1

The choice of semantics as a methodological question

Wilfrid Hodges

March 2013

http://wilfridhodges.co.uk/semantics14.pdf

2

Peter S-H kindly invited me to express

> "opinions that strongly differ from the those of the majority of people at the conference."

Not easy since I never worked in proof theory, semantic or otherwise.

Instead I will try to raise some issues about what proof-theoretic semantics needs to do in order to work as a semantics.

Peter suggested that since I'm a model theorist, model theory might be a way in.

I doubt it. Model theorists do use one form of semantics as a technical tool, but they have nothing to say about semantics in general.

There is also a subject called
'model-theoretic semantics'.
It is not done by model theorists,
and I have no opinions for or against it.

So don't expect model theory to appear below.

Peter says (Stanford Encyclopedia):

> "the meaning of a term should be explained by reference to the way it is used in our language".

I take this as a starting point. More precisely

> *The semantics of a language L is a part of the body of information about L that people need to have in order to use the language.*

It's the part of that body that deals with *meanings*. Later we will pin this down more precisely.

Question: What is the language?

If purely formal language, then no real meanings (any more than in tic tac toe).

If as a regimented fragment abstracted from English (or Swedish), then the task is to give meanings of logical words of English (or Swedish). These words can be used outside ‘logical’ contexts.

I’ll assume the latter,
so we are into natural language semantics.

A semantics of a language L consists of information.

Information never comes raw; it always has to be packaged up in some *form of representation.*

Questions about the ontology and epistemology of meanings have no purchase until a form of representation has been fixed.
(Though you might want to reject a form of representation because it presupposes an ontology that you dislike.)

If I understand right, a claim of proof-theoretic semantics is that the meanings of logical words of English can be adequately represented within certain formalised fragments abstracted from English.

To assess this claim, we can start by proving it, and then see what is unsatisfactory in the proof.

Thought-experiment.

Suppose Ororob is a Brazilian language not yet deciphered.

A textbook of natural deduction written in Ororob comes to light.

Logicians recognise the patterns, and on the basis of them are able to decipher the language.
They translate the logical terms by English words obeying the same rules.

If this is possible, then it is *possible* to read off the meanings of logical words from their natural deduction rules.
Hence the meanings *can* be represented in the rules.

But note:

- the lack of clarity about *how the information is encoded*, and
- the essential use of an indeterminate amount of *background information*.

We will analyse these points.

Natural languages are open-ended and massively complex.
Two questions that constantly arise in choosing a form of representation:

Modularity.
: Which parts to separate out as autonomous?

General versus special.
: How to balance general rules against special instances,
given that both are needed?

Example: describing plurals of nouns:

1. Turkish: noun $\Rightarrow$ noun + 'ℓ' + vowel + 'r'.
   A vowel harmonisation rule gives the vowel.
2. Swahili: each noun has two forms, singular and plural, which must be learned separately (though there are some regular patterns).
3. Arabic: two types of plural, 'sound plurals' like Turkish and 'broken plurals' like Swahili.

Note the different general/special balances.

It's now common to separate off questions about how morphemes of L are combined into words and sentences, as the *syntax* of L. Then:

> *The semantics of a language L consists of the information about L that people need to have in order to use L, given the syntax of L.*

There are issues about the autonomy of syntax in natural language, but in formal languages it's clearly autonomous.

Besides hiving off autonomous modules,
we can also *reduce* a semantics.

We do this by identifying parts of the language usage that can be inferred from other parts by *general rules.*

> *Example: The speaker must know what sentence is appropriate to express her intention.*
> *The hearer must know how (in normal circumstances) to infer the speaker's intention from the sentence used.*

The speaker-side information about L covers everything needed for the hearer-side.
So semantics can be purely speaker-side
(plus general rules of interpretation).

A more contentious reduction: can we also eliminate reference to the speaker's intention, by general rules about use of descriptive content?

Example (logic meeting in Europe c. 20 years ago):

- *Senior logician to junior logician* 'I will do everything in my power to prevent you being appointed to the position you have applied for.'
- *Junior logician* 'You have a large red nose.'

I believe it's a consensus of linguists and lexicographers that phrases can have politeness implications beyond their descriptive content.

E.g. Russian imperfective imperatives more polite than perfective ones.
Use of metaphor can make criticism less sharp.
Tristram Shandy aims to make the word 'whiskers' obscene without altering its descriptive content.

I'll ignore politeness implications of logical words. But this needs justification. If formal languages are actually used, there could be politeness issues.

Other problematic aspects still present in some formal languages:

Imperatives, questions
: Present in programming and database languages.
Is their semantics reducible?
Is it worth it?
Does proof-theoretic semantics have anything to say about these?

Open-endedness
: Infinitely many sentences, so general/special decisions required.

Pause

Proof-theoretic semantics is about the meanings of some *words*.
But the information is packaged as facts about certain *sentences* containing the words.
What is the connecting link?

This is part of a very general phenomenon in semantics, and we can treat it with general machinery.

Most languages have a limited number of words and a limited number of syntactic constructions.
But these generate an infinite number of sentences.

In theory we can package the semantics as

1. meanings of single words, and
2. meanings of syntactic constructions, represented by operations on meanings.

In practice semanticists tend not to do this.
One reason: part of the meaning of a word is hidden inside meanings of syntactic constructions that involve it.

Instead one identifies for each word the primary applications ('ursprüngliche Anwendungsweise', Frege's phrase) in which the word occurs.

E.g. for 'mother' a primary application might be 'X is the mother of Y'.
We give a meaning for the phrase,
as a function of what meaning is put at 'X' and 'Y'.

The primary applications have to be *representative* in the sense that all uses of the word can be inferred from the meanings given to these contexts.

General rules will be needed to extend from primary applications to other contexts.

On this approach, the lexicon will assign a meaning not to 'and',
but to 'ϕ and ψ' as a function of the meanings of ϕ and ψ.

General rules already exist for inferring the use of 'and' in other contexts (e.g. 'black and white').

If properly done, this approach removes the need for semantic rules for semantic constructions.

But there are still regularities
that need to be stated as general rules,
not repeated for each case in the lexicon.

Example: Coercion rules in programming languages.
I don't know whether proof-theoretic semantics has any view on these.

25

Footnote: One might take as primary application for 'meaning' itself the context

$$X \text{ is the meaning of } Y.$$

But this phrase can't have a canonical explication, because one must choose a form of representation.

This fact may lie behind Dummett's 1974 claim (otherwise mysterious to me) that the task of a theory of meaning is to explain the phrase

$$X \text{ knows the meaning of } Y.$$

26

Pause again

We return to the question: Can the semantics of logical words be reduced to their use in a formalised logical language?

We saw that the answer is Yes if we are careless about general rules needed for the reduction.

Proof theorists as logicians can plausibly hand over some parts of the reduction to general lingustics (e.g. reduction to primary applications).
But somebody should check it out.

There remain questions *within logic*.
E.g. can we show the equivalence of a proof rule semantics and another semantics expressed in significantly different terms, using precisely formulated translation rules?
Are there some impossibility results?

For example, what are the obstacles to showing that the natural deduction rules for $\wedge$ express the same information about the meaning of $\wedge$ as a truth table?

The main obstacle is the question of encoding. Exactly what information do the proof rules and the truth tables express?
For example do these two tables present the same information about $\wedge$?

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F iff π is irrational

Same truth function, different algorithm?

It seems plausible that this and corresponding questions on the proof-theory side *could* be answered in ways that allow a proof that the proof semantics and the truth-table semantics give *exactly the same information.*

Could such a proof of equivalence be

- canonical?
- useful?
- generalisable?

On the proof theory side,
a question that needs an answer is how the proof rules are intended to relate to use of language.

For example, does having a formula on a line of a proof correspond in some way to stating the formula?

In practice we hardly ever make bare statements. Even in contexts of pure reasoning, we relate them with 'Then', 'But', 'Suppose', 'I grant that', 'I think I can show that', 'I claim that' etc. etc.

I illustrate with a treatment of assumptions, which is in late Frege but I was surprised to find it already in Ibn Sīnā (11th c. Persia).

On this view, a move of the form
'Suppose ϕ' or 'Let ...' is always a shorthand, designed to avoid an 'ungeheurige Länge' in proofs.

When we say 'Suppose ϕ', we intend that 'If ϕ then' should be *understood* at the beginning of all relevant propositions down to the point where the assumption is discharged. The device allows us to avoid having to repeat ϕ every time it is used.

So we should read

$$\dfrac{\begin{matrix}[\phi] \\ \vdots \\ \psi\end{matrix}}{\phi \to \psi} \qquad \text{as} \qquad \dfrac{\begin{matrix}\phi \to \phi \\ \vdots \\ \phi \to \psi\end{matrix}}{\phi \to \psi}$$

where now the top line is an axiom
and the bottom step falls away.
A general metarule asserts that for every step
$\Delta, \alpha \vdash \beta$ we have a step $\Delta, (\phi \to \alpha) \vdash (\phi \to \beta)$.

This is a claim about what kind of contentful argument
is expressed by the natural deduction rules.

It is directly relevant to
how we can read the proof rule as carrying
information about the meaning of $\to$.

Summary of main issues:

- With respect to what language?
- Do the uses in derivations determine more general uses?
  Make this precise, at least within logic.
- How to decode the information from the proof rules?
- Given different codings within logic, prove results showing equivalence or impossibility of equivalence.