 12 depends on 13 (say) is incomprehensible.

Nevertheless we do have a fregean extension of truth for first order logic enriched with this notion.
So far nothing has been done about carrying this over to natural languages.