Models in science and technology

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I. Some history

The history of a word isn't always helpful
for understanding its use today.

But for the word *model* it is.

Classical Latin: *modulus* means measuring device.

Late Latin allowed the form *modellus*, hence Italian *modello*,
hence English *model*.

Meanwhile Old French shortened *modulus* to *modle*,
which the English found easier to pronounce as *mould*.

Renaissance scholars introduced the form *module*
in deference to the Latin original.
(Latin was still widely used.)

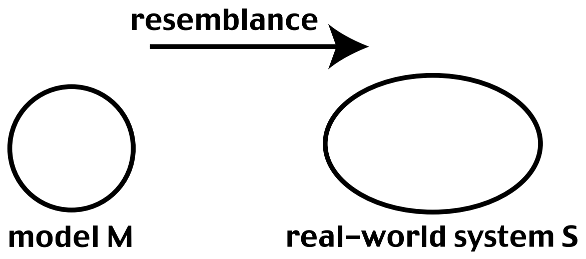
Till the late 17th century,
model and *module* were interchangeable.
Mould may have separated earlier.
Boumans (1999) speaks of *mathematical moulding*
as a part of mathematical modelling.

Other words have appeared for similar concepts
(e.g. *exemplar*, *Bild*).

Around 1950, logic, statistics, economics, physics, philosophy of science all embraced the word *model*.

A mould for moulding cheese, or for printing a letter, has a shape.

This shape is the same as that of the moulded cheese or the printed letter.



Already we have the basics:

- A mould or model M and a subject (or system or situation) S which is described by M;
- the mould M is 'accessible' in a way that S is not.

Question One. What part of the model carries the data?

Answer: (for cheese mould) The internal surface of the mould;
(for printing type) The raised surface of the letter.

Question Two. How is the data translated from M to S?

Answer: (for both) Physical contact.

Question Three. How is M more accessible than S?

Answer: We already have M and we have to make S.

An early generalisation is the painter's or builder's model.

The *data carrier* is the visible surface of the model.

The *translation* is by the eye of the artist
or the measuring rule of the builder.

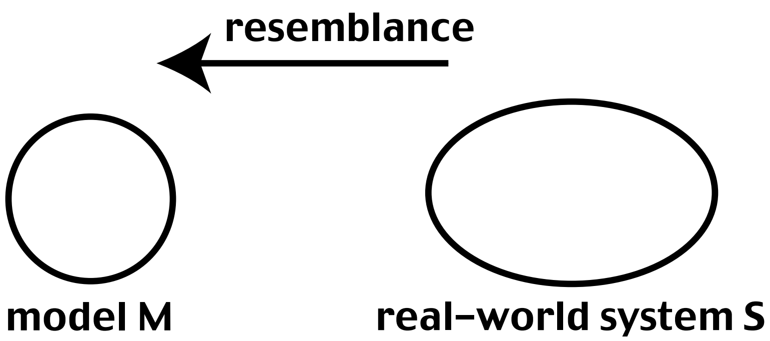
The *accessibility* of the model is
(for the artist) that nature provides Naomi Campbell and Claudia Schiffer;
(for the builder) that the model is smaller and easily adjusted.

Shakespeare *Much ado about nothing* (1613) I.3:

> *Borachio* I can give you intelligence of an intended marriage.
> *Don John* Will it serve for any model to build mischief on?

Note both the literal sense (model of a building) and the metaphor (modelling one's behaviour on something).

Another early generalisation is where S already exists
and we construct M as an aid to understanding S.



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Francis Bacon *Novum Organum* (1620) i.124:

> We are establishing a true copy (*exemplar*) of the world in the human mind . . . This can be done only by dissecting and diligently anatomising the world. We altogether condemn those unfit models (*modulos ineptos*) and caricatures of 'worlds' that the human imaginations of philosophers construct.

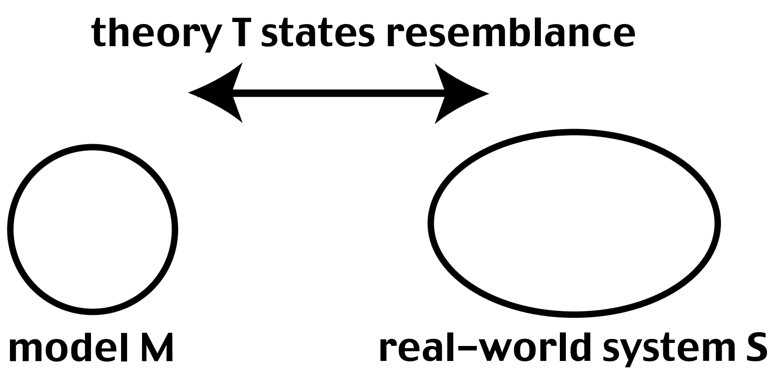
II. The variety of models

We give a preliminary classification of models in terms of the way the information carries over from them to the system S.

It's helpful to classify models broadly into three kinds:

- Theory-like models.
- Picture-like models (ranging from graphs to 3-dimensional working models).
- (Intangible mathematical) structure-like models.

Scientists and engineers have used all three kinds in various combinations.



For a theory-like model, the translation from model M to system S is that the model is *true of* the system.

For a picture-like model, the translation is that the system is *similar to* the model.
Pictures can rely on conventions.
The more they do, the more they are like theories.
Graphs of functions are very like theories.

For a structure-like model there is no direct translation.
We shall have to explain this below.

The question of direction
(model-to-system or system-to-model)
doesn't always have a clear answer.

Example: To guide forestry,
we model the behaviour of the forest.
Then in the model we change some parameters,
to see what will happen
if we impose the same changes on the forest.

III. Engineering specifications

These are specifications of systems to be built,
so the direction is mainly from model to system.

The customer and the engineer draw up a contract that
specifies the system S to be built.

Writing the specification and turning it into the system
is often a complex process involving several different
descriptions at different levels.

The first step (often called *modelling*) is to establish
exactly what the customer wants.
Today this is sometimes done with UML
(Unified Modelling Language),
a language designed for this purpose.

UML uses both text and pictures, so a UML description
is a mixture of theory-like and picture-like.

Next, the engineer adjusts the specification
from a description of the customer's needs
to a detailed description of the final system.

This can be done in stages, sometimes called *refinements*.

These descriptions are normally theory-like,
and often written in a suitable formal language.

The descriptions form a linear sequence
from 'abstract' to 'concrete'.

Each description in turn is *verified* to check that
the system which it describes will deliver
what was required by the previous description,
i.e. that the description *implements* the previous description.

The process of implementing is done partly by *interpreting*
formal expressions in the earlier description.

It's helpful to think of an interpretation as a lookup table
supplying meanings for uninterpreted symbols:

l	the distance from A to B

At a later stage we might make the further interpretation:

distance	distance in centimetres
A	the top of the pendulum
B	the centre of gravity of the bob of the pendulum

The final implementation is to turn the last description into a working system.

Sometimes the engineer will turn an earlier description into a ‘toy’ working system, called a *prototype*.
The prototype is a model of the final system.
It’s picture-like: it shows how the final system will operate by working *like* the final system (but maybe with features missing).

Software engineers sometimes use a type of specification called an *abstract machine*.
(Example: the abstract state machines of Gurevich and Börger, now supported by Microsoft.)

Abstract machines are abstract mathematical structures.
Since they are invisible and intangible,
they need to be described precisely in a language.
This description is what translates across to the final system;
i.e. the abstract model works *through* a theory-like model.
This is the typical situation with abstract models.

In what sense is the abstract model ‘accessible’?

From an abstract state machine website: ASMs are good because “one needs to be able to see the correspondence between specification and reality directly, by inspection”.

We can’t literally inspect something abstract.
But this is an appeal to what the Germans call *Anschauung*,
our mental ability to see relationships in our mind’s eye.

The ASM description is accessible because
we can *visualise* what it describes.

IV. Scientific models

These are much more complicated and varied than engineering models,
partly because of the range of systems covered.

Traditionally, a set of sentences true of S is called a *theory of* S, or an *axiom system for* S. Label it T.

We now understand that this is naive.
A scientific theory applying to S is generally not just a set of true statements about S.

There are several kinds of gap between theory and system.
Ways of filling these gaps are often called *models*;
they are usually theory-like.

(a) Unlike the engineering situation,
the starting theory is generally not about the particular system we are interested in.
Instead it is about the world in general.

So we need a lower-level theory that fits closer to the particular system.

As with engineering refinements,
we may find ourselves using a sequence of progressively more specific descriptions.

I think the sequences tend to be longer in physics,
where the most abstract theories are extremely abstract,
and shorter in economics where the general theories are already about fairly concrete systems.

This may be why we freely talk of Keynes' *model*,
but not of Newton's *model*.

(b) The system S may contain many features not described in the theory.
These other features may have the effect of making the theory's statements true only on good days
and to a certain approximation.

There can be other reasons for using a theory that is only approximately true of the system.
For example the consequences of the theory can be easier to compute than those of a more accurate one.

There is often a trade-off between accessibility and accuracy.

(c) Conversely the theory may talk about kinds of object not visible in the system.

Examples: real numbers, fibre bundles, electrons, ...

These often serve the purpose of making the description more 'accessible'.

The question 'What part of the model carries the data?' now becomes the philosophical problem of realism.

(d) A scientific theory T generally contains uninterpreted expressions.

Example T is the classical linear oscillator:

$$m\frac{d^2x(t)}{dt^2} = -kx(t), \quad m, k > 0.$$

Here $x(t)$, m, k need interpretation.
An interpretation making the equation and inequalities true is called a *semantic model* of T.

For instance M below is a semantic model of T:

m	1
k	4
$x(t)$	$\sin(2t)$

This applies to our pendulum S if the following is also a semantic model of T:

m	mass of S's bob in grams
k	$g \times \dfrac{\text{mass}}{\text{length}}$ of S (in grams, cm)
$x(t)$	$\sin(2t)$ when t counts in seconds the time after 12.30 pm on 4 April 2004.

The interpretation rules leading from more abstract to more concrete descriptions are part of the shared expertise of the scientific community.

For example quantum theory includes not just the Schrödinger equation
but also the techniques used to find the Hamiltonian of an ammonia molecule (for example).

The theory T, interpreted as above,
makes a statement about our pendulum S.
If this statement is true (to the required accuracy),
then T is in Bacon's sense a model of S.

It is more 'accessible' than S in several senses:
we can transmit it by e-mail,
work out its consequences, etc.

But our semantic model M is also a model of S:
it describes S by obeying the same mathematical law as S
(so T itself translates from M to S).
It is more accessible than S by being simpler to describe,
as an interpretation of T.

The interpretation M is a fairly typical example of a *structure-like* model.
It consists of assignments of mathematical objects to linguistic expressions.
We can regard it as a description of S only if
we include with it the rules for working out what parts of S correspond to the parts of M.
As with the engineering models, a structure-like semantic model describes the system S *only through a linguistic description*.

V. Model theory and the semantic conception of theories

Within logic during 1900–1950 there was a slow shift
from calling M a model of S,
to calling M a (semantic) model of T,
and then to calling S too a (semantic) model of T.
Hence the confusion about whether T is a model of S or vice versa.
In philosophy of science I recommend not using the logicians' terminology except in purely technical discussions of models of formal languages.

The semantic model M above is very dependent on the language of the theory T.
This is a basic feature of all semantic models:
they are always assignments of objects to uninterpreted expressions E.

Translating from such a model to a system S *always* involves interpreting those same expressions E in S.
So it is dubious whether semantic models are intermediate between the theory T and S in any useful sense.

The semantic conception of theories (e.g. Suppe)

is basically the observation that a theory with uninterpreted formal terms gets its significance from its semantic models, i.e. those interpretations of its terms that make it true.

Suppe also proposes to *identify* the theory with the class of its models.
This is gratuitous. To state the theory, we need to restore its linguistic form.

The suggestion (Giere and others) that semantic models are ‘non-linguistic’ is bizarre.

Morrison and Morgan (1999):

> According to Alfred Tarski (1936), a famous twentieth-century logician, a model is a non-linguistic entity.

In fact the paper of Tarski that they refer to never mentions models, and is heavily syntactic throughout.
Much later Tarski came to speak of the (semantic) ‘models’ of an uninterpreted formal theory.

Patrick Suppes justifies the use of semantic models in science (1960):

> . . . the concept of model used by mathematical logicians is the basic and fundamental concept of model needed for an exact statement of any branch of empirical science.

This is sound (as Suppes reliably is).
To understand what a formal theory is, we need to understand what constitutes interpreting it,
and what constitutes a true interpretation.

What is more debatable is how far semantic models are useful in the practice of science.

Also to give the bare form of a logician’s interpretation is a long way from describing the richness of techniques available for interpreting in a particular science,
and the variety of criteria to test the usefulness of an interpretation.