Compositionality:

Its history and formalism

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http://wilfridhodges.co.uk/semantics13.pdf

Too large a subject for one hour. See:

Markus Werning, Wolfram Hinzen,
Edouard Machery eds.,
The Oxford Handbook of Compositionality,
Oxford University Press 2012.

Includes chapters by
Wilfrid Hodges,
Theo M. V. Janssen,
Dag Westerståhl.

1. Aristotelian compositionality

- Al-Fārābī (10th c)
- Ibn Sīnā, Al-Jurjānī (11th c)
- Abelard (12th c)
- Leibniz (17th/18th c)
- Frege (19th c)

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How to transfer meanings from your mind to that of Professor Arnauld: a video guide

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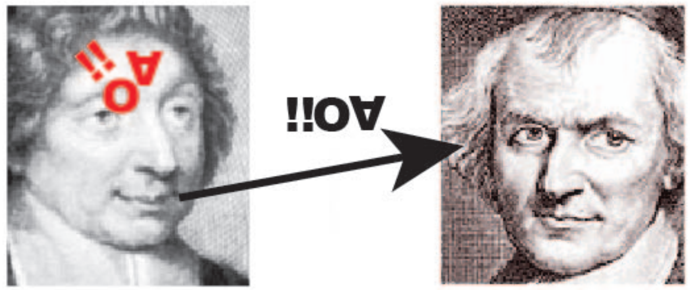

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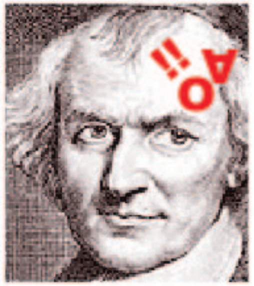

Meanings are built up from atomic meanings by repeated attachments.
So meanings have *parts that are meanings*.
Meanings can be arbitrarily complex.

The encoding of meanings into language
is defined by recursion on complexity:

- Single words encode atomic meanings.
- Syntactic constructions encode attachments.

Some differences of opinion.

(1) *Frege* seems to assume there is a decoding map from phrases to meanings.

Ibn Sīnā denies this: the shared context of speech allows the speaker to leave out meanings that the hearer can reconstruct.

Al-Jurjānī is explicit that there is a decoding map, but he is talking about poetry.

(2) *The Westerners* (but not the Arabs) infer a substitution rule:

> When two phrases have the same construction
> and corresponding words in them
> have the same meanings,
> the two phrases have the same meaning.
> (*Abelard*)

Should apply also to replacing one subphrase by another with the same meaning.

Aristotelian compositionality offers an explanation of why language has the structure that it has: this structure copies the structure of compound meanings.

A very influential idea.

Leonard Bloomfield, *Language* (1933), while pioneering analysis of sentences into *constituents*, implies that the constituents are those parts of the sentence that 'have a meaning'.

2. Transition through Tarski and Chomsky

Tarski presented his truth definition in stages. The earliest in a paper of 1931 on definitions in $\mathbb{R}$.

He defines the class of (first-order) definable relations on $\mathbb{R}$. He takes each relation to be a set of functions $f : N \to \mathbb{R}$ for some fixed finite set N of natural numbers.

Reading i as representing the variable v_i, the relation can be seen as a set of assignments of real numbers to a finite set of variables.

The basic relations correspond to atomic formulas with no nested function symbols.

The remaining relations are constructed by 'fundamental operations', which correspond to $\neg, \vee, \wedge, \exists, \forall$.

So we can see the relation defined by a formula ϕ as a sort of *meaning* of ϕ.
This gives a map from formulas to meanings, though in 1931 Tarski doesn't mention the map.

Comparing with Aristotelian compositionality:

- No idea that meanings have parts that are meanings.
- No map from meanings to phrases.
- An implicit 'decoding' map from phrases to meanings.
- Meanings only for formulas, not e.g. for quantifiers.

Next stage the full Tarski truth definition (1933):

- Formulas are defined explicitly *without reference to meanings.* ('Autonomous syntax')
- The 'fundamental operations' become operations on formulas, not on meanings.
- The decoding map from formulas to meanings becomes explicit (almost—see below).

Tarski's decoding map assigns to formula ϕ not a set R of assignments f to free variables of ϕ, but a metaformula expressing '$f \in R$'.

Reason: to minimise the set-theoretic assumptions.

To mathematicians, obvious that a definition of R and a definition of '$f \in R$' carry same information. But it was not obvious to all linguists.

Also, for linguists Tarski's fundamental operation

$$(\phi, \psi) \ \mapsto \ (\phi \wedge \psi)$$

would be better understood as ternary:

$$(\phi, \wedge, \psi) \ \mapsto \ (\phi \wedge \psi).$$

Similarly with his other fundamental operations.

This will be important for later applications, where we are interested in finding the meanings of $\wedge$ and $\forall$.

The next steps are not all in the written record.
Some relevant dates:

- 1932– Quine at Harvard, lectures on logic.
- 1945 Chomsky begins study of linguistics and philosophy at Penn Univ under Zellig Harris.
- 1951 Harris describes theoretical procedure for discovering syntax of a natural language without going via meanings.
- 1955 Chomsky appointed at MIT after PhD for which he studied partly at Harvard.
- 1955 Chomsky proposes criteria for identifying constituents without reference to meaning.

- 1957 Chomsky proposes that 'syntactic framework' supports 'semantic description'.
- 1963 Katz and Fodor (MIT) show how syntax can support semantics through Tarski-style 'projection rules' that are 'compositional' (first recorded use of this term).
- 1965 Barbara Hall Partee completes PhD with Chomsky and goes to UCLA.
- 1968 Montague at UCLA adapts Tarski truth definition to fragments of English, assigning 'intensions'. Partee recognises the procedure as 'compositional'.

Katz and Fodor didn't say what features they intended by 'compositional'.

Fodor has since reverted to a more Aristotelian position that 'thoughts' are compositional.

The OED's best offering for 'compositional':

> 1984 Times Lit. Suppl. 14 Sept. 1012/3
> Elgar, compositionally, had virtually to start from scratch.

3. PTW compositionality formalised

Partee's definition of 'compositionality' appears e.g. in Partee, Ter Meulen and Wall *Mathematical Methods in Linguistics* (1990). Slightly adjusted:

> The meaning of a compound expression is determined by the meanings of its immediate constituents and the syntactic rule by which they are combined.

Pauline Jacobson has proposed the name 'strong direct compositionality' for this notion.

This assumes autonomous syntax.

But note that for a linguist, two identically written words, even of the same grammatical type, can be syntactically distinct:

- kibble$_1$ *n.* stick or cudgel.
- kibble$_2$ *n.* wooden tub for carrying metal ore.

For Apollonius Dyscolus c. AD 150, and for linguists ever since, syntax is about how a word or phrase fits into larger phrases, not about how it is built up from letters.

Hodges has proposed an axiomatic framework:

Definition. A *constituent structure* is an ordered pair of sets $(\mathbb{E}, \mathbb{F})$, where the elements of $\mathbb{E}$ are called the *expressions* and the elements of $\mathbb{F}$ are called the *frames*, such that the four conditions below hold. (e, f etc. are expressions. F, $G(\xi)$ etc. are frames.)

1. $\mathbb{F}$ is a set of nonempty partial functions on $\mathbb{E}$.

('Nonempty' means their domains are not empty.)

We say e is a *constituent* of f if f is $G(e)$ for some frame $G(\xi)$.

2. (Nonempty Composition) If $F(\xi_1, \ldots, \xi_n)$ and $G(\eta_1, \ldots, \eta_m)$ are frames, $1 \leqslant i \leqslant n$ and there is an expression

$$F(e_1, \ldots, e_{i-1}, G(f_1, \ldots, f_m), e_{i+1}, \ldots, e_n),$$

then

$$F(\xi_1, \ldots, \xi_{i-1}, G(\eta_1, \ldots, \eta_m), \xi_{i+1}, \ldots, \xi_n)$$

is a frame.

Note: If $H(\xi)$ is $F(G(\xi))$ then the existence of $H(f)$ implies the existence of $G(f)$.

3. (Nonempty Substitution) If $F(e_1, \ldots, e_n)$ is an expression, $n > 1$ and $1 \leqslant i \leqslant n$, then

$$F(\xi_1, \ldots, \xi_{i-1}, e_i, \xi_{i+1}, \ldots, \xi_n)$$

is a frame.

4. (Identity) There is a frame $1(\xi)$ such that for each expression e, $1(e) = e$.

A *meaning function* on the constituent structure $(\mathbb{E}, \mathbb{F})$ is a map μ with domain $\mathbb{E}$.

We say e and f are μ-*synonymous* if $\mu(e) = \mu(f)$.

We say that μ is (PTW)-*compositional* if the two equivalent conditions below hold.
Note that these conditions use only μ-synonymy.

1. For every frame $F(\xi_1, \ldots, \xi_n)$, if $F(e_1, \ldots, e_n), F(f_1, \ldots, f_n)$ exist and

$$\mu(e_1) = \mu(f_1), \ \ldots, \ \mu(e_n) = \mu(f_n)$$

then $\mu(F(e_1, \ldots, e_n)) = \mu(F(f_1, \ldots, f_n))$.

2. For every frame $F(\xi_1, \ldots, \xi_n)$ there is a function h_F such that for every expression $F(e_1, \ldots, e_n)$,

$$\mu(F(e_1, \ldots, e_n)) = h_F(\mu(e_1), \ldots, \mu(e_n)).$$

We can also define what it is for e to be an *immediate constituent* of f.

Then assuming that the constituent structure is well-founded in a suitable sense,
the conditions above are equivalent to
the special case where $e_1, \ldots, e_n$ are
immediate constituents of $F(e_1, \ldots, e_n)$.

I omit details.

4. The fregean cover

A common semantic problem: Given a language L and a semantics for a subset X of the expressions of L, extend the semantics to all of L.

E.g. if L is a formal language and we have a definition of $M \models \phi$ for sentences ϕ of L, maybe by games played on ϕ, how can we extend to an interpretation of formulas with free variables?

Similar questions arise in natural language field work, e.g. to describe the meanings of affixes in agglutinative languages.

So we have a partial semantics μ defined on X. How to lift to a semantics ν on all of $\mathbb{E}$?

Reasonable requirements on ν:

- ν is compositional.
- If $\nu(e_1) \neq \nu(e_2)$ then there is a frame $F(\xi)$ such that either just one of $F(e_1), F(e_2)$ is in X, or both are in X and $\mu(F(e_1)) \neq \mu(F(e_2))$.
- If $\nu(e_1) = \nu(e_2)$ and $e_1, e_2 \in X$ then $\mu(e_1) = \mu(e_2)$.

The second condition says ν is *fully abstract with respect to* μ.

When these conditions are met, we call ν a (the?) *fregean cover* of μ. (Frege: 'Always look for the meaning of a word in the interconnections of the sentence containing the word.')

Given μ, we can always define an equivalence relation $\equiv_\mu$ on $\mathbb{E}$:

$$e_1 \equiv_\mu e_2 \text{ iff } \begin{array}{l} \text{for every frame } F(\xi), \\ F(e_1) \in X \Leftrightarrow F(e_2) \in X, \text{ and} \\ \text{if } F(e_1) \in X \text{ then } \mu(F(e_1)) = \mu(F(e_2)). \end{array}$$

Define ν so that $\nu(e_1) = \nu(e_2)$ iff $e_1 \equiv_\mu e_2$.

We can show:

- If every expression is a constituent of an expression in X, then ν is a fregean cover of μ.
- If moreover for all $e_1, e_2 \in X$ and all $F(\xi)$,

  $$\begin{array}{l} \mu(e_1) = \mu(e_2) \text{ and } F(e_1) \in X \;\Rightarrow \\ F(e_2) \in X \text{ and } \mu(F(e_1)) = \mu(F(e_2)) \end{array}$$

  then ν can be chosen as an extension of μ.

It may not be easy to describe ν concretely.

Partial anticipations:

Lappin and Zadrozny (preprint, 2000) gave the fregean cover of a total semantics.

The map $\mu \mapsto \nu$ generalises the *Leibniz operator* of Blok and Pigozzi (1986) in universal algebra of logics. The relation $\equiv_\mu$ generalises their *Leibniz congruence*.

Tarski's truth definition for first-order logic is the fregean cover of its restriction to sentences (maybe after trivial adjustments).

For IF-type logics with a game semantics for sentences,
a concrete description of a fregean cover
in terms of sets of assignments
led Jouko Väänänen to the discovery of
Dependence Logic.

Besides Werning, Hinzen & Machery and other books cited above, note:

- Chris Barker and Pauline Jacobson eds., *Direct Compositionality*, Oxford University Press 2007.
- W. Blok and D. Pigozzi, 'Protoalgebraic logics', *Studia Logica* 45 (1986) 337–369.
- Hidé Ishiguro, *Leibniz's Philosophy of Logic and Language*, Cambridge University Press, 2nd edition 1990: Ch. II on substitution salva veritate.
- J. Katz and J. Fodor, 'The structure of a semantic theory', *Language* 39 (1963) 170–210.

- Alfred Tarski, *Logic, Semantics, Metamathematics*, ed. Corcoran, Hackett 1983, chapters 'On definable sets of real numbers' (1931), 'The concept of truth in formalized languages' (expansion of 1933 paper).

Some of the history has parallels in some much earlier Indian linguistics, though there was no cross-influence. See

- Brendon Gillon, 'Pāṇini's Aṣṭādhyāyī and linguistic theory', *Journal of Indian Philosophy* 35 (2007) 445–468.